Find The Slope And Y Intercept Of The Line 5x-9y=45

Find The Slope And Y Intercept Of The Line 5x-9y=45 is a fundamental problem in algebra that helps us understand the characteristics of a linear equation. Understanding how to determine the slope and y-intercept from an equation allows students and mathematicians to graph lines accurately and analyze their behavior. In this article, we will walk through the process of finding both the slope and y-intercept of the line given by the equation 5x - 9y = 45, explore the concepts behind these calculations, and provide step-by-step instructions for solving similar problems.

Understanding the Equation of a Line

Before diving into the specific problem, it’s essential to understand the general form of a linear equation and how it relates to the slope and y-intercept.

The Slope-Intercept Form

The most common form for analyzing lines is the slope-intercept form:

$$ y = mx + b $$

where:

- m is the slope of the line, indicating its steepness and direction.
- b is the y-intercept, the point where the line crosses the y-axis.

Knowing how to convert any linear equation into this form makes it easy to identify the slope and y-intercept directly.

Standard Form of a Line

The given equation, 5x - 9y = 45, is in standard form:

$$ Ax + By = C $$

where:

- $A = 5$
- $B = -9$
- $C = 45$

To find the slope and y-intercept, it’s often easiest to manipulate the standard form into slope-intercept form.

Converting the Equation to Slope-Intercept Form

The goal here is to solve for y in terms of x:

$$ 5x - 9y = 45 $$

Step-by-Step Solution

1. Isolate the term with y:
Subtract 5x from both sides:
$$ -9y = -5x + 45 $$

2. Divide both sides by -9:
$$ y = \frac{-5x + 45}{-9} $$

3. Simplify the fraction:
$$ y = \frac{-5x}{-9} + \frac{45}{-9} $$
$$ y = \frac{5x}{9} - 5 $$

Now, the equation in slope-intercept form is:
$$ y = \frac{5}{9}x - 5 $$

Summary:
- The slope $m = \frac{5}{9}$
- The y-intercept $b = -5$

Interpreting the Slope and Y-Intercept

Having converted the equation, we can interpret what these values mean in terms of the line’s behavior.

The Slope $m = \frac{5}{9}$

- The slope indicates that for every 9 units moved horizontally to the right (positive x-direction), the line rises by 5 units.
- Since the slope is positive, the line inclines upward from left to right.

The Y-Intercept $b = -5$

- The y-intercept is the point where the line crosses the y-axis, which occurs at:
$$ (0, -5) $$
- This point can be plotted directly on a graph and serves as a starting point for sketching the line.

Graphing the Line

Using the slope and y-intercept, you can graph the line accurately:

1. Plot the y-intercept point at (0, -5).

2. Use the slope to find another point: from (0, -5), move 9 units to the right (x=9), and move 5

units up (since the slope is positive), reaching (9, 0).

3. Draw a straight line through these points.

This graphical approach provides a visual understanding of the line's position and inclination.

Additional Tips for Finding Slope and Y-Intercept

To assist in similar problems, here are some practical tips:

1. Always convert to slope-intercept form

This makes identifying the slope and intercept straightforward.

2. Pay attention to signs

- A positive slope indicates an upward trend.
- A negative slope indicates a downward trend.
- The y-intercept’s sign shows whether it crosses above or below the x-axis.

3. Use the standard form to slope-intercept conversion

If the equation is given in standard form:
- Isolate y: $By = -Ax + C$
- Divide by B to solve for y.

4. Verify your results

Check by plugging in the y-intercept into the original equation:

$$5(0) - 9(-5) = 45$$

$$0 + 45 = 45$$

which confirms the y-intercept is correct.

Practice Problems

To reinforce understanding, try solving these similar problems:

- Find the slope and y-intercept of $3x + 4y = 12$.
- Determine the slope and y-intercept of $2x - y = 7$.

- Given the line in standard form $-x + 2y = 8$, find its slope and y-intercept.

Example Solution for $3x + 4y = 12$
- Convert to slope-intercept form:
$$4y = -3x + 12$$
$$y = -\frac{3}{4}x + 3$$
- Slope: $-\frac{3}{4}$
- Y-intercept: 3

Example Solution for $2x - y = 7$
- Convert:
$$-y = -2x + 7$$
$$y = 2x - 7$$
- Slope: 2
- Y-intercept: -7

By practicing these conversions, you develop a strong intuition for analyzing linear equations.

Conclusion

Understanding how to find the slope and y-intercept from any linear equation is a crucial skill in algebra. For the equation 5x - 9y = 45, converting it into slope-intercept form reveals that the slope is $\frac{5}{9}$ and the y-intercept is -5. These values not only help in graphing the line but also provide insight into its behavior and position. Mastering this process enables you to analyze linear relationships efficiently and accurately, which is fundamental in various fields such as mathematics, physics, economics, and engineering. Keep practicing with different equations to strengthen your skills and deepen your understanding of linear functions.

Frequently Asked Questions

What is the slope of the line given by the equation 5x - 9y = 45?

The slope of the line is 5/9.

How do I find the y-intercept of the line 5x - 9y = 45?

To find the y-intercept, set x = 0 and solve for y: 5(0) - 9y = 45, which gives y = -5. So, the y-intercept is (0, -5).

What is the slope-intercept form of the line 5x - 9y = 45?

Rearranged into slope-intercept form: y = (5/9)x - 5.

How can I determine both the slope and y-intercept from the equation 5x - 9y = 45?

Rewrite the equation in slope-intercept form to identify the slope as 5/9 and y-intercept as -5.

Is the slope of the line 5x - 9y = 45 positive or negative?

The slope is positive, specifically 5/9.

What is the significance of the y-intercept in the line 5x - 9y = 45?

The y-intercept, at (0, -5), indicates where the line crosses the y-axis.

Can I graph the line 5x - 9y = 45 using the slope and y-intercept?

Yes, by plotting the y-intercept at (0, -5) and using the slope 5/9 to find another point, you can accurately graph the line.

Additional Resources

Find The Slope And Y Intercept Of The Line 5x - 9y = 45

Understanding the fundamental features of a linear equation is essential in algebra and analytical geometry. Among the most critical aspects are the slope and the y-intercept, which provide vital information about the line's orientation and position on the coordinate plane. In this article, we will explore how to determine these features from the given linear equation, specifically focusing on the equation 5x - 9y = 45. Through a detailed, step-by-step approach, alongside explanations of underlying concepts, readers will gain a comprehensive understanding of how the slope and y-intercept are derived, their significance, and their applications in various mathematical and real-world contexts.

Understanding the Equation of a Line

Before diving into the specifics of finding the slope and y-intercept, it is important to establish a clear understanding of what a linear equation represents and how it is generally expressed.

The Standard Form of a Linear Equation

The equation 5x - 9y = 45 is presented in what is known as the standard form of a linear equation:

$$Ax + By = C$$

where:

- A, B, and C are constants,
- x and y are variables representing coordinates on the Cartesian plane.

For the given equation:

- $A = 5$,
- $B = -9$,
- $C = 45$.

This form is useful for quickly identifying the coefficients associated with each variable, but it is often more practical to work with the slope-intercept form for graphing and analysis.

The Slope-Intercept Form

The slope-intercept form of a linear equation is expressed as:

$$y = mx + b$$

where:

- m is the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

Converting the standard form to this form allows us to directly identify the slope and y-intercept, making it easier to analyze and graph the line.

Transforming 5x - 9y = 45 into Slope-Intercept Form

To find the slope and y-intercept of the given line, the primary step is to algebraically manipulate the equation into the slope-intercept form.

Step-by-Step Conversion Process

Let's consider the original equation:

$$5x - 9y = 45$$

The goal is to solve for y:

1. Isolate the term with y:

$$ -9y = -5x + 45 $$

2. Divide through by the coefficient of y, which is -9:

$$ y = \frac{-5x + 45}{-9} $$

3. Simplify the fraction:

$$ y = \frac{-5x}{-9} + \frac{45}{-9} $$

$$ y = \frac{5x}{9} - 5 $$

Now, the equation is in the slope-intercept form:

$$ y = \frac{5}{9}x - 5 $$

Identifying the Slope and Y-Intercept

With the equation now in the form $ y = mx + b $, the slope and y-intercept are immediately evident.

The Slope (m)

From the simplified form:

$$ y = \frac{5}{9}x - 5 $$

the coefficient of x:

$$ m = \frac{5}{9} $$

Interpretation:

- The slope $ \frac{5}{9} $ indicates that for every increase of 1 unit in x, y increases by $\frac{5}{9}$ units.
- The positive sign of the slope signifies that the line inclines upwards from left to right, characteristic of a positive correlation between x and y.

The Y-Intercept (b)

The constant term:

$$b = -5$$

Interpretation:

- The y-intercept at -5 means the line crosses the y-axis at the point $(0, -5)$.
- This is the value of y when $x = 0$.

Graphical Significance of the Slope and Y-Intercept

Visualizing the line based on the slope and y-intercept enhances understanding of these concepts.

Plotting the Y-Intercept

- Since the y-intercept occurs at $(0, -5)$, plotting this point on the coordinate plane provides a starting point for graphing.
- This point marks where the line crosses the y-axis.

Using the Slope to Find Additional Points

- The slope $\frac{5}{9}$ indicates that moving from the y-intercept, for every 9 units moved horizontally to the right, the line moves upward by 5 units.
- Conversely, moving to the left by 9 units would cause a downward movement of 5 units, due to the positive slope.

Example:

Starting at $(0, -5)$:

- Move 9 units right in x:

$$x = 0 + 9 = 9$$

- Move 5 units up in y:

$$y = -5 + 5 = 0$$

- The second point on the line is $(9, 0)$.

Plotting these points, the line can be drawn accurately, illustrating the linear relationship.

Applications and Broader Significance

Knowing how to find the slope and y-intercept from an equation like $5x - 9y = 45$ is more than a mathematical exercise; it has practical applications across various disciplines.

In Mathematics and Education

- Graphing Lines: The process simplifies the graphing of lines, an essential skill in coordinate geometry.
- Understanding Relationships: The slope indicates the nature of relationships between variables, such as positive or negative correlation.
- Analyzing Data: Linear models are used in statistics to interpret data trends.

In Science and Engineering

- Modeling Phenomena: Linear equations model relationships such as speed over time, cost versus quantity, or chemical reactions.
- Design and Optimization: Engineers use slope and intercepts to optimize systems and understand constraints.

In Economics and Business

- Cost and Revenue Models: Calculating break-even points involves understanding the intercepts.
- Supply and Demand Analysis: Linear equations model how quantity and price interact.

Common Challenges and Tips for Students

While converting equations and identifying slopes and intercepts seem straightforward, students often encounter challenges. Here are some tips:

- Always check your algebra during conversion to prevent errors.
- Remember the importance of signs: Negative signs in the original equation can change the slope's sign.
- Practice with different forms: Be comfortable converting between standard, slope-intercept, and point-slope forms.
- Visualize points: Plotting the y-intercept and using the slope to find other points helps confirm calculations.

Conclusion: The Power of Simple Algebraic Manipulation

The process of finding the slope and y-intercept of the line represented by $5x - 9y = 45$ exemplifies how straightforward algebraic manipulation unlocks significant insights into a line's behavior. By converting the standard form into slope-intercept form, we identify:

- Slope (m): $\frac{5}{9}$,
- Y-intercept (b): -5.

These elements facilitate graphing, analysis, and understanding of the line’s geometric and real-world implications. Mastery of these techniques forms a foundation for more advanced topics in mathematics, data analysis, science, and engineering, illustrating that even complex systems often hinge on simple, fundamental principles. Whether used in academic contexts or practical applications, recognizing and calculating these features enhances analytical skills and deepens comprehension of linear relationships.

In summary, the line described by the equation $5x - 9y = 45$ has a slope of $\frac{5}{9}$, indicating an upward incline, and crosses the y-axis at -5, providing a clear starting point for visualization and further analysis. This process underscores the importance of algebraic techniques in deciphering the characteristics of linear functions, reinforcing their central role across sciences and mathematics.

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