Find The Slope Of Theline Perpendicular To3x + Y = 16

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Understanding how to find the slope of a line perpendicular to a given line is a fundamental concept in coordinate geometry. Whether you're a student tackling algebra problems or a math enthusiast exploring the intricacies of slopes and lines, this guide will walk you through the process step-by-step. In this article, we will focus on the specific problem: Find The Slope Of Theline Perpendicular To 3x + Y = 16. By the end, you'll have a clear understanding of how to approach such problems and the principles behind the calculations.

Understanding the Equation of a Line

Before diving into the specifics of perpendicular lines, it's essential to understand the general form of a linear equation and how to interpret it.

Standard Form of a Line

The most common form of a line's equation in two-dimensional space is:

\[Ax + By = C \]

where A, B, and C are constants.

In our case, the given line's equation is:

\[3x + Y = 16 \]

which can be rewritten as:

\[3x + y = 16 \]

Converting to Slope-Intercept Form

To find the slope of a line, it's most straightforward to convert the equation into the slope-intercept form:

\[y = mx + b \]

where:
- \(m \) is the slope,
- \(b \) is the y-intercept.

Let's do this for the given line:

$$3x + y = 16$$

Subtract 3x from both sides:

$$y = -3x + 16$$

Now, it's in the form $y = mx + b$, and the slope m of the original line is -3.

What is a Perpendicular Line?

In coordinate geometry, two lines are perpendicular if they intersect at a right angle (90 degrees). A key property of perpendicular lines is the relationship between their slopes.

Relationship Between Slopes of Perpendicular Lines

- If the slope of one line is m, then the slope of a line perpendicular to it is $-\frac{1}{m}$.
- This reciprocal relationship, with a sign change, ensures the lines are orthogonal.

For example:
- If a line has slope 2, the perpendicular line has slope $-\frac{1}{2}$.
- If a line has slope -3, the perpendicular line's slope is $-\frac{1}{-3} = \frac{1}{3}$.

Calculating the Slope of the Perpendicular Line

Given the original line:

$$y = -3x + 16$$

the slope m is:

$$m = -3$$

The slope of any line perpendicular to this is:

$$m_{\perp} = -\frac{1}{m} = -\frac{1}{-3} = \frac{1}{3}$$

Therefore, the slope of the line perpendicular to the given line is $\boxed{\frac{1}{3}}$.

Summary of Steps to Find the Perpendicular Slope

To summarize, here are the steps to find the slope of a line perpendicular to any given line:

1. Convert the given line's equation into slope-intercept form $y = mx + b$.
2. Identify the slope m of the original line.
3. Calculate the negative reciprocal of m to find the perpendicular slope: $m_{\perp} = -\frac{1}{m}$.

Applying these steps to our specific problem:

- Original line: $3x + y = 16$
- Converted: $y = -3x + 16$
- Slope: -3
- Perpendicular slope: $\frac{1}{3}$

Practical Examples and Applications

Knowing how to determine the slope of a perpendicular line has numerous applications in fields such as engineering, architecture, physics, and even computer graphics. Here are some common scenarios:

Example 1: Finding the Equation of a Perpendicular Line

Suppose you need to find the equation of a line that passes through a point (x_1, y_1) and is perpendicular to the line $3x + y = 16$.

Steps:
1. Find the perpendicular slope, which we've determined as $\frac{1}{3}$.
2. Use the point-slope form:

$$y - y_1 = m_{\perp} (x - x_1)$$

3. Plug in the point and slope to get the specific line.

Example 2: Calculating Angles Between Lines

The slopes of perpendicular lines are related to the angles they form with the x-axis. The tangent of the angle θ that a line makes with the x-axis is equal to its slope:

$$ \tan \theta = m $$

Since perpendicular lines have slopes m and $-\frac{1}{m}$, the angles they form satisfy:

$$ \theta_1 + \theta_2 = 90^\circ $$

This property is useful in geometric proofs and design.

Additional Tips for Solving Line Slope Problems

- Always check the form of the line's equation before attempting to find the slope.
- If the line is vertical, its slope is undefined, and the perpendicular line will be horizontal with a slope of 0.
- For horizontal lines, the perpendicular line will be vertical with an undefined slope.
- Remember that the negative reciprocal is only valid for non-zero slopes.

Common Mistakes to Avoid

- Confusing the slope of the original line with that of the perpendicular line.
- Forgetting to convert the equation into slope-intercept form.
- Not considering special cases like vertical or horizontal lines.
- Using incorrect reciprocal or sign changes.

Conclusion

Finding the slope of a line perpendicular to a given line is a straightforward process once you understand the relationship between slopes. For the line $3x + y = 16$, the steps are:

- Convert to slope-intercept form:

$$ y = -3x + 16 $$

- Identify the slope:

$$ m = -3 $$

- Determine the perpendicular slope:

$$ m_{\perp} = \frac{1}{3} $$

This slope, $\frac{1}{3}$, describes any line that is perpendicular to the original. Whether you're solving homework problems, designing geometric models, or analyzing physical systems, mastering this concept is essential for working with lines and angles in the coordinate plane. Keep practicing with different equations to solidify your understanding and become proficient in handling perpendicular line problems.

Frequently Asked Questions

What is the slope of the line given by the equation 3x + y = 16?

First, rewrite the equation in slope-intercept form (y = mx + b). Subtract 3x from both sides: y = -3x + 16. The slope (m) is -3.

How do you find the slope of a line perpendicular to 3x + y = 16?

Find the slope of the original line, then take its negative reciprocal. Since the slope of the original line is -3, the perpendicular slope is 1/3.

What is the slope of the line perpendicular to the line 3x + y = 16?

The slope of the perpendicular line is 1/3.

How do you write the equation of a line perpendicular to 3x + y = 16 passing through a point (x_1, y_1)?

Use the point-slope form: $y - y_1 = m (x - x_1)$, where m is 1/3, the perpendicular slope.

Why is the slope of the perpendicular line to 3x + y = 16 equal to 1/3?

Because the original line's slope is -3, and the slope of a perpendicular line is the negative reciprocal of -3, which is 1/3.

Can you convert the original line 3x + y = 16 into slope-intercept form?

Yes. Subtract 3x from both sides: y = -3x + 16.

What is the importance of finding the perpendicular slope in coordinate geometry?

It helps in determining the equation of a line perpendicular to a given line, useful in problems involving orthogonality and constructing perpendicular lines.

If a line has a slope of 2, what is the slope of its perpendicular line?

The slope of the perpendicular line is -1/2, which is the negative reciprocal of 2.

Additional Resources

Find The Slope Of The Line Perpendicular To 3x + y = 16

Understanding the concept of slopes and their significance in geometry and algebra is fundamental to mastering the study of lines in coordinate geometry. When analyzing a linear equation such as 3x + y = 16, a key question often posed is: how do we determine the slope of a line that is perpendicular to this given line? This inquiry not only aids in graphing lines accurately but also deepens comprehension of the relationships between different lines in the Cartesian plane. In this article, we explore the process of finding the slope of a line perpendicular to the given equation, elucidating the mathematical principles involved, and providing a comprehensive guide for students, educators, and enthusiasts alike.

Understanding the Basics: What Is a Slope?

Before delving into the specifics of perpendicular lines, it is imperative to establish a clear understanding of what a slope represents in the context of straight lines.

Definition of Slope

The slope of a line, often denoted by the letter 'm', measures the steepness or inclination of that line. It quantifies how much y (the vertical coordinate) changes concerning a change in x (the horizontal coordinate). Mathematically, the slope is expressed as:

\[m = \frac{\Delta y}{\Delta x} \]

where Δy is the change in y-values and Δx is the change in x-values between two points on the line.

Geometric Interpretation

Graphically, the slope indicates whether a line rises or falls as it moves from left to right:
- A positive slope indicates the line ascends.
- A negative slope indicates the line descends.
- A zero slope is horizontal.
- An undefined slope (vertical line) indicates an infinite steepness.

Rewriting the Equation in Slope-Intercept Form

The given line is expressed as:

$$3x + y = 16$$

To analyze its slope, it's most straightforward to convert this equation into the slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept.

Conversion Process

Starting with the given equation:

$$3x + y = 16$$

Solve for y:

$$y = -3x + 16$$

This form reveals directly that the slope of the given line is:

$$m = -3$$

Key insight: The slope of the original line is -3.

Determining the Slope of a Perpendicular Line

Having established that the slope of the original line is -3, the next step is to find the slope of a line perpendicular to it.

The Concept of Perpendicular Lines

Perpendicular lines are lines that intersect at a right angle (90 degrees). A crucial property in coordinate geometry is that the slopes of two perpendicular lines are negative reciprocals of each other.

Negative reciprocal refers to:
- Taking the reciprocal of the original slope.
- Changing its sign.

Mathematically:

$$\text{If } m_1 = -3, \text{ then the perpendicular slope } m_2 = -\frac{1}{m_1}$$

Calculating:

$$m_2 = -\frac{1}{-3} = \frac{1}{3}$$

Result: The slope of the line perpendicular to the original line is $\frac{1}{3}$.

Why Do Slopes of Perpendicular Lines Follow This Rule?

This reciprocal relationship arises from the geometric requirement that two lines intersect at a right angle. When the slopes m_1 and m_2 satisfy:

$$m_1 \times m_2 = -1$$

then the lines are perpendicular.

Verifying:

$$(-3) \times \frac{1}{3} = -1$$

which confirms the perpendicularity.

Implications and Applications of the

Perpendicular Slope

Knowing the slope of a perpendicular line has practical and theoretical implications across various fields such as engineering, physics, computer graphics, and architecture.

Graphing Perpendicular Lines

- Once the perpendicular slope is determined, the next step involves constructing the perpendicular line on a coordinate plane.
- If a specific point through which the perpendicular line passes is known, the equation of that line can be written using point-slope form.

Example: Constructing a Perpendicular Line

Suppose the perpendicular line passes through the point $(2, 5)$. Its equation is:

$$ y - y_1 = m(x - x_1) $$

Plugging in the point $(2, 5)$ and the slope $\frac{1}{3}$:

$$ y - 5 = \frac{1}{3}(x - 2) $$

This equation can be expanded and rearranged into slope-intercept form for easier graphing.

Applications in Real-World Scenarios

- Design and Architecture: Ensuring right angles between structural components.
- Navigation and Robotics: Calculating paths that are perpendicular to existing routes.
- Physics: Analyzing forces acting at right angles, such as electric and magnetic fields.
- Computer Graphics: Rotating objects or aligning elements at right angles.

Additional Mathematical Considerations

While the core process involves finding the negative reciprocal of the original slope, several nuances can influence calculations and interpretations.

Vertical and Horizontal Lines

- If the original line is horizontal ($m=0$), the perpendicular line is vertical, which has an undefined slope.
- Conversely, if the original line is vertical (undefined slope), the perpendicular line is horizontal ($m=0$).

In our case, since the original line has a slope of -3, neither of these special cases applies, simplifying the process.

Confirming the Perpendicular Relationship

- Always verify that the product of the slopes equals -1. This ensures the lines are truly perpendicular.
- This principle is fundamental in solving geometry problems involving right angles.

Summary and Final Remarks

In conclusion, the process of finding the slope of a line perpendicular to a given line encompasses several key steps:
1. Convert the given equation to slope-intercept form to identify its slope.
2. Recognize that perpendicular lines have slopes that are negative reciprocals.
3. Calculate the negative reciprocal of the original slope to find the perpendicular slope.

Applying these principles to the specific equation $3x + y = 16$, we find:
- The original line's slope: -3.
- The slope of the perpendicular line: $\frac{1}{3}$.

This straightforward yet powerful method exemplifies the elegance of algebraic relationships in geometric contexts. Whether for academic purposes or practical applications, understanding how to find perpendicular slopes enhances one's ability to analyze and interpret the relationships between lines in the coordinate plane.

As mathematical concepts continue to underpin technological advancements and scientific endeavors, mastery of fundamental ideas like slopes and perpendicularity remains essential. With this knowledge, students and professionals can confidently approach a wide array of problems involving lines, angles, and spatial reasoning, fostering both analytical skills and geometric intuition.

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In summary:

- The given line: \(3x + y = 16\)
- Slope of the given line: \(-3\)
- Slope of the perpendicular line: \(\boxed{\frac{1}{3}}\)

Understanding and applying these concepts forms a cornerstone of analytical
geometry, opening doors to more advanced topics and real-world problem-
solving.
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