Find The Solution Set For The Equation |6x-6|+4=8

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Understanding how to solve absolute value equations like $|6x - 6| + 4 = 8$ is fundamental in algebra. These types of equations often appear in various mathematical problems and real-world applications where the magnitude of a quantity is involved, regardless of its sign. In this comprehensive guide, we will explore the step-by-step process to find the solution set of the given equation, provide insights into the properties of absolute value functions, and offer tips to approach similar problems effectively.

Understanding Absolute Value Equations

What Is an Absolute Value?

Absolute value, denoted by $|x|$, represents the distance of a number x from zero on the number line. By definition:

- If $x \geq 0$, then $|x| = x$
- If $x < 0$, then $|x| = -x$

This concept is crucial because it deals with magnitude without regard to direction, which makes absolute value equations unique and sometimes tricky to solve.

General Form of Absolute Value Equations

Absolute value equations typically take the form:

$$|A x + B| = C$$

where A, B, and C are constants.

To solve such equations, we leverage the property:

$$|Y| = C \quad \Rightarrow \quad Y = C \quad \text{or} \quad Y = -C$$

provided that $C \geq 0$.

This principle forms the backbone of solving equations involving absolute values — by splitting the equation into two separate cases.

Step-by-Step Solution for |6x - 6| + 4 = 8

Let's apply these principles to the specific equation:

$$|6x - 6| + 4 = 8$$

Step 1: Isolate the Absolute Value

First, we want to isolate the absolute value expression on one side of the equation. Currently, the equation is:

$$ |6x - 6| + 4 = 8 $$

Subtract 4 from both sides to isolate $|6x - 6|$:

$$ |6x - 6| = 8 - 4 $$

$$ |6x - 6| = 4 $$

Step 2: Set Up Two Equations Based on the Absolute Value Property

Since $|6x - 6| = 4$, it implies:

$$ 6x - 6 = 4 \quad \text{or} \quad 6x - 6 = -4 $$

Let's solve each case separately.

Case 1: $6x - 6 = 4$

Add 6 to both sides:

$$ 6x = 4 + 6 $$

$$ 6x = 10 $$

Divide both sides by 6:

$$ x = \frac{10}{6} = \frac{5}{3} $$

Case 2: $6x - 6 = -4$

Add 6 to both sides:

$$ 6x = -4 + 6 $$

$$ 6x = 2 $$

Divide both sides by 6:

$$ x = \frac{2}{6} = \frac{1}{3} $$

Solution Set

The solutions to the original equation are:

$$ x = \frac{5}{3} \quad \text{and} \quad x = \frac{1}{3} $$

Expressed as a set:

$$ \boxed{ \left\{ \frac{1}{3}, \frac{5}{3} \right\} } $$

Verifying the Solutions

It's always good practice to verify solutions by substituting them back into the original equation to ensure they satisfy it.

Verification for $x = 1/3$

Calculate $|6x - 6| + 4$:

$$ 6 \times \frac{1}{3} - 6 = 2 - 6 = -4 $$

$$ | -4 | + 4 = 4 + 4 = 8 $$

Matches the right side; solution is valid.

Verification for $x = 5/3$

Calculate $|6x - 6| + 4$:

$$ 6 \times \frac{5}{3} - 6 = 10 - 6 = 4 $$

$$ |4| + 4 = 4 + 4 = 8 $$

Matches the right side; solution is valid.

Graphical Interpretation of the Equation

Graphing the equation $|6x - 6| + 4 = 8$ can provide visual insight into its solutions.

Graph of $|6x - 6| + 4$

- The graph of $|6x - 6|$ is a V-shaped graph with its vertex at the point where the expression inside the absolute value equals zero, i.e., $6x - 6 = 0$.

- Solving for x:

$$ 6x - 6 = 0 \Rightarrow x = 1 $$

- The vertex of the graph is at $(1, 4)$, because when $x = 1$, $|6(1) - 6| + 4 = 0 + 4 = 4$.

Line $y = 8$

- The right side of the original equation is a horizontal line at $y = 8$.

- The solutions to the equation correspond to the x-coordinates where the graph of $|6x - 6| + 4$ intersects $y = 8$.

- From the algebraic solution, these are at $x = 1/3$ and $x = 5/3$, which can be verified visually on the graph.

Tips for Solving Absolute Value Equations

Solving absolute value equations can sometimes involve more complex steps, especially with nested or more complicated expressions. Here are some tips:

- Always isolate the absolute value expression before solving.
- Remember that $|Y| = C$ implies $Y = C$ or $Y = -C$, provided $C \geq 0$.
- Check for extraneous solutions, especially when dealing with equations that involve additional operations or restrictions.
- Graphical methods can be helpful for visual understanding, especially for equations involving absolute values and linear functions.
- For equations with multiple absolute value expressions, consider solving each case separately and then combining solutions.

Common Mistakes to Avoid

When solving absolute value equations, some common pitfalls include:

- Failing to split into two cases when taking the absolute value property.
- Forgetting to check solutions in the original equation, which might be extraneous solutions introduced during algebraic manipulations.
- Misapplying the absolute value property when the right side is negative; remember that $|Y| = C$ only makes sense for $C \geq 0$.
- Incorrectly combining solutions from different cases or missing solutions due to algebraic errors.

Extensions and Related Problems

Once comfortable with solving equations like $|6x - 6| + 4 = 8$, you can explore more complex problems:

- Equations with nested absolute values, e.g., $||x| - 3| = 2$
- Absolute value inequalities, e.g., $|x - 2| < 5$
- Absolute value equations involving quadratic or higher-degree expressions
- Word problems translating real-world scenarios into absolute value equations

Conclusion

In summary, solving the equation $|6x - 6| + 4 = 8$ involves isolating the absolute value expression, applying the fundamental property that $|Y| = C$ implies $Y = C$ or $Y = -C$, and then solving the resulting linear equations. The solutions are $x = 1/3$ and $x = 5/3$, which can be verified by substitution. Understanding the properties of absolute value functions, practicing various types of problems, and visualizing solutions graphically can significantly improve your proficiency in handling similar equations.

Mastering these techniques not only enhances your algebra skills but also prepares you for more advanced topics involving absolute values and their applications in mathematics and science.

Frequently Asked Questions

What is the first step to solve the equation $|6x - 6| + 4 = 8$?

Subtract 4 from both sides to isolate the absolute value: $|6x - 6| = 4$.

How do you solve $|6x - 6| = 4$?

Set up two equations: $6x - 6 = 4$ and $6x - 6 = -4$, then solve each separately.

What are the solutions for $6x - 6 = 4$?

Add 6 to both sides: $6x = 10$, then divide both sides by 6: $x = 10/6 = 5/3$.

What are the solutions for $6x - 6 = -4$?

Add 6 to both sides: $6x = 2$, then divide both sides by 6: $x = 2/6 = 1/3$.

What is the solution set for the equation $|6x - 6| + 4 = 8$?

The solution set is $\{ x = 5/3, x = 1/3 \}$.

Are there any extraneous solutions in this problem?

No, both solutions satisfy the original equation after substitution.

How can I verify the solutions for the equation?

Substitute each solution back into the original equation to check if it holds true.

Additional Resources

Find The Solution Set For The Equation |6x - 6| + 4 = 8

Mathematics often challenges us to decipher patterns, unravel complexities, and arrive at precise solutions that reveal the underlying structure of problems. One such problem, seemingly straightforward at first glance, involves an absolute value equation that invites a methodical approach to find its solution set. This article explores the problem statement: Find The Solution Set For The Equation |6x - 6| + 4 = 8, dissecting it with thorough analysis, step-by-step solutions, and insights into the nature of absolute value equations.

Understanding the Equation

Before delving into solution strategies, it is essential to understand the components of the given equation:

$|6x - 6| + 4 = 8$

This equation comprises an absolute value expression and a constant. The absolute value function, denoted as $|\cdot|$, returns the non-negative magnitude of its argument, regardless of whether the argument is positive or negative.

Key Components and Their Significance

- Absolute Value Expression: $|6x - 6|$
Represents the distance of the algebraic expression $6x - 6$ from zero on the number line.
- Constants: +4 and = 8
These constants serve as boundary conditions and shifts, influencing the magnitude and the solution set.

Step-by-Step Solution Approach

The process of solving an absolute value equation involves considering the definition of the absolute value function and analyzing the equation in separate cases based on the argument's sign.

1. Isolate the Absolute Value

Given the equation:

$|6x - 6| + 4 = 8$

Subtract 4 from both sides:

$|6x - 6| = 8 - 4$
$|6x - 6| = 4$

This simplifies the problem to solving:

$|6x - 6| = 4$

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2. Recall the Definition of Absolute Value

The absolute value equation $|A| = B$ (where $B \geq 0$) is equivalent to:

$A = B$ OR $A = -B$

Applying this to our problem:

$6x - 6 = 4$ OR $6x - 6 = -4$

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3. Solve Each Case Separately

Case 1: $6x - 6 = 4$

Add 6 to both sides:

$6x = 4 + 6$
$6x = 10$

Divide both sides by 6:

$x = 10 / 6$
$x = 5 / 3$

Case 2: $6x - 6 = -4$

Add 6 to both sides:

$6x = -4 + 6$
$6x = 2$

Divide both sides by 6:

$x = 2 / 6$
$x = 1 / 3$

Solution Set and Verification

The solutions derived are:

$x = 5/3$ and $x = 1/3$

To ensure these solutions satisfy the original equation, substitute each back into the original.

Verification for x = 5/3

Calculate |6x - 6| + 4:

6(5/3) - 6 = 10 - 6 = 4
|4| + 4 = 4 + 4 = 8

Matches the right side of the original equation.

Verification for x = 1/3

Calculate |6x - 6| + 4:

6(1/3) - 6 = 2 - 6 = -4
|-4| + 4 = 4 + 4 = 8

Again, matches the right side.

Both solutions are valid.

Special Considerations and Common Pitfalls

While the solution appears straightforward, several points warrant attention, especially for students and practitioners:

1. Domain Restrictions

Absolute value equations generally do not impose restrictions beyond the real numbers unless explicitly stated. The solutions here are within the real domain, and both solutions are valid.

2. Extraneous Solutions

In some equations involving absolute value, extraneous solutions can arise when solving algebraically. Here, verification confirms both solutions are valid.

3. Multiple Cases and Sign Analysis

Always consider both cases derived from the absolute value definition. Missing a case can result in incomplete solution sets.

4. Handling Constants and Simplification

Ensure constants are correctly moved and simplified to avoid algebraic errors.

Alternative Methods and Generalization

While the above method is direct and effective for this problem, other techniques and general principles can be applied.

Graphical Interpretation

Plotting the functions $y = |6x - 6| + 4$ and $y = 8$ reveals their intersection points, which correspond to solutions. The absolute value graph is a V-shaped curve, and the line $y=8$ intersects it at the solutions calculated.

Method Extension to Similar Equations

For equations of the form $|ax + b| + c = d$:

- Isolate the absolute value.
- Solve the two linear equations: $ax + b = d - c$ and $ax + b = -(d - c)$.
- Verify solutions.

Implications and Broader Context

Understanding how to solve absolute value equations is fundamental in algebra, serving as a building block for more complex topics such as inequalities, piecewise functions, and real-world modeling.

Key takeaways:

- Absolute value equations often have two solutions, corresponding to the positive and negative cases.
- Verification is vital to confirm solutions satisfy the original equation.
- The solution set can be expressed explicitly or graphically, depending on context.

Conclusion

The comprehensive analysis of Find The Solution Set For The Equation $|6x - 6| + 4 = 8$ underscores the importance of systematic problem-solving approaches in algebra. By isolating the absolute value, applying the definition, solving the resulting linear equations, and verifying solutions, we find that:

Solution Set: $x = 1/3, 5/3$

Both solutions satisfy the original equation, illustrating the symmetry and structure inherent in absolute value equations. Mastery of these techniques not only enhances algebraic proficiency but also prepares learners for tackling more complex mathematical challenges with confidence and precision.

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