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Introduction

Mathematics often involves working with complex algebraic expressions, especially when dealing with functions and their evaluations at specific points. The given function:

$$
Y = \frac{(x+3)(x^2 + 2x + 5)}{3x^2+1}
$$

serves as a prime example of a rational function where the numerator and denominator are polynomial expressions. Calculating the value of Y at a particular value of x, specifically at $x=2$, requires a systematic approach—substituting the value into the function, simplifying the expression, and ensuring that the calculation is accurate.

This comprehensive guide aims to walk through the process step-by-step, explaining key concepts such as polynomial expansion, substitution, and simplification. Additionally, we will explore the importance of understanding rational functions in algebra and their applications in various fields such as physics, engineering, and data analysis.

Understanding the Function Y

The Structure of the Function

The function:

$$
Y = \frac{(x+3)(x^2 + 2x + 5)}{3x^2 + 1}
$$

has three core components:
- Numerator: a product of a linear binomial $(x+3)$ and a quadratic polynomial $(x^2 + 2x + 5)$.
- Denominator: a quadratic polynomial $3x^2 + 1$.

Significance of Each Part

- Linear term $(x+3)$: Influences the overall degree and roots of the numerator.
- Quadratic $(x^2 + 2x + 5)$: Adds complexity and potential for roots or zeros.
- Quadratic denominator $3x^2 + 1$: Ensures the function’s domain excludes points where denominator equals zero, which in this case, does not occur for real x since $3x^2 + 1 > 0$ always.

Understanding these components helps in analyzing the behavior of the function, especially as x varies and at specific points like $x=2$.

Step-by-Step Calculation of $Y(2)$

Step 1: Substituting $x=2$

The first step is to replace every occurrence of x in the function with 2:

$$
Y(2) = \frac{(2+3)(2^2 + 2 \times 2 + 5)}{3 \times 2^2 + 1}
$$

Step 2: Simplify numerator components

Simplify the linear term:

$$
2+3 = 5
$$

Simplify the quadratic term inside the numerator:

$$
2^2 + 2 \times 2 + 5 = 4 + 4 + 5 = 13
$$

Calculate numerator:

$$
5 \times 13 = 65
$$

Step 3: Simplify denominator

Calculate each part:

$$
3 \times 2^2 + 1 = 3 \times 4 + 1 = 12 + 1 = 13
$$

Step 4: Final calculation

Now, combine numerator and denominator:

$$
Y(2) = \frac{65}{13} = 5
$$

Thus, the value of the function at $x=2$ is 5.

Additional Insights into Rational Function Evaluation

The importance of substitution

Substituting specific values into functions like Y is fundamental in understanding their behavior at particular points. This process involves replacing the variable with the number of interest, then simplifying step by step to arrive at the result.

Key points when evaluating rational functions:

- Check the denominator: Ensure it does not equal zero at the specific point to avoid undefined expressions.
- Perform polynomial operations carefully: Expansion, factoring, or simplifying polynomials can be error-prone if not done systematically.
- Verify your calculations: Always double-check each step, especially during substitution and arithmetic operations.

Domain considerations

For the given function:

$$
Y = \frac{(x+3)(x^2 + 2x + 5)}{3x^2+1}
$$

the denominator $3x^2 + 1$ is never zero for real numbers because $3x^2 \geq 0$, so $3x^2 + 1 > 0$. Therefore, the function is defined for all real x. When evaluating at $x=2$, the function is well-defined and the calculation is valid.

Applications of Similar Calculations in Real-World Scenarios

Understanding how to evaluate functions like Y at specific points has practical implications across various disciplines:

1. Engineering and Physics

- Calculating the value of a function at a specific point can model physical phenomena, such as the stress at a particular point in a structure or the velocity of an object at a given moment.

2. Data Analysis and Modeling

- Rational functions are used in curve fitting to model relationships in data. Evaluating the function at specific points helps in understanding the model's predictions and behavior.

3. Economics and Finance

- Cost, revenue, or profit functions often involve rational expressions. Calculating their values at specific quantities informs decision-making.

4. Calculus and Mathematical Analysis

- Evaluating functions at points lays the foundation for understanding limits, continuity, and derivatives, which are fundamental concepts in calculus.

Common Mistakes to Avoid

While calculating $Y(2)$, some common pitfalls include:

- Incorrect substitution: Mixing up the value of x during substitution.
- Arithmetic errors: Miscalculations during polynomial expansion or simplification.
- Neglecting domain restrictions: Overlooking points where the denominator might be zero.
- Order of operations: Failing to follow PEMDAS (Parentheses, Exponents, Multiplication and Division, Addition and Subtraction) correctly.

Ensuring careful and systematic work minimizes these errors and leads to accurate results.

Summary and Key Takeaways

- The given function $Y = \frac{(x+3)(x^2 + 2x + 5)}{3x^2+1}$ was evaluated at $x=2$.
- Substituting $x=2$ simplified the function to $\frac{65}{13}$, which equals 5.
- The process involved careful substitution, polynomial simplification, and division.
- Rational functions are essential in modeling real-world scenarios, and precise evaluation at specific points enhances understanding of their behavior.
- Always verify that the denominator does not equal zero at the point of evaluation to ensure the function is defined there.

Final thoughts

Mastering the evaluation of rational functions like Y at specific points is a crucial skill in algebra and higher mathematics. It provides insights into the function's behavior, supports problem-solving across various scientific fields, and reinforces fundamental algebraic techniques. With practice, performing such calculations becomes more straightforward, paving the way for tackling more complex functions and mathematical models with confidence.

Keywords: rational function, polynomial, function evaluation, algebra, substitution, polynomial simplification, domain, calculus, applications in science and engineering

Frequently Asked Questions

How do I evaluate Y(2) for the function $Y = (x+3)(x^2 + 2x + 5) / (3x^2 + 1)$?

To evaluate Y(2), substitute $x=2$ into the function: $Y(2) = (2+3)(2^2 + 22 + 5) / (32^2 + 1)$. Simplify numerator and denominator step by step to find the value.

What is the first step to calculate Y(2) in the given expression?

The first step is to substitute $x=2$ into the expression: compute $(2+3)$, $(2^2 + 22 + 5)$, and $(32^2 + 1)$.

How do I simplify the numerator when calculating Y(2)?

Calculate $(x+3)$ at $x=2$: $2+3=5$. Then compute $(x^2 + 2x + 5)$ at $x=2$: $4 + 4 + 5=13$. Multiply these results: $5\ 13=65$.

What is the value of the denominator when x=2 in the given function?

At $x=2$, compute $3\ (2)^2 + 1$: $34 + 1=12 +1=13$.

How do I finalize the calculation of Y(2)?

Divide the numerator result (65) by the denominator result (13): $Y(2) = 65 / 13 = 5$.

What is the value of Y at x=2 for the given function?

Y(2) equals 5 after substituting and simplifying the expression.

Can I verify the calculation of Y(2) by plugging in values directly?

Yes, by substituting $x=2$ directly into the original function and simplifying step by step, you can verify the result is 5.

Are there any common mistakes to avoid when calculating Y(2)?

Yes, ensure correct substitution, order of operations, and simplify numerator and denominator separately before division to avoid mistakes.

What is the importance of breaking down the calculation into steps for Y(2)?

Breaking down ensures accuracy, clarity, and helps identify any errors during the substitution and simplification process.

How can I generalize this method to find Y at any x-value?

Substitute the desired x-value into the expression, evaluate numerator and denominator separately, then divide to find Y at that point.

Additional Resources

Mathematical Analysis and Calculation of Y(2) for the Given Function

Understanding and evaluating complex algebraic functions is fundamental in mathematics, especially in calculus, algebra, and applied mathematics. The problem at hand involves a rational function:

$$ Y = \frac{(x + 3)(x^2 + 2x + 5)}{3x^2 + 1} $$

and requires calculating the value of Y at $x=2$, i.e., $Y(2)$. To thoroughly analyze this function, we will explore its algebraic structure, properties, potential simplifications, and then perform a detailed calculation of $Y(2)$.

Introduction to the Function

The function provided is a rational expression, which typically involves a numerator and denominator that are polynomial expressions. Rational functions are significant in various mathematical fields, including calculus, algebra, and engineering, due to their diverse behaviors such as asymptotes, intercepts, and discontinuities.

The given function:

$$ Y(x) = \frac{(x + 3)(x^2 + 2x + 5)}{3x^2 + 1} $$

can be broken down into parts to understand its structure better.

Structural Breakdown of the Function

Numerator Analysis

The numerator is a product of two polynomials:

$$ (x + 3) \quad \text{and} \quad (x^2 + 2x + 5) $$

- Linear factor: $x + 3$ — a simple first-degree polynomial with a root at $x = -3$.
- Quadratic factor: $x^2 + 2x + 5$ — a second-degree polynomial. Its discriminant:

$$
\Delta = (2)^2 - 4 \times 1 \times 5 = 4 - 20 = -16 < 0
$$

indicates it has no real roots, meaning it's always positive for real x. Its minimum value can be found by completing the square or using calculus.

Denominator Analysis

The denominator is:

$$ 3x^2 + 1 $$

which is a quadratic with all positive coefficients, ensuring it is always positive for all real x. Its minimum value occurs at $x = 0$:

$$
3 \times 0^2 + 1 = 1
$$

which indicates the function is defined for all real x (no asymptotes due to the denominator being zero).

Simplification of the Function

Before substituting $x=2$, simplifying the function can make the calculation cleaner.

Step 1: Expand the numerator

Let's expand $(x + 3)(x^2 + 2x + 5)$:

$$
\begin{aligned}
(x + 3)(x^2 + 2x + 5) &= x \times (x^2 + 2x + 5) + 3 \times (x^2 + 2x + 5) \\
&= x^3 + 2x^2 + 5x + 3x^2 + 6x + 15 \\
&= x^3 + (2x^2 + 3x^2) + (5x + 6x) + 15 \\
&= x^3 + 5x^2 + 11x + 15
\end{aligned}
$$

Thus, the numerator simplifies to:

$$
x^3 + 5x^2 + 11x + 15
$$

Step 2: Write the entire function in simplified form

The function now becomes:

$$
Y(x) = \frac{x^3 + 5x^2 + 11x + 15}{3x^2 + 1}
$$

Calculating $Y(2)$: Step-by-Step Process

Now, with the simplified form, we can directly substitute $x=2$ into the expression.

Step 1: Compute numerator at $x=2$

$$
\begin{aligned}
\text{Numerator} &= (2)^3 + 5 \times (2)^2 + 11 \times 2 + 15 \\
&= 8 + 5 \times 4 + 22 + 15 \\
&= 8 + 20 + 22 + 15 \\
&= (8 + 20) + (22 + 15) \\
&= 28 + 37 \\
&= 65
\end{aligned}
$$

Step 2: Compute denominator at $x=2$

$$
3 \times (2)^2 + 1 = 3 \times 4 + 1 = 12 + 1 = 13
$$

Step 3: Final calculation of $Y(2)$

$$
Y(2) = \frac{65}{13} = 5
$$

Result: $Y(2) = 5$

Deeper Insights and Properties of the Function

While the primary goal was to compute $Y(2)$, exploring the function's properties provides a richer understanding.

Behavior at Infinity

As $x \to \pm \infty$:

$$
Y(x) \sim \frac{x^3}{3x^2} = \frac{x^3}{3x^2} = \frac{x}{3}
$$

which implies:

- For large positive x, $Y(x) \to +\infty$.
- For large negative x, $Y(x) \to -\infty$.

The function exhibits linear growth at infinity, with the end behavior dominated by the degree 3 numerator and degree 2 denominator.

Intercepts

- Y-intercept: at $x=0$:

$$
Y(0) = \frac{0^3 + 5 \times 0^2 + 11 \times 0 + 15}{3 \times 0^2 + 1} = \frac{15}{1} = 15
$$

- X-intercepts: where numerator equals zero:

$$
x^3 + 5x^2 + 11x + 15 = 0
$$

This cubic does not factor easily, but numerical methods or graphing can approximate roots.

Asymptotic Behavior

Since the degree of numerator (3) exceeds that of the denominator (2), the function has an oblique (slant) asymptote. Polynomial division can find this asymptote.

Polynomial Division to Find the Oblique Asymptote

Dividing numerator $x^3 + 5x^2 + 11x + 15$ by denominator $3x^2 + 1$:

1. Divide x^3 by $3x^2$:

$$
\frac{x^3}{3x^2} = \frac{1}{3} x
$$

2. Multiply back:

$$
\frac{1}{3} x \times (3x^2 + 1) = x^3 + \frac{1}{3} x
$$

3. Subtract:

$$
(x^3 + 5x^2 + 11x + 15) - (x^3 + \frac{1}{3} x) = 5x^2 + (11x - \frac{1}{3} x) + 15
$$

$$
= 5x^2 + \left(11 - \frac{1}{3}\right) x + 15 = 5x^2 + \frac{32}{3} x + 15
$$

4. Divide $5x^2$ by $3x^2$:

$$
\frac{5x^2}{3x^2} = \frac{5}{3}
$$

5. Multiply back:

$$
\frac{5}{3} \times (3x^2 + 1) = 5x^2 + \frac{5}{3}
$$

6. Subtract:

$$
(5x^2 + \frac{32}{3} x + 15) - (5x^2 + \frac{5}{3}) = \frac{32}{3} x + 15 - \frac{5}{3}
$$

$$
= \frac{32}{3} x + \left(15 - \frac{5}{3}\right) = \frac{32}{3} x + \frac{45}{3} - \frac{5}{3} = \frac{32}{3} x + \frac{40}{3}
$$

This remainder indicates the oblique asymptote is:

$$
Y(x) \approx \frac{1}{3} x + \frac{5}{3}
$$

as $x \to \pm \infty$.

Graphical Interpretation

Plotting the function or analyzing its graph

[Given Y X 3 X 2 2x 5 3x 2 1 Calculate Y 2]

Given Y X 3 X 2 2x 5 3x 2 1 Calculate Y 2

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