Graph-3x2 + 12y2 = 84. What Are The Domain And Range?

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Understanding the domain and range of a graph is fundamental in algebra and calculus, as it helps us interpret the behavior and limitations of the equations. In this article, we will explore the specific equation $3x^2 + 12y^2 = 84$, analyze its geometric characteristics, and determine its domain and range. This equation represents a conic section—specifically, an ellipse—and understanding its properties provides valuable insights into its graphing and applications.

Understanding the Equation $3x^2 + 12y^2 = 84$

Before diving into the domain and range, it's essential to understand the form and nature of the given equation.

Identifying the Type of Conic Section

The equation:

$$3x^2 + 12y^2 = 84$$

is quadratic in both variables and resembles the standard form of an ellipse:

$$ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 $$

To confirm this, we can manipulate the given equation into the standard form.

Rearranging into Standard Form

Divide both sides of the equation by 84:

$$ \frac{3x^2}{84} + \frac{12y^2}{84} = 1 $$

Simplify each term:

$$ \frac{x^2}{28} + \frac{y^2}{7} = 1 $$

Expressed as:

$$ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 $$

we identify:

$$ a^2 = 28 \quad \Rightarrow \quad a = \sqrt{28} \approx 5.29 $$
$$ b^2 = 7 \quad \Rightarrow \quad b = \sqrt{7} \approx 2.65 $$

Therefore, the equation describes an ellipse centered at the origin with semi-major axis $a \approx 5.29$ along the x-axis and semi-minor axis $b \approx 2.65$ along the y-axis.

Plotting the Ellipse: Visual Interpretation

Understanding the shape and extent of the ellipse is crucial for determining its domain and range.

Ellipse Characteristics

- The ellipse is centered at the origin (0,0).
- It extends horizontally from $-a$ to a, i.e., approximately -5.29 to 5.29.
- It extends vertically from $-b$ to b, i.e., approximately -2.65 to 2.65.

Visualizing this, the graph appears as an elongated ellipse along the x-axis with a narrower height along the y-axis.

Determining the Domain of the Equation

The domain of a function or relation describes all possible x-values for which the equation yields real y-values.

Methodology for Finding the Domain

Given the equation:

$$\frac{x^2}{28} + \frac{y^2}{7} = 1$$

or equivalently,

$$ 3x^2 + 12y^2 = 84 $$

We fix an x-value and analyze whether the corresponding y-values are real.

1. Isolate y^2:

$$ \frac{y^2}{7} = 1 - \frac{x^2}{28} $$

2. Rewrite:

$$ y^2 = 7 \left(1 - \frac{x^2}{28} \right) $$

3. For y^2 to be non-negative (since $y^2 \geq 0$), the right side must be ≥ 0:

$$ 7 \left(1 - \frac{x^2}{28} \right) \geq 0 $$

Dividing both sides by 7:

$$ 1 - \frac{x^2}{28} \geq 0 $$

4. Rearranged:

$$ \frac{x^2}{28} \leq 1 $$

Multiply both sides by 28:

$$ x^2 \leq 28 $$

5. Take the square root:

$$ -\sqrt{28} \leq x \leq \sqrt{28} $$

Since $\sqrt{28} \approx 5.29$, the domain is:

$$ \boxed{ -\sqrt{28} \leq x \leq \sqrt{28} } $$

or approximately:

$$ -5.29 \leq x \leq 5.29 $$

Therefore, the domain of the ellipse is:

- x-values from $-\sqrt{28}$ to $\sqrt{28}$, approximately -5.29 to 5.29.

Determining the Range of the Equation

The range describes all possible y-values for which the equation is valid, given the x-values within the domain.

Methodology for Finding the Range

Starting again from:

$$ y^2 = 7 \left(1 - \frac{x^2}{28} \right) $$

Since $y^2 \geq 0$:

$$ y^2 \leq 7 \left(1 - \frac{x^2}{28} \right) $$

The maximum value of y^2 occurs at the minimal x^2, which is zero ($x=0$):

$$ y^2_{\max} = 7 \left(1 - 0 \right) = 7 $$

Similarly, the minimum y-value occurs at the maximum x-value ($x=\pm\sqrt{28}$), where:

$$ y^2 = 7 \left(1 - \frac{28}{28} \right) = 0 $$

Thus, y ranges from:

$$ -\sqrt{7} \leq y \leq \sqrt{7} $$

since $y^2 \geq 0$ and y can be positive or negative.

Therefore, the range of the ellipse is:

- y-values from $-\sqrt{7}$ to $\sqrt{7}$, approximately -2.65 to 2.65.

Summary of Domain and Range

Property	Values	Approximate Values

Domain (x-values)	$-\sqrt{28}$ to $\sqrt{28}$	-5.29 to 5.29
Range (y-values)	$-\sqrt{7}$ to $\sqrt{7}$	-2.65 to 2.65

Implications for Graphing the Equation

Knowing the domain and range allows for precise plotting of the ellipse, ensuring the graph accurately reflects the limits of the equation.

Graphing Tips

- Plot the center at (0,0).
- Mark key points at the extremities:
  - Horizontal endpoints: $(-5.29, 0)$ and $(5.29, 0)$.
  - Vertical endpoints: $(0, -2.65)$ and $(0, 2.65)$.
- Sketch a smooth, symmetrical ellipse passing through these points.

Applications of the Equation and Its Domain and Range

Understanding the domain and range of an ellipse like this has practical applications in various fields:

Physics and Engineering

- Modeling orbital paths where elliptical orbits are common.
- Designing lenses or reflective surfaces with elliptical shapes.

Mathematics and Geometry

- Analyzing conic sections and their properties.
- Solving optimization problems constrained by elliptical boundaries.

Computer Graphics

- Rendering elliptical shapes accurately.
- Implementing collision detection within elliptical boundaries.

Conclusion

The equation $3x^2 + 12y^2 = 84$ describes an ellipse centered at the origin with semi-major axis approximately 5.29 along the x-axis and semi-minor axis approximately 2.65 along the y-axis. Its domain and range are directly linked to these axes: the domain spans from $-\sqrt{28}$ to $\sqrt{28}$, and the range from $-\sqrt{7}$ to $\sqrt{7}$. Understanding these limits is crucial for graphing, analyzing, and applying the ellipse in various scientific and mathematical contexts. Whether for academic study or practical application, knowing the domain and range of this conic section enhances comprehension of its geometric properties and real-world relevance.

Frequently Asked Questions

What type of conic section is represented by the equation $3x^2 + 12y^2 = 84$?

The equation represents an ellipse because both x^2 and y^2 terms are positive and the equation can be rewritten in standard form to identify it as an ellipse.

How do you find the domain of the equation $3x^2 + 12y^2 = 84$?

To find the domain, solve for x in terms of y or analyze the equation to determine the possible x-values. Since the equation describes an ellipse, the domain is all x-values where the expression under the square root is non-negative, resulting in x-values between $-\sqrt{(28)}$ and $\sqrt{(28)}$.

What is the range of the equation $3x^2 + 12y^2 = 84$?

The range of y-values for the ellipse is from $-\sqrt{(7/2)}$ to $\sqrt{(7/2)}$, because solving for y gives $y = \pm\sqrt{[(84 - 3x^2)/12]}$, which is real only within these bounds.

How can we rewrite the equation $3x^2 + 12y^2 = 84$ in standard form?

Divide both sides by 84 to get $(x^2/28) + (y^2/7) = 1$, which is the standard form of an ellipse centered at the origin.

What are the vertices of the ellipse given by $3x^2 + 12y^2 = 84$?

The vertices are at the points $(0, \pm\sqrt{(7/2)})$, corresponding to the maximum and minimum y-values, and at $(\pm\sqrt{(28)}, 0)$, corresponding to the x-intercepts.

How do you determine the foci of the ellipse $3x^2 + 12y^2 = 84$?

After rewriting in standard form, identify a and b from the denominators. Then, calculate $c = \sqrt{(a^2 - b^2)}$ to find the foci at $(\pm c, 0)$ or $(0, \pm c)$, depending on the ellipse's orientation.

Are the domain and range of the ellipse finite or infinite?

Both the domain and range are finite because the ellipse is bounded, with specific maximum and minimum x and y values determined by the equation.

Additional Resources

Graph-3x2 + 12y2 = 84. What Are The Domain And Range? is a compelling topic that delves into the fascinating world of conic sections, specifically the ellipse represented by this equation. Understanding the graph of this equation involves analyzing its shape, orientation, and the key properties such as domain and range. This article offers a comprehensive review of the graph of the equation, providing insights into how to interpret and analyze such curves, along with practical applications and mathematical significance.

Introduction to the Equation

The equation $-3x^2 + 12y^2 = 84$ describes a conic section, more specifically, an ellipse. To analyze its graph effectively, it is crucial to understand the form of the equation, its features, and how it relates to the general form of an ellipse.

The given equation can be rewritten for clarity:

$-3x^2 + 12y^2 = 84$

Dividing through by 84 to normalize the equation:

$(-3x^2)/84 + (12y^2)/84 = 1$

which simplifies to:

$- x^2 / 28 + y^2 / 7 = 1$

Or, written as:

$x^2 / 28 - y^2 / 7 = -1$

However, since the standard form of an ellipse involves positive denominators, it is better to manipulate the original to achieve standard form:

Divide the original equation by 84:

$(-3x^2 + 12y^2) / 84 = 1$

which simplifies to:

$- x^2 / 28 + y^2 / 7 = 1$

Now, multiply both sides by -1 to get a positive coefficient for x^2:

$x^2 / 28 - y^2 / 7 = -1$

Alternatively, rearranged to the form:

$x^2 / 28 + y^2 / 7 = 1$

But note that the original form suggests the conic is a hyperbola rather than an ellipse because of the negative coefficient of x^2. To clarify, rewriting the original:

$-3x^2 + 12y^2 = 84$

becomes:

$3x^2 - 12y^2 = -84$

Dividing through by -84:

$(3x^2)/-84 + (-12y^2)/-84 = 1$

which simplifies to:

$- x^2 / 28 + y^2 / 7 = 1$

Now, the form:

$x^2 / 28 - y^2 / 7 = -1$

indicates a hyperbola, because of the difference of squares structure with opposite signs.

Key Point: The equation represents a hyperbola, not an ellipse, because of the negative sign between the terms.

Understanding the Graph of the Equation

Identifying the Type of Conic

The form of the equation, after simplification, reveals that it is a hyperbola:

$x^2 / a^2 - y^2 / b^2 = -1$

or equivalently,

$- x^2 / a^2 + y^2 / b^2 = 1$

depending on how the equation is manipulated. The signs indicate the conic's nature.

In our case, the original form:

$-3x^2 + 12y^2 = 84$

can be rewritten as:

$3x^2 - 12y^2 = -84$

Dividing through by -84:

$- x^2 / 28 + y^2 / 7 = 1$

which indicates a hyperbola opening along the x-axis because the positive term is with x^2.

Features of the Hyperbola:

- It opens horizontally along the x-axis.
- The vertices are located at points on the x-axis.
- It has asymptotes that define its slopes.

Plotting the Hyperbola

To plot the hyperbola accurately:

- Find the vertices: for hyperbolas of the form $x^2 / a^2 - y^2 / b^2 = 1$, the vertices are at $(\pm a, 0)$.
- Identify the foci: located at $(\pm c, 0)$, where $c^2 = a^2 + b^2$.
- Draw the asymptotes: lines passing through the center with slopes $\pm b/a$.

Calculations:

$a^2 = 28$, so a □ □28 □ 5.29

$b^2 = 7$, so b □ □7 □ 2.65

$c^2 = a^2 + b^2 = 28 + 7 = 35$

c □ □35 □ 5.92

Vertices are at:

$(\pm a, 0)$ □ approximately $(\pm 5.29, 0)$

Foci are at:

(±c, 0) □ approximately (±5.92, 0)

Asymptotes are lines:

y = ±(b/a) x = ±(2.65 / 5.29) x □ ±0.5 x

Graph features:

- The hyperbola opens left and right.
- The vertices lie at approximately (±5.29, 0).
- The asymptotes intersect at the center (0,0).

Domain and Range of the Hyperbola

Understanding the Domain

The domain of a graph refers to all possible x-values for which the function (or the curve) is defined.

For hyperbolas of the form $x^2 / a^2 - y^2 / b^2 = 1$, the hyperbola is defined for all x-values satisfying the inequality:

|x| □ a

because the hyperbola does not exist between -a and a, where the expression under the square root becomes negative in the corresponding y-form.

From the calculations:

a □ 5.29

Thus, the domain:

x □ (-□, -5.29] □ [5.29, □)

This means the hyperbola extends infinitely along the X-axis outside the vertices, but not between -a and a.

Pros of this domain:

- Clear boundaries for where the graph exists.
- Facilitates understanding of the hyperbola's extent along the x-axis.

Cons:

- The domain excludes the interval between -a and a, which might seem unintuitive at first glance.

Understanding the Range

The range indicates all possible y-values for the hyperbola.

Since the hyperbola opens along the x-axis, for each x-value outside the domain, there are corresponding y-values.

For hyperbolas of the form $x^2 / a^2 - y^2 / b^2 = 1$, the range is all real y-values because the y-value extends to infinity.

However, to see the specific y-values at the vertices:

At $x = \pm a$,

$x^2 / a^2 = 1$

then,

$- y^2 / b^2 = 0 \Rightarrow y = 0$

which confirms vertices at $(\pm a, 0)$.

For $|x| > a$,

$y = \pm (b / a) \sqrt{(x^2 - a^2)}$

which indicates y can be any real number as x moves away from the vertices.

Thus, the range is:

$y \in (-\infty, \infty)$

Advantages:

- The range encompasses all real numbers, indicating the hyperbola extends infinitely upward and downward.

Disadvantages:

- For those unfamiliar with hyperbolas, understanding why y can take any value might require additional explanation about the structure of the curve.

Key Features and Properties of the Graph

Vertices

- Located at (±a, 0), approximately (±5.29, 0).
- These are the points where the hyperbola is closest to the center along the x-axis.

Foci

- Located at (±c, 0), approximately (±5.92, 0).
- The foci determine the hyperbola's shape and eccentricity.

Asymptotes

- Lines passing through the center with slopes ±b/a, approximately ±0.5.
- They guide the shape of the hyperbola and are crucial for understanding its curvature.

Center

- Located at (0, 0), the intersection point of the asymptotes and the hyperbola's symmetry point.

Practical Applications of the Hyperbola

Hyperbolas appear in various real-world contexts, including:

- Physics: Describing the paths of objects under inverse-square law forces, such as gravitational or electrostatic fields.
- Engineering: Antenna design, where hyperbolas are used to shape reflectors for signal focusing.
- Navigation: Hyperbolic positioning systems like LORAN, which utilize hyperbolas for triangulation.
- Astronomy: Modelling orbits and celestial mechanics.

Understanding the domain and range helps in applying the hyperbola to these fields effectively.

Conclusion and Summary

Analyzing the equation $-3x^2 + 12y^2 = 84$ reveals that it represents a hyperbola with its center at the origin, opening along the x-axis. The key features such as vertices, foci, asymptotes, and the extent of the graph are determined by the coefficients of the x^2 and y^2 terms. The domain is restricted to x-values outside the interval [-a, a], approximately [-5.29, 5.29], with the hyperbola existing for $|x| \geq a$. Conversely, the range spans all real y-values,

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