Graph The Function G(x) = 2x2 + 8x - 1

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Understanding the graph of the quadratic function $G(x) = 2x^2 + 8x - 1$ is essential in algebra and calculus, as it offers insights into the behavior of quadratic equations, their roots, vertex, and symmetry. This comprehensive guide will explore the various aspects of this function, including its shape, key features, graphing techniques, and real-world applications. Whether you're a student learning about parabolas or a data analyst interpreting quadratic models, this article provides a detailed, SEO-optimized overview.

Introduction to Quadratic Functions

Quadratic functions are polynomial functions of degree two, generally expressed as:

$$ y = ax^2 + bx + c $$

where:
- $a \neq 0$
- b and c are real numbers

The graph of a quadratic function is a parabola, which opens upward if $a > 0$ and downward if $a < 0$. The function $G(x) = 2x^2 + 8x - 1$ falls into this category with $a = 2$, indicating an upward-opening parabola.

Understanding the Function $G(x) = 2x^2 + 8x - 1$

This quadratic function models a wide range of phenomena, from projectile motion to profit maximization in economics. To analyze its graph comprehensively, we need to determine its key features—vertex, axis of symmetry, roots, y-intercept, and domain and range.

Standard Form of the Quadratic Function

The given function is in standard form:

$$ G(x) = ax^2 + bx + c $$
$$ G(x) = 2x^2 + 8x - 1 $$

This form makes it straightforward to identify coefficients and analyze the parabola's shape and position.

Vertex Form Conversion

Converting the standard form into vertex form, $y = a(x - h)^2 + k$, makes it easier to locate the vertex.

Steps to convert G(x) to vertex form:

1. Factor out the leading coefficient from the quadratic terms:

$$G(x) = 2(x^2 + 4x) - 1$$

2. Complete the square inside the parentheses:

- Take half of the coefficient of x, which is 4, divide by 2: 2
- Square it: $2^2 = 4$

3. Add and subtract this square inside the parentheses:

$$G(x) = 2(x^2 + 4x + 4 - 4) - 1$$

4. Rewrite as:

$$G(x) = 2[(x + 2)^2 - 4] - 1$$

5. Distribute:

$$G(x) = 2(x + 2)^2 - 8 - 1$$

6. Simplify:

$$G(x) = 2(x + 2)^2 - 9$$

Vertex form:

$$G(x) = 2(x + 2)^2 - 9$$

This reveals the vertex at $(-2, -9)$.

Key Features of the Graph of $G(x) = 2x^2 + 8x - 1$

Understanding the features of the parabola is crucial for sketching and analyzing the graph.

Vertex

- Located at $(-2, -9)$
- The vertex represents the minimum point of the parabola because $a = 2 > 0$

Axis of Symmetry

- The vertical line passing through the vertex:

$$x = -2$$

- It divides the parabola into two mirror-image halves.

Y-Intercept

- The point where the graph crosses the y-axis ($x = 0$):

$$G(0) = 2(0)^2 + 8(0) - 1 = -1$$

- Coordinates: $(0, -1)$

Roots or X-Intercepts

- The points where the graph crosses the x-axis ($y=0$)
- Found by solving:

$$2x^2 + 8x - 1 = 0$$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Plugging in $a=2$, $b=8$, $c=-1$:

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(-1)}}{2 \times 2}$$
$$x = \frac{-8 \pm \sqrt{64 + 8}}{4}$$
$$x = \frac{-8 \pm \sqrt{72}}{4}$$
$$x = \frac{-8 \pm 6\sqrt{2}}{4}$$
$$x = -2 \pm \frac{3\sqrt{2}}{2}$$

Approximate roots:

- $x \approx -2 + 2.1213 = 0.1213$
- $x \approx -2 - 2.1213 = -4.1213$

X-intercepts:

$$(0.12, 0) \quad \text{and} \quad (-4.12, 0)$$

Graphing the Function $G(x) = 2x^2 + 8x - 1$

Creating an accurate graph involves plotting key features and understanding the parabola's shape.

Step-by-Step Graphing Process

1. Plot the vertex at $(-2, -9)$.
2. Draw the axis of symmetry at $x = -2$.
3. Plot the y-intercept at $(0, -1)$.
4. Calculate and plot x-intercepts at approximately $(0.12, 0)$ and $(-4.12, 0)$.
5. Determine additional points by choosing x-values around the vertex (e.g., $x = -1, -3$) and computing corresponding y-values.
6. Sketch the parabola, ensuring it opens upward and is symmetric about the axis.

Characteristics of the Graph

- Opens upward due to $a=2 > 0$.
- The parabola is narrow compared to wider parabolas with smaller a.
- The vertex is the minimum point.
- X-intercepts are real and distinct, indicating the parabola crosses the x-axis at two points.

Applications of the Quadratic Function $G(x) = 2x^2 + 8x - 1$

Quadratic functions like $G(x) = 2x^2 + 8x - 1$ have diverse applications across various fields.

1. Physics and Engineering

- Modeling projectile motion, where the height of an object over time follows a quadratic pattern.
- Designing parabolic reflectors and satellite dishes for optimal signal

reflection.

2. Economics and Business

- Maximizing profit or minimizing cost, where the profit function is quadratic.
- Analyzing revenue and cost functions to determine optimal pricing strategies.

3. Biology and Ecology

- Population models where growth follows quadratic trends under certain conditions.

4. Mathematics Education

- Teaching fundamental concepts like vertex, roots, symmetry, and graphing techniques.

Advanced Analysis: Derivatives and Vertex Optimization

In calculus, analyzing the function's derivatives reveals the nature of its extrema.

First Derivative

$$ G'(x) = \frac{d}{dx} [2x^2 + 8x - 1] = 4x + 8 $$

Setting $G'(x) = 0$:

$$ 4x + 8 = 0 \Rightarrow x = -2 $$

This confirms the vertex's x-coordinate, as the critical point occurs at $x = -2$.

Second Derivative

$$ G''(x) = \frac{d}{dx} [4x + 8] = 4 $$

Since $G''(x) > 0$, the critical point at $x = -2$ is a minimum.

Impacts of Changing Coefficients on the Graph

Altering the coefficients a, b, and c affects the parabola’s shape and position.

- Increasing a: makes the parabola narrower and steeper.
- Decreasing a: makes it wider.
- Changing b: shifts the axis of symmetry horizontally.
- Changing c: shifts the parabola vertically without altering its shape.

Understanding these influences helps in modeling real-world problems accurately.

Summary and Key Takeaways

- The quadratic function $G(x) = 2x^2 + 8x - 1$ is an upward-opening parabola with a vertex at \((-

Frequently Asked Questions

What is the general shape of the graph of $G(x) = 2x^2 + 8x - 1$?

Since the leading coefficient (2) is positive, the graph is a parabola opening upward.

How do I find the vertex of the parabola $G(x) = 2x^2 + 8x - 1$?

Use the vertex formula $x = -b/(2a)$. Here, $a=2$ and $b=8$, so $x = -8/(22) = -8/4 = -2$. Plug $x = -2$ into $G(x)$ to find y: $G(-2) = 2(-2)^2 + 8(-2) - 1 = 2(4) - 16 - 1 = 8 - 16 - 1 = -9$. The vertex is at $(-2, -9)$.

What is the axis of symmetry for the parabola $G(x) = 2x^2 + 8x - 1$?

The axis of symmetry is the vertical line $x = -b/(2a)$. For this function, $x = -8/(22) = -2$.

What is the y-intercept of $G(x) = 2x^2 + 8x - 1$?

The y-intercept occurs when $x=0$. Plugging in, $G(0) = 2(0)^2 + 8(0) - 1 = -1$. So, the y-intercept is at $(0, -1)$.

How do I find the roots (x-intercepts) of $G(x) = 2x^2 + 8x - 1$?

Set $G(x) = 0$ and solve for x: $2x^2 + 8x - 1 = 0$. Use the quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}]/(2a)$. Here, $a=2$, $b=8$, $c=-1$. Discriminant $D = 8^2 - 42(-1) = 64 + 8 = 72$. Roots: $x = [-8 \pm \sqrt{72}]/4 = [-8 \pm 6\sqrt{2}]/4 = [-8/4] \pm [6\sqrt{2}/4] = -2 \pm (3\sqrt{2})/2$.

What is the discriminant of the quadratic $G(x) = 2x^2 + 8x - 1$, and what does it tell us?

The discriminant $D = 72$. Since it's positive, the quadratic has two real roots.

How does the coefficient 'a' affect the graph of $G(x) = 2x^2 + 8x - 1$?

The coefficient 'a' = 2 determines the parabola's opening direction (upward) and its width; larger $|a|$ makes the parabola narrower.

Can I determine the maximum or minimum value of $G(x)$?

Since the parabola opens upward ($a > 0$), it has a minimum point at the vertex. The minimum value is $y = -9$ at $x = -2$.

How do I graph $G(x) = 2x^2 + 8x - 1$ accurately?

Plot the vertex at $(-2, -9)$, mark the y-intercept at $(0, -1)$, find the roots using the quadratic formula, and plot these points. Draw a smooth parabola passing through them, ensuring symmetry about the axis $x = -2$.

Additional Resources

Graph The Function $G(x) = 2x^2 + 8x - 1$: A Comprehensive Analysis

Understanding the behavior of quadratic functions is a fundamental aspect of algebra and calculus that opens doors to numerous applications in science, engineering, and everyday problem-solving. The function $G(x) = 2x^2 + 8x - 1$ serves as an excellent example to explore the key features of a quadratic graph, including its shape, vertex, axis of symmetry, and intercepts. In this detailed guide, we will dissect the graph of G(x), providing step-by-step insights to deepen your understanding of quadratic functions and their graphical representations.

Introduction to Quadratic Functions

Quadratic functions are polynomial functions of degree two, generally expressed in the form:

$f(x) = ax^2 + bx + c$

where:
- a, b, and c are constants,
- $a \neq 0$.

The graph of a quadratic function is a parabola, which can open upwards or downwards depending on the sign of a. For $G(x) = 2x^2 + 8x - 1$, since $a = 2 > 0$, the parabola opens upward.

Key Features of the Graph of G(x)

To understand the graph of $G(x) = 2x^2 + 8x - 1$, we analyze several core components:

1. Vertex of the Parabola

The vertex is the highest or lowest point on the parabola, depending on its orientation. For an upward-opening parabola, the vertex is the minimum point.

Finding the Vertex:

The x-coordinate of the vertex is given by:

xv = -b / (2a)

For G(x):

xv = -8 / (2 2) = -8 / 4 = -2

To find the y-coordinate:

$G(-2) = 2(-2)^2 + 8(-2) - 1$
$= 2(4) + (-16) - 1$
$= 8 - 16 - 1$
$= -9$

Vertex Coordinates:

(-2, -9)

2. Axis of Symmetry

The axis of symmetry is a vertical line that passes through the vertex, dividing the parabola into mirror images.

Equation:

$x = xv = -2$

3. Y-Intercept

The y-intercept occurs when $x = 0$:

$G(0) = 2(0)^2 + 8(0) - 1 = -1$

Y-intercept:

(0, -1)

4. X-Intercepts (Roots or Zeros)

Find the x-values where $G(x) = 0$:

$2x^2 + 8x - 1 = 0$

Using the quadratic formula:

$x = [-b \pm \sqrt{b^2 - 4ac}] / (2a)$

Plugging in the values:

$x = [-8 \pm \sqrt{8^2 - 4\ 2\ (-1)}] / (2\ 2)$

$x = [-8 \pm \sqrt{64 + 8}] / 4$

$x = [-8 \pm \sqrt{72}] / 4$

$x = [-8 \pm 6\sqrt{2}] / 4$

Simplify:

$x = [-8 / 4] \pm [6\sqrt{2} / 4]$
$x = -2 \pm (3\sqrt{2}) / 2$

Approximate values:

$x \approx -2 + 2.1213 / 2 \approx -2 + 1.0607 \approx -0.9393$

$x \approx -2 - 2.1213 / 2 \approx -2 - 1.0607 \approx -3.0607$

X-intercepts:

Approximately at (-3.06, 0) and (-0.94, 0)

Graphical Representation and Interpretation

Visualizing $G(x) = 2x^2 + 8x - 1$ involves plotting its key features and sketching the parabola:

- The vertex at (-2, -9) indicates the lowest point.
- The axis of symmetry at $x = -2$.
- The y-intercept at (0, -1).
- The x-intercepts at approximately (-3.06, 0) and (-0.94, 0).

Given the positive leading coefficient ($a = 2$), the parabola opens upward, with the arms extending infinitely in both directions.

Step-by-Step Graph Construction

Step 1: Plot the Vertex

Mark the point (-2, -9) on the coordinate plane.

Step 2: Draw the Axis of Symmetry

Draw a dashed vertical line at $x = -2$.

Step 3: Plot Intercepts

- Plot the y-intercept at (0, -1).
- Plot the approximate x-intercepts at (-3.06, 0) and (-0.94, 0).

Step 4: Select Additional Points

Choose x-values around the vertex to get a smooth curve:

- For example, $x = -3$, $x = -1$, $x = 1$

Calculate their corresponding y-values:

- G(-3):

$2(-3)^2 + 8(-3) -1 = 2(9) - 24 - 1 = 18 - 24 - 1 = -7$

- G(-1):

$2(-1)^2 + 8(-1) -1 = 2(1) - 8 - 1 = 2 - 8 - 1 = -7$

- G(1):

$2(1)^2 + 8(1) - 1 = 2(1) + 8 - 1 = 2 + 8 - 1 = 9$

Plot these points:

(-3, -7), (-1, -7), (1, 9)

Step 5: Sketch the Parabola

Connect all points smoothly, ensuring the parabola opens upward, with symmetry about $x = -2$.

Applications and Implications

Understanding the graph of $G(x) = 2x^2 + 8x - 1$ is not just an academic exercise. Such quadratic functions model real-world phenomena such as projectile motion, profit maximization in economics, and optimization problems.

Real-World Examples:

- Physics: The trajectory of an object under gravity (parabolic path).
- Economics: Cost or revenue functions with quadratic relationships.
- Engineering: Structural load distributions modeled by quadratic equations.

Summary of Key Takeaways

- The vertex provides the minimum point at (-2, -9).
- The axis of symmetry is the vertical line $x = -2$.
- The x-intercepts are approximately at (-3.06, 0) and (-0.94, 0).
- The y-intercept is at (0, -1).
- The graph opens upward due to the positive leading coefficient.

Final Thoughts

Mastering the analysis of quadratic functions like $G(x) = 2x^2 + 8x - 1$ equips you with essential tools for mathematical modeling, problem-solving, and critical thinking. By understanding how to identify key features, interpret the shape, and sketch the graph, you develop a deeper intuition for the behavior of parabolas and their applications across various fields.

Whether you're tackling algebra homework, preparing for calculus, or applying mathematics in real-world scenarios, this comprehensive approach ensures you grasp the foundational concepts and can confidently analyze quadratic `functions.`

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