If 1-cosA = 1/2 , Then Find The Value Of SinA.

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Understanding the relationship between the trigonometric functions sine and cosine is fundamental in trigonometry. Problems that involve given values of these functions often require the application of identities, algebraic manipulation, and sometimes the use of special angles to determine unknown quantities. One such problem is: If 1 - cosA = 1/2, then find the value of sinA. This question not only tests your ability to manipulate trigonometric expressions but also deepens your understanding of identities and the Pythagorean theorem.

In this comprehensive article, we will explore the problem step-by-step, covering the necessary background, relevant identities, solution methods, and related concepts. By the end, you will have a clear understanding of how to approach such problems and how to apply the fundamental principles of trigonometry to find the value of sinA when given a specific expression involving cosA.

Understanding the Problem

The problem states: If 1 - cosA = 1/2, then find the value of sinA.

This involves a direct relationship between the cosine function and the sine function at a specific angle A. The goal is to find the value of sinA based on the given expression involving cosA.

Key Concepts and Identities in Trigonometry

Before solving, it is essential to understand some fundamental identities and concepts.

1. Pythagorean Identity

The core identity connecting sine and cosine functions is:

\[
\sin^2A + \cos^2A = 1
\]

This identity is pivotal in relating the two functions and solving for unknowns.

2. Range of Trigonometric Functions

- \(\cosA\) can range from -1 to 1.
- \(\sinA\) can range from -1 to 1.

Knowing these ranges helps determine possible solutions after calculation.

3. Algebraic Manipulation

Often, problems involving trigonometric functions require algebraic rearrangement to isolate the desired variable.

Step-by-Step Solution Approach

Let's proceed to solve the problem systematically.

Step 1: Express \(\cosA\) using the given equation

Given:

\[
1 - \cosA = \frac{1}{2}
\]

Rearranged to find \(\cosA\):

\[
\cosA = 1 - \frac{1}{2} = \frac{1}{2}
\]

So,

\[
\boxed{\cosA = \frac{1}{2}}

\]

Step 2: Use the Pythagorean identity to find \(\sinA\)

From the identity:

\[
\sin^2A + \cos^2A = 1
\]

Substitute \(\cosA = \frac{1}{2}\):

\[
\sin^2A + \left(\frac{1}{2}\right)^2 = 1
\]
\[
\sin^2A + \frac{1}{4} = 1
\]
\[
\sin^2A = 1 - \frac{1}{4} = \frac{3}{4}
\]

Therefore:

\[
\sinA = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}
\]

Final Result and Interpretation

Based on the calculations, the possible values for \(\sinA\) are:

\[
\boxed{
\sinA = \pm \frac{\sqrt{3}}{2}
}
\]

This indicates two possible solutions for \(\sinA\), corresponding to different quadrants where \(\cosA = \frac{1}{2}\):

- When \(\cosA = \frac{1}{2}\), A could be in the first quadrant (\(0^\circ < A < 90^\circ\)) where \(\sinA > 0\), or in the fourth quadrant (\(270^\circ <

A < 360^\circ\)) where \(\sinA < 0\).

Quadrant Analysis:

- First Quadrant (\(0^\circ < A < 90^\circ\)): \(\sinA = \frac{\sqrt{3}}{2}\)
- Fourth Quadrant (\(270^\circ < A < 360^\circ\)): \(\sinA = -\frac{\sqrt{3}}{2}\)

Visualizing the Solution with a Unit Circle

The unit circle provides a geometric interpretation of the values:

- \(\cosA = \frac{1}{2}\) corresponds to angles where the x-coordinate of the point on the circle is \(\frac{1}{2}\).
- These angles are \(A = 60^\circ\) (or \(\pi/3\) radians) and \(300^\circ\) (or \(5\pi/3\) radians).
- At \(60^\circ\):

\[
\sinA = \frac{\sqrt{3}}{2}
\]

- At \(300^\circ\):

\[
\sinA = -\frac{\sqrt{3}}{2}
\]

Thus, the solutions align with the geometric interpretation.

Additional Topics and Related Problems

Understanding this problem opens doors to various related concepts in trigonometry.

1. Solving for \(\tanA\)

Once \(\sinA\) and \(\cosA\) are known, \(\tanA\) can be easily calculated:

\[
\tanA = \frac{\sinA}{\cosA}

\]

For $A = 60^\circ$:

$$
\tan60^\circ = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}
$$

For $A = 300^\circ$:

$$
\tan300^\circ = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}
$$

2. Inverse Trigonometric Functions

Understanding how to find angles from given sine or cosine values:

- $\sin A = \frac{\sqrt{3}}{2}$ implies $A = 60^\circ$ or 120° (since sine is positive in first and second quadrants).
- $\cos A = \frac{1}{2}$ implies $A = 60^\circ$ or 300°.

Common Mistakes to Avoid

When solving such problems, be cautious of:

- Ignoring the $\pm$ sign: Both positive and negative solutions are valid unless specified.
- Confusing angles in different quadrants: Recall the signs of sine and cosine in each quadrant.
- Forgetting the range of functions: Ensure solutions are within the standard range or specified domain.

Practice Problems for Mastery

To reinforce understanding, consider practicing the following:

1. Given that $\sin A = \frac{3}{5}$, find $\cos A$ and verify the Pythagorean identity.

2. If $\tanA = 1$, find all possible values of $\sinA$ and $\cosA$.

3. Prove that $\sin^2A + \cos^2A = 1$ for angles in different quadrants.

4. Given $\secA = 2$, find $\sinA$ and $\tanA$.

Summary and Conclusion

In conclusion, when given that:

$$
1 - \cosA = \frac{1}{2}
$$

we immediately deduce:

$$
\cosA = \frac{1}{2}
$$

Applying the Pythagorean identity yields:

$$
\sinA = \pm \frac{\sqrt{3}}{2}
$$

The sign depends on the quadrant where the angle A resides. The solutions are consistent with the unit circle and fundamental trigonometric identities.

This problem exemplifies the importance of understanding identities, algebraic manipulation, and geometric interpretation in trigonometry. Mastery of these concepts enables solving a wide range of problems involving angles and their trigonometric functions efficiently and accurately.

Remember: Always verify your solutions within the context of the problem and the domain of the functions involved.

Frequently Asked Questions

If 1 - cos A = 1/2, what is the value of sin A?

The value of sin A is $1/\sqrt{2}$ or approximately 0.707.

Given 1 - cos A = 1/2, how can we find sin A using trigonometric identities?

From 1 - cos A = 1/2, we get cos A = 1/2. Using $\sin^2 A + \cos^2 A = 1$, then $\sin A = \pm\sqrt{(1 - (1/2)^2)} = \pm\sqrt{(1 - 1/4)} = \pm\sqrt{(3/4)} = \pm(\sqrt{3}/2)$.

What are the possible values of sin A if 1 - cos A = 1/2?

The possible values of sin A are $\pm\sqrt{3}/2$, i.e., approximately ±0.866.

If cos A = 1/2, what are the corresponding values of sin A?

Corresponding values of sin A are $\pm\sqrt{3}/2$, since $\sin^2 A = 1 - \cos^2 A = 1 - (1/2)^2 = 3/4$.

How do the values of A relate to standard angles in this problem?

Since cos A = 1/2, A could be 60° or 300° (or $\pi/3$ or $5\pi/3$ radians). For these angles, sin A is $\sqrt{3}/2$ or $-\sqrt{3}/2$ respectively.

Is the value of sin A positive or negative in this case?

It depends on the quadrant: sin A is positive in the first quadrant (A = 60°) and negative in the fourth quadrant (A = 300°).

Can 1 - cos A = 1/2 be used to find other trigonometric functions of A?

Yes, knowing cos A allows calculation of sin A, and subsequently tangent, cotangent, secant, and cosecant as needed.

What is the significance of the given equation in solving trigonometric problems?

It helps identify the specific angles and their sine values, which are fundamental in solving various trigonometric equations and real-world applications.

Additional Resources

If 1 - cos A = 1/2, then find the value of sin A is a classic trigonometric problem that often appears in mathematics exams and practice tests. This question is not only fundamental for understanding the relationships between sine and cosine functions but also offers insight into the application of trigonometric identities. Solving such problems enhances one's grasp of the unit circle, identities, and the interplay between different trigonometric functions, making it a valuable exercise for students and enthusiasts alike.

Understanding the Problem

The problem states: If 1 - cos A = 1/2, then find the value of sin A. At first glance, this appears straightforward, but it requires a systematic approach to decipher the relationship between the given expression and the desired quantity.

The key here is to recognize that the given expression involves the cosine of angle A, and we need to find the sine of the same angle. Since sine and cosine are related through fundamental identities, the problem invites us to employ these identities for a solution.

Fundamental Trigonometric Identities Involved

Before diving into the solution, it is essential to recall some core identities:

Pythagorean Identity

- \[
\sin^2 A + \cos^2 A = 1
\]
This identity relates sine and cosine directly, allowing us to find one if the other is known.

Half-Angle and Double-Angle Formulas

- \[
1 - \cos A = 2 \sin^2 \frac{A}{2}
\]
- \[

$$
1 + \cos A = 2 \cos^2 \frac{A}{2}
$$
These formulas are particularly useful when dealing with expressions involving $1 - \cos A$.

Step-by-Step Solution Approach

To find $\sin A$ given $1 - \cos A = \frac{1}{2}$, let's follow a systematic approach.

Step 1: Express $1 - \cos A$ using the half-angle formula

Recall that:
$$
1 - \cos A = 2 \sin^2 \frac{A}{2}
$$
Given:
$$
1 - \cos A = \frac{1}{2}
$$
Thus:
$$
2 \sin^2 \frac{A}{2} = \frac{1}{2}
$$
Dividing both sides by 2:
$$
\sin^2 \frac{A}{2} = \frac{1}{4}
$$

Step 2: Find $\sin \frac{A}{2}$

Since $\sin^2 \frac{A}{2} = \frac{1}{4}$, then:
$$
\sin \frac{A}{2} = \pm \frac{1}{2}
$$

The sign depends on the quadrant of $A/2$. Without additional information about the angle A, both positive and negative solutions are possible. However, in most contexts, angles are considered within the principal range $0^\circ \leq A < 360^\circ$, and particular emphasis is placed on positive values unless specified otherwise.

Determining the Range of $\sin A$

To find $\sin A$, we need to relate $\sin A$ to $\sin \frac{A}{2}$.

Step 3: Use the double-angle formula for sine

Recall:
$$
\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}
$$
We already have $\sin \frac{A}{2} = \pm \frac{1}{2}$.

Next, we need $\cos \frac{A}{2}$. Using the Pythagorean identity:
$$
\cos^2 \frac{A}{2} = 1 - \sin^2 \frac{A}{2}
$$
which gives:
$$
\cos^2 \frac{A}{2} = 1 - \left(\pm \frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}
$$
Therefore:
$$
\cos \frac{A}{2} = \pm \frac{\sqrt{3}}{2}
$$

Again, the sign depends on the quadrant of $\frac{A}{2}$.

Calculating $\sin A$

Now, applying the double-angle formula:
$$
\sin A = 2 \times \left(\pm \frac{1}{2}\right) \times \left(\pm \frac{\sqrt{3}}{2}\right)
$$
Multiplying:
$$
\sin A = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times (\pm \times \pm)
$$
Simplify:

$$
\sin A = 1 \times \frac{\sqrt{3}}{2} \times (\pm \times \pm)
$$

$$
\sin A = \frac{\sqrt{3}}{2} \times (\pm \times \pm)
$$

The product of the signs determines the sign of $\sin A$:

- If both $\sin \frac{A}{2}$ and $\cos \frac{A}{2}$ are positive, then $\sin A = \frac{\sqrt{3}}{2}$.
- If one is positive and the other negative, then $\sin A = - \frac{\sqrt{3}}{2}$.

Significance of Quadrants and Final Values

The signs of $\sin \frac{A}{2}$ and $\cos \frac{A}{2}$ depend on the quadrant in which $\frac{A}{2}$ lies.

- If $\sin \frac{A}{2} = \frac{1}{2}$ and $\cos \frac{A}{2} = \frac{\sqrt{3}}{2}$:

$\frac{A}{2}$ is in the first quadrant (0° to 90°), so A is in the first or second quadrant (0° to 180°).

$\sin A = \frac{\sqrt{3}}{2}$

- If $\sin \frac{A}{2} = -\frac{1}{2}$ and $\cos \frac{A}{2} = \frac{\sqrt{3}}{2}$:

$\frac{A}{2}$ is in the fourth quadrant (270° to 360°), which is less common for typical angle ranges, but still valid mathematically.

$\sin A = - \frac{\sqrt{3}}{2}$

Similarly, other combinations produce corresponding signs.

Final Answer and Interpretation

The possible values for $\sin A$ are:

$$
\boxed{
\sin A = \pm \frac{\sqrt{3}}{2}
}
$$

The sign depends on the specific quadrant of A.

In most standard problems, unless otherwise specified, the principal solution is:

$$
\boxed{
\sin A = \frac{\sqrt{3}}{2}
}
$$

which corresponds to angles where $\sin A$ is positive, typically in the first or second quadrants.

Additional Insights and Common Pitfalls

Pros of this approach:

- Utilizes fundamental identities, making the solution systematic and reliable.
- Demonstrates the power of half-angle formulas in solving inverse problems.
- Reinforces understanding of the unit circle and quadrant considerations.

Cons / Challenges:

- The sign ambiguity can sometimes lead to confusion if the quadrant isn't specified.
- Misapplication of identities could lead to incorrect solutions.
- Assumes familiarity with half-angle formulas; beginners might need extra practice.

Features:

- Clear step-by-step methodology.
- Emphasizes the importance of understanding the geometric interpretation of identities.
- Offers a comprehensive solution that can be adapted for similar problems involving other trigonometric functions.

Conclusion

The problem "If 1 - cos A = 1/2, then find the value of sin A" beautifully illustrates how fundamental trigonometric identities interconnect and can be employed to derive unknown values. By leveraging the half-angle formula, we deduced that:

$$
\sin A = \pm \frac{\sqrt{3}}{2}
$$

depending on the quadrant. This problem not only enhances problem-solving skills but also deepens one's understanding of the relationships between sine and cosine functions within the unit circle framework. Mastering such problems prepares students for a wide range of advanced topics in trigonometry and calculus, making it a vital part of mathematical education.

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