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Introduction

Mathematics often presents us with intriguing equations that challenge our understanding and problem-solving skills. One such equation is:

$$ 4x^2 - x - 4 = 0 $$

This quadratic equation serves as a foundation for exploring various algebraic manipulations and concepts. The primary goal here is to determine the value of the expression:

$$ x - \frac{1}{x} $$

This problem not only reinforces the application of quadratic equations but also introduces the idea of transforming expressions to simplify complex algebraic fractions. Whether you're a student preparing for exams, a teacher designing problem sets, or an enthusiast exploring algebraic relationships, understanding how to approach such questions is essential.

In this article, we will delve into the detailed steps to find the value of $x - \frac{1}{x}$ given the quadratic equation, exploring multiple methods including substitution, algebraic manipulation, and the use of quadratic formulas. We will also discuss related concepts for a comprehensive understanding of the problem.

Understanding the Given Quadratic Equation

The Quadratic Equation: $4x^2 - x - 4 = 0$

This is a standard quadratic form:

$$ ax^2 + bx + c = 0 $$

where:

- $a = 4$
- $b = -1$

- $c = -4$

Our goal is to find the value of an expression involving x, but first, we need to understand the nature of x by solving the quadratic.

Solving the Quadratic Equation

Quadratic equations can be solved using various methods:

- Factoring
- Completing the square
- Quadratic formula

Given the coefficients, the most straightforward method here is to apply the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Applying the Quadratic Formula

Let's substitute the known values:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 4 \times (-4)}}{2 \times 4}$$

Simplify numerator:

$$x = \frac{1 \pm \sqrt{1 - 4 \times 4 \times (-4)}}{8}$$

Calculate the discriminant:

$$D = 1 - 4 \times 4 \times (-4)$$

$$D = 1 - (-64)$$

$$D = 1 + 64 = 65$$

Therefore,

$$x = \frac{1 \pm \sqrt{65}}{8}$$

The solutions are:

$$ x = \frac{1 + \sqrt{65}}{8} \quad \text{or} \quad x = \frac{1 - \sqrt{65}}{8} $$

Finding the Value of $x - \frac{1}{x}$

Now that we know the solutions for x, we can proceed to find the value of the expression:

$$ x - \frac{1}{x} $$

This expression involves both x and its reciprocal, which suggests that algebraic manipulation can lead us to a simplified form.

Approach 1: Direct Substitution

Using the solutions for x, substitute into the expression:

$$ x - \frac{1}{x} $$

but this might be cumbersome as it involves radicals. Alternatively, an algebraic approach that does not depend on the specific roots is preferred.

Approach 2: Algebraic Manipulation

We can manipulate the given quadratic equation to relate to $x - \frac{1}{x}$.

Deriving $x - \frac{1}{x}$ from the Quadratic Equation

Step 1: Express $x + \frac{1}{x}$

It is often easier to work with sums or differences involving x and $\frac{1}{x}$. To find $x - \frac{1}{x}$, we can consider the related expression:

$$ x + \frac{1}{x} $$

which can be linked to the quadratic equation.

Step 2: Multiply the original equation by $\frac{1}{x}$

Starting from:

$$ 4x^2 - x - 4 = 0 $$

Divide through by x (assuming $x \neq 0$):

$$ 4x - 1 - \frac{4}{x} = 0 $$

Rearranged as:

$$ 4x - \frac{4}{x} = 1 $$

Divide both sides by 4:

$$ x - \frac{1}{x} = \frac{1}{4} $$

This is a pivotal step: we've directly expressed $x - \frac{1}{x}$ in terms of a constant.

Final Answer

From the above derivation, the value of $x - \frac{1}{x}$ is:

$$ \boxed{\frac{1}{4}} $$

This remarkable result shows that regardless of which root x takes (since the derivation is based on the original quadratic), the value of $x - \frac{1}{x}$ is $\frac{1}{4}$.

Additional Insights and Verification

Verifying the Result

To ensure the correctness of this conclusion, let's verify by substituting the roots back into the expression.

For $x = \frac{1 + \sqrt{65}}{8}$:

Calculate $x - \frac{1}{x}$:

- $x = \frac{1 + \sqrt{65}}{8}$
- $\frac{1}{x} = \frac{8}{1 + \sqrt{65}}$

Rationalize the denominator:

$$ \frac{8}{1 + \sqrt{65}} \times \frac{1 - \sqrt{65}}{1 - \sqrt{65}} = \frac{8(1 - \sqrt{65})}{(1)^2 - (\sqrt{65})^2} = \frac{8(1 - \sqrt{65})}{1 - 65} = \frac{8(1 - \sqrt{65})}{-64} $$

Simplify numerator:

$$ \frac{8(1 - \sqrt{65})}{-64} = -\frac{8(1 - \sqrt{65})}{64} = -\frac{1 - \sqrt{65}}{8} $$

Now, compute:

$$ x - \frac{1}{x} = \frac{1 + \sqrt{65}}{8} - \left(-\frac{1 - \sqrt{65}}{8} \right) = \frac{1 + \sqrt{65}}{8} + \frac{1 - \sqrt{65}}{8} = \frac{(1 + \sqrt{65}) + (1 - \sqrt{65})}{8} $$

Simplify numerator:

$$ 1 + \sqrt{65} + 1 - \sqrt{65} = 2 $$

Therefore:

$$ x - \frac{1}{x} = \frac{2}{8} = \frac{1}{4} $$

Similarly, for the other root, the value will also be $\frac{1}{4}$, confirming the consistency.

Conclusion

In this comprehensive exploration, we examined the quadratic equation:

$$ 4x^2 - x - 4 = 0 $$

and derived the solutions using the quadratic formula as:

$$ x = \frac{1 \pm \sqrt{65}}{8} $$

The key insight was to manipulate the original equation to directly find the value of the expression $x - \frac{1}{x}$. By multiplying the original equation by $\frac{1}{x}$ and simplifying, we found:

$$ x - \frac{1}{x} = \frac{1}{4} $$

This demonstrates the power of algebraic manipulation and substitution in solving equations involving complex expressions. The result is elegant and confirms that regardless of which root x represents, the value of $x - \frac{1}{x}$ remains constant at $\frac{1}{4}$.

Related Concepts and Tips for Students

- Quadratic Equations: Always consider solving quadratic equations using the quadratic formula, especially when factoring isn't straightforward.
- Expression Manipulation: When dealing with expressions like \(x - \frac{1}{x} \), try to relate them to the original equation or to known identities.
- Rationalization: Rationalizing denominators is crucial when simplifying expressions involving radicals.
- Verification: Always verify your solutions by substituting back into the original expressions to confirm correctness.
- Common Mistakes: Be cautious with signs, especially when applying the quadratic formula and rationalizing denominators.

Final Thoughts

Understanding how to manipulate quadratic equations and related expressions is fundamental in algebra. The ability to derive the value of complex expressions like \(x - \frac{1}{x} \) from given equations enhances problem-solving skills and deepens mathematical understanding. This approach not only applies to this specific problem but also prepares you for more advanced topics involving algebraic identities, equations, and functions.

Whether you're preparing

Frequently Asked Questions

Given the quadratic equation 4x^2 - x - 4 = 0, how do we find the value of (x - 1/x)?

First, solve for x using the quadratic formula, then compute (x - 1/x) with the obtained roots or express it in terms of x's sum and product to find its value.

Can we find the value of (x - 1/x) without explicitly solving for x in the quadratic equation 4x^2 - x - 4 = 0?

Yes, by using relationships between roots and coefficients, such as sum and product of roots, to express (x - 1/x) in terms of these quantities.

What is the sum and product of the roots for the quadratic 4x^2 - x - 4 = 0?

Sum of roots (x + y) = 1/4, and product of roots (xy) = -1.

How can we express (x - 1/x) in terms of x's sum and product?

Note that (x - 1/x) = (x^2 - 1)/x. Using the roots, you can relate this to the sum and product to find its value without solving for x explicitly.

What is the value of (x - 1/x) if x satisfies 4x^2 - x - 4 = 0?

The value of (x - 1/x) is $\pm\sqrt{5}$, depending on the root chosen, as derived from the quadratic relationships.

Is the value of (x - 1/x) the same for both roots of the quadratic equation?

No, since the roots are different, (x - 1/x) will generally have different values for each root, which are $\pm\sqrt{5}$.

How do you verify the value of (x - 1/x) for solutions of 4x^2 - x - 4 = 0?

By solving the quadratic for x, then substituting back into (x - 1/x), or by using algebraic identities to find the consistent value across roots, confirming it equals $\pm\sqrt{5}$.

Additional Resources

If 4x^2 - x - 4 = 0, Then Find The Value Of (x - 1/x)

Understanding the relationship between a quadratic equation and the expression involving a reciprocal, such as (x - 1/x), is a classic problem that combines algebraic manipulation with deeper insights into quadratic roots. The equation $4x^2 - x - 4 = 0$ presents an excellent case for exploring how the roots of a quadratic can be used to evaluate more complex expressions. This problem not only tests your ability to manipulate equations but also encourages strategic substitution and understanding of symmetrical relationships between roots and coefficients. In this comprehensive review, we will delve into solving the quadratic, deriving the value of (x - 1/x), and exploring various methods, advantages, and potential pitfalls associated with such problems.

Understanding the Given Quadratic Equation

The starting point for solving this problem is to analyze the quadratic equation:

$$ 4x^2 - x - 4 = 0 $$

This is a standard quadratic equation with coefficients:

- $a = 4$
- $b = -1$
- $c = -4$

The first step is to recall the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

which will help us find the roots explicitly if needed. Alternatively, properties of roots such as sum and product can be leveraged to simplify the process of finding the value of (x - 1/x) without directly computing the roots.

Solving the Quadratic Equation

Using the Quadratic Formula

Applying the quadratic formula:

$$
x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 4 \times (-4)}}{2 \times 4}
$$

Simplify numerator:

$$
x = \frac{1 \pm \sqrt{1 - (-64)}}{8}
$$

Since subtracting a negative is adding:

$$
x = \frac{1 \pm \sqrt{1 + 64}}{8} = \frac{1 \pm \sqrt{65}}{8}
$$

Thus, the roots are:

$$
x_1 = \frac{1 + \sqrt{65}}{8}, \quad x_2 = \frac{1 - \sqrt{65}}{8}
$$

While explicitly calculating roots is possible, in many problems involving expressions like (x - 1/x), it's more efficient to use relationships between roots rather than their explicit values.

Using Sum and Product of Roots

From the quadratic, we know:

- Sum of roots: $x_1 + x_2 = -\frac{b}{a} = -\frac{-1}{4} = \frac{1}{4}$
- Product of roots: $x_1 \times x_2 = \frac{c}{a} = \frac{-4}{4} = -1$

These relationships are crucial in simplifying the target expression without explicitly solving for the roots.

Finding the Value of (x - 1/x)

The problem asks for the value of $x - \frac{1}{x}$. To evaluate this expression, especially given the quadratic roots, a strategic approach is to manipulate the expression algebraically and relate it to known sums and products of roots.

Method 1: Using Algebraic Manipulation

Starting with:

$$
x - \frac{1}{x}
$$

Combine into a single fraction:

$$
\frac{x^2 - 1}{x}
$$

Note that:

$$
x^2 - 1 = (x - 1)(x + 1)
$$

But directly substituting roots might be complex. A better approach is to find $x - \frac{1}{x}$ in terms of $x + \frac{1}{x}$, which can be related to the sum and product of roots.

Alternatively, square the expression:

$$
\left(x - \frac{1}{x} \right)^2 = x^2 - 2 + \frac{1}{x^2}
$$

which simplifies to:

$$
x^2 + \frac{1}{x^2} - 2
$$

If we can find $x^2 + \frac{1}{x^2}$, then we can determine $x - \frac{1}{x}$.

Method 2: Using the Quadratic Equation to Find $x + \frac{1}{x}$

We can relate $x + \frac{1}{x}$ to the roots:

- From the quadratic, the roots satisfy the original equation.
- Rewrite the original quadratic in terms of x:

$$
4x^2 - x - 4 = 0
$$

Divide through by x (assuming $x \neq 0$):

$$
4x - 1 - \frac{4}{x} = 0
$$

Rearranged:

$$
4x - \frac{4}{x} = 1
$$

Divide both sides by 4:

$$
x - \frac{1}{x} = \frac{1}{4}
$$

This is an elegant shortcut! The key insight is that the original quadratic's coefficients lead to this relation directly. Therefore:

$$
x - \frac{1}{x} = \frac{1}{4}
$$

This means the value of $x - \frac{1}{x}$ is $\boxed{\frac{1}{4}}$.

Verification and Alternative Approaches

While the shortcut above provides a quick answer, it's instructive to verify or explore alternative methods to reinforce understanding.

Verification via Roots

Recall the roots:

$$
x_{1,2} = \frac{1 \pm \sqrt{65}}{8}
$$

Calculate $x - 1/x$ for each root:

$$
x_{1} - \frac{1}{x_{1}} = \frac{1 + \sqrt{65}}{8} - \frac{1}{\frac{1 + \sqrt{65}}{8}} = \frac{1 + \sqrt{65}}{8} - \frac{8}{1 + \sqrt{65}}
$$

Rationalize the denominator:

$$
\frac{8}{1 + \sqrt{65}} \times \frac{1 - \sqrt{65}}{1 - \sqrt{65}} = \frac{8(1 - \sqrt{65})}{(1)^2 - (\sqrt{65})^2} = \frac{8(1 - \sqrt{65})}{1 - 65} = \frac{8(1 - \sqrt{65})}{-64} = -\frac{1 - \sqrt{65}}{8}
$$

Now:

$$
x_{1} - \frac{1}{x_{1}} = \frac{1 + \sqrt{65}}{8} - \left(-\frac{1 - \sqrt{65}}{8} \right) = \frac{1 + \sqrt{65}}{8} + \frac{1 - \sqrt{65}}{8} = \frac{(1 + \sqrt{65}) + (1 - \sqrt{65})}{8} = \frac{2}{8} = \frac{1}{4}
$$

Similarly, for x_2, the expression yields the same value, confirming the validity of the shortcut approach.

Features, Pros, and Cons of the Methods

Features of the Shortcut Method (Dividing the Quadratic by x):

- Pros:
- Quick and efficient.
- Requires no explicit calculation of roots.
- Leverages algebraic relationships directly from the original equation.
- Cons:
- Requires the assumption $x \neq 0$.
- May not be intuitive for beginners unfamiliar with dividing equations.

Features of the Root-Based Calculation:

- Pros:

- Provides explicit verification.
- Reinforces understanding of roots and their relationships.
- Cons:
- More algebraically intensive.
- Less efficient for quick solutions.

Conclusion and Final Remarks

The problem of determining $x - \frac{1}{x}$ when $4x^2 - x - 4 = 0$ beautifully illustrates the power of algebraic manipulation and the importance of understanding the relationships between roots and coefficients. By leveraging the quadratic's structure, we discovered that:

$$
x - \frac{1}{x} = \frac{1}{4}
$$

This concise result exemplifies how sometimes a strategic algebraic step—dividing the original quadratic by x—can yield solutions more straightforwardly than direct root calculation.

In summary:

- The roots of the quadratic are $\frac{1 \pm \sqrt{65}}{8}$.
- The value of $x - 1/x$ can be obtained elegantly by dividing the quadratic by x, leading to a simple relation.
- The final answer is

If 4x2 X 4 0 Then Find The Value Of X 1 X

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