If A1=1 And An =-5an-1 Find The Value Of A6

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Understanding how to find the value of A6 in a recursive sequence defined by the relation $A_n = -5A_{n-1}$, given that $A_1=1$, is an essential concept in sequences and series in mathematics. This problem involves recognizing the pattern of the sequence, deriving a general formula, and then calculating the specific term A6. In this article, we will explore step-by-step how to approach this problem, including understanding recursive relations, deriving explicit formulas, and calculating specific sequence values.

Understanding Recursive Sequences

What Is a Recursive Sequence?

A recursive sequence is a sequence in which each term is defined in terms of one or more previous terms. The defining relation, called the recurrence relation, provides a way to generate successive terms from initial values.

For example, the sequence given by:
- $A_1 = 1$
- $A_n = -5A_{n-1}$ for $n > 1$

is a recursive sequence because each term after the first is generated by multiplying the previous term by -5.

The Importance of Initial Conditions

The initial term $A_1=1$ is critical because it allows the entire sequence to be constructed precisely. Without this initial condition, the sequence would be incomplete. The initial value serves as the seed from which all subsequent terms grow or diminish according to the recurrence relation.

Deriving a General Formula for A_n

Analyzing the Pattern

Let's examine the first few terms to recognize the pattern:
- $A_1 = 1$

- $A_2 = -5 A_1 = -5 \ 1 = -5$
- $A_3 = -5 A_2 = -5 (-5) = 25$
- $A_4 = -5 A_3 = -5 \ 25 = -125$
- $A_5 = -5 A_4 = -5 (-125) = 625$

Notice the pattern:
- The signs alternate between positive and negative.
- The absolute values follow powers of 5: 1, 5, 25, 125, 625, which are 5^0, 5^1, 5^2, 5^3, 5^4.

This suggests that the sequence can be expressed in a general formula involving powers of 5 and alternating signs.

Formulating the Explicit Expression

From the pattern observed:
- $A_1 = 1 = (-1)^0 5^{0}$
- $A_2 = -5 = (-1)^1 5^{1}$
- $A_3 = 25 = (-1)^2 5^{2}$
- $A_4 = -125 = (-1)^3 5^{3}$
- $A_5 = 625 = (-1)^4 5^{4}$

Thus, the general term A_n can be written as:
$$A_n = (-1)^{n-1} \times 5^{n-1}$$

This formula allows us to compute any term directly without recursively calculating all previous terms.

Calculating A6 Using the Derived Formula

Applying the Formula

Using:
$$A_n = (-1)^{n-1} \times 5^{n-1}$$
we can find A_6:
$$A_6 = (-1)^{6-1} \times 5^{6-1}$$
$$A_6 = (-1)^5 \times 5^5$$

Calculate each part:
- $(-1)^5 = -1$
- $5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$

Therefore:
$$A_6 = -1 \times 3125 = -3125$$

Final Value of A6

The value of A6 in the sequence is:
A6 = -3125

Summary and Key Takeaways

- Recursive sequences are defined by a relation that expresses each term in terms of previous terms.
- Initial conditions are crucial for generating the sequence accurately.
- Identifying patterns in the sequence can help derive a general explicit formula.
- In this specific sequence, the explicit formula is $A_n = (-1)^{n-1} \times 5^{n-1}$.
- Using the formula, we found that A6 = -3125.

Additional Tips for Solving Similar Problems

Identify the Pattern

Always start by computing the first few terms to see if there's a recognizable pattern in signs, magnitudes, or both.

Derive the Explicit Formula

Once the pattern is clear, aim to formulate an explicit expression for the nth term. This makes calculating specific terms much easier.

Check Your Work

Verify your formula by calculating the first few terms and ensuring they match the sequence's initial terms.

Conclusion

Understanding how to find specific terms in recursive sequences is a valuable skill in mathematics. By analyzing the sequence's pattern, deriving a general explicit formula, and applying it accurately, you can efficiently find any term, including A6 in this case. The sequence defined by $A_1=1$ and $A_n = -5A_{n-1}$ leads to the explicit formula $A_n = (-1)^{n-1} \times 5^{n-1}$, and thus, the value of A6 is -3125. Mastering these techniques enhances problem-solving skills in algebra and discrete mathematics, providing a strong foundation for more complex sequences and series problems.

Frequently Asked Questions

What is the given recurrence relation in the problem?

The recurrence relation is A1 = 1 and An = -5 A(n-1) for n > 1.

How do you find the value of A6 using the recurrence relation?

You start with A1 = 1 and repeatedly apply the relation An = -5 A(n-1) to find subsequent terms until A6.

What is the value of A2 in the sequence?

A2 = -5 A1 = -5 1 = -5.

How do we compute A3 given A2?

A3 = -5 A2 = -5 (-5) = 25.

What is the pattern for the sequence A1, A2, A3, ...?

The sequence follows A(n) = (-5)^(n-1) A1, which simplifies to A(n) = (-5)^(n-1).

What is the explicit formula for A6 based on the recurrence?

A6 = (-5)^(6-1) = (-5)^5.

What is the numerical value of A6?

A6 = (-5)^5 = -3125.

Additional Resources

If A1=1 And An = -5A(n-1) Find The Value Of A6

Introduction

In mathematical analysis, recursive sequences often serve as fundamental building blocks for understanding complex patterns and behaviors within algebra and calculus. One such intriguing sequence is defined by an initial value and a recurrence relation, which can reveal rich properties upon exploration. The problem at hand involves a sequence $\{A_n\}$ with a given initial term and a recursive formula:

$$
A_1 = 1 \quad \text{and} \quad A_n = -5A_{n-1} \quad \text{for} \quad n \geq 2
$$

The task is to determine A_6. While at first glance, this appears to be a straightforward recursive problem, delving deeper reveals insights into sequence behaviors, methods for solving recurrence relations, and understanding patterns in such sequences. This article provides a thorough exploration of this sequence, from basic definitions to detailed calculations, offering a comprehensive understanding suitable for students, educators, and enthusiasts alike.

Understanding the Sequence and Its Recurrence Relation

Initial Definitions and Context

The sequence $\{A_n\}$ is defined as follows:

- The first term: $A_1 = 1$
- The recursive rule: $A_n = -5A_{n-1}$ for $n \geq 2$

This recurrence relation indicates that each term is obtained by multiplying the previous term by -5. Such sequences are classified as linear homogeneous recurrence relations with constant coefficients.

Understanding the nature of the recurrence is essential because it hints at the sequence's behavior—whether it grows exponentially, oscillates, or converges. Since each term depends linearly on the previous one with a constant multiplier, the sequence will exhibit exponential growth or decay, combined with oscillation due to the negative sign.

Solving the Recurrence Relation

Method 1: Iterative Calculation

The most straightforward approach to find specific terms is to compute iteratively:

1. $A_1 = 1$
2. $A_2 = -5 \times A_1 = -5 \times 1 = -5$
3. $A_3 = -5 \times A_2 = -5 \times -5 = 25$
4. $A_4 = -5 \times A_3 = -5 \times 25 = -125$
5. $A_5 = -5 \times A_4 = -5 \times -125 = 625$
6. $A_6 = -5 \times A_5 = -5 \times 625 = -3125$

From this calculation, the value of A_6 is directly obtained as -3125.

Method 2: Closed-Form Expression

While iterative calculation is effective for small n, a closed-form expression helps analyze the sequence's behavior for larger n.

Given the recurrence:

$$
A_n = -5A_{n-1}
$$

and initial condition $A_1 = 1$, we recognize this as a geometric sequence. The general form of a geometric sequence is:

$$
A_n = A_1 \times r^{n-1}
$$

where r is the common ratio.

In this case, the common ratio:

$$
r = -5
$$

Therefore, the explicit formula for the sequence is:

$$
A_n = 1 \times (-5)^{n-1} = (-5)^{n-1}
$$

This formula allows for direct computation of any term without recursive calculation.

Verification of the Closed-Form Solution

To ensure the correctness of the explicit formula, we verify it for the first few terms:

- $A_1 = (-5)^{1-1} = (-5)^0 = 1$ (matches initial condition)
- $A_2 = (-5)^{2-1} = (-5)^1 = -5$ (matches iterative calculation)
- $A_3 = (-5)^{2} = 25$
- $A_4 = (-5)^3 = -125$
- $A_5 = (-5)^4 = 625$
- $A_6 = (-5)^5 = -3125$

The explicit formula aligns perfectly with the iterative approach, confirming its validity.

Implications and Sequence Behavior

Pattern Recognition

The sequence exhibits a pattern:

- Alternating sign: positive, negative, positive, negative, ...
- Exponential growth in magnitude: 5^{n-1}

This alternating and exponential nature is typical of geometric sequences with negative ratios.

Behavior for Large n

As n increases:

- The magnitude of A_n grows exponentially as 5^{n-1}.
- The sign alternates due to the negative base -5.

This impacts the sequence's convergence properties, as it diverges to infinity in magnitude, oscillating in sign.

Calculating A_6: Final Result

Using the explicit formula:

$$
A_6 = (-5)^{6-1} = (-5)^5
$$

Calculating:

$$
(-5)^5 = -5 \times -5 \times -5 \times -5 \times -5
$$

Step-by-step:

1. $-5 \times -5 = 25$
2. $25 \times -5 = -125$
3. $-125 \times -5 = 625$
4. $625 \times -5 = -3125$

Hence,

$$
A_6 = -3125
$$

Summary and Key Takeaways

- The recursive sequence is defined by $A_1=1$ and $A_n = -5A_{n-1}$.
- The sequence can be expressed explicitly as $A_n = (-5)^{n-1}$.
- Calculating A_6 yields -3125.

- The sequence demonstrates exponential growth in magnitude and alternating signs, typical of geometric sequences with negative ratios.

Additional Considerations and Extensions

While the problem confines itself to a specific sequence, understanding such recurrence relations opens pathways to broader topics:

- Characteristic equations: For second-order or higher recurrence relations, solution methods often involve characteristic equations.
- Generating functions: These can be used for more complex sequences to analyze their properties.
- Applications: Recursive sequences model real-world phenomena like population growth, financial calculations, and computer algorithms.

Conclusion

The sequence defined by $A_1=1$ and $A_n = -5A_{n-1}$ exemplifies the elegance of recursive sequences and their explicit solutions. By recognizing the pattern as a geometric sequence, we derived a closed-form expression that simplifies computation and analysis. The specific task of finding A_6 demonstrates the power of algebraic methods in solving recurrence relations efficiently, leading to the definitive answer:

$$
\boxed{A_6 = -3125}
$$

This exploration underscores the importance of understanding recurrence relations not only in pure mathematics but also in practical modeling and problem-solving contexts across science and engineering disciplines.

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