

. If $\cos A = \frac{3}{5}$,find The Value Of $9 + 9 \tan A$

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Introduction

In trigonometry, understanding the relationships between angles and side lengths in triangles is essential. Problems involving the calculation of trigonometric functions like tangent, sine, and cosine often appear in various mathematical contexts, from basic geometry to advanced calculus. One common type of problem requires finding the value of an expression involving tangent when the cosine of an angle is known.

In this article, we will explore how to find the value of $(9 + 9 \tan A)$ given that $(\cos A = \frac{3}{5})$. We will walk through the entire process step-by-step, providing clear explanations, formulas, and methods to solve similar problems efficiently.

Understanding the Problem

Given:

$$\cos A = \frac{3}{5}$$

Find:

$$9 + 9 \tan A$$

Step 1: Recall Basic Trigonometric Definitions

Before solving, let's revisit the fundamental trigonometric functions relevant to the problem:

- Cosine ($\cos A$): Ratio of the adjacent side to the hypotenuse in a right triangle.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- Sine ($\sin A$): Ratio of the opposite side to the hypotenuse.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

- Tangent (tan A): Ratio of the opposite to adjacent sides.

$$\tan A = \frac{\sin A}{\cos A}$$

Step 2: Visualize the Triangle

Since $\cos A = \frac{3}{5}$, we can imagine a right triangle where:

- The adjacent side to angle A is 3 units.
- The hypotenuse is 5 units.

Using the Pythagorean theorem, we can find the opposite side:

$$\begin{aligned} \text{opposite} &= \sqrt{\text{hypotenuse}^2 - \text{adjacent}^2} = \sqrt{5^2 - 3^2} = \\ &= \sqrt{25 - 9} = \sqrt{16} = 4 \end{aligned}$$

Therefore, in the right triangle:

- Adjacent side = 3
- Opposite side = 4
- Hypotenuse = 5

Step 3: Calculate $\sin A$

Using the opposite side and hypotenuse:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5}$$

Step 4: Calculate $\tan A$

Using the sine and cosine values:

$$\begin{aligned} \tan A &= \frac{\sin A}{\cos A} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4/5}{3/5} = \\ &= \frac{4/5 \times 5/3}{1} = \frac{4}{3} \end{aligned}$$

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Thus:

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$$\tan A = \frac{4}{3}$$

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Step 5: Find the Value of $(9 + 9 \tan A)$

Substitute $(\tan A = \frac{4}{3})$:

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$$9 + 9 \times \frac{4}{3} = 9 + 9 \times \frac{4}{3}$$

\]

Simplify:

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$$9 + \left(9 \times \frac{4}{3} \right) = 9 + \left(\frac{9 \times 4}{3} \right) = 9 + \left(\frac{36}{3} \right) = 9 + 12 = 21$$

\]

Answer:

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$$\boxed{21}$$

\]

Additional Insights: Why Is This Method Useful?

Understanding how to derive sine and tangent from cosine is a foundational skill in trigonometry. This problem demonstrates the importance of:

- Using the Pythagorean theorem to find missing sides.
- Converting between different trigonometric functions.
- Simplifying algebraic expressions to reach the final answer.

This approach can be applied to a wide variety of problems involving angles and side lengths, especially when certain trigonometric ratios are given.

Common Mistakes to Avoid

- Assuming the signs of trigonometric functions: Always consider the quadrant in which the angle resides. For this problem, since all sides are positive, we assume (A) is in the

first quadrant where all trig functions are positive.

- Incorrect calculation of the hypotenuse: Remember to use the Pythagorean theorem correctly.
- Forgetting to simplify fractions: Always simplify your tangent and other ratios before substituting into expressions.

Practice Problems

To reinforce the concepts discussed, try solving these similar problems:

1. If $\sin A = \frac{12}{13}$, find $\cos A$ and then evaluate $(5 + 5 \tan A)$.
2. Given $\tan A = \frac{7}{24}$, find $\sin A$ and $\cos A$, then compute $(10 + 10 \tan A)$.
3. If $\cos A = \frac{5}{13}$, find $\sin A$ and $\tan A$, then evaluate $(15 + 15 \tan A)$.

Summary

- Given $\cos A = \frac{3}{5}$, we found $\sin A = \frac{4}{5}$ using the Pythagorean theorem.
- From $\sin A$ and $\cos A$, we calculated $\tan A = \frac{4}{3}$.
- Substituting $\tan A$ into the expression $(9 + 9 \tan A)$, the final value is 21.

This systematic approach highlights the importance of understanding basic trigonometric identities and relationships, enabling you to solve a wide range of problems efficiently and accurately.

Final Thoughts

Mastering trigonometric ratios and their applications is crucial for success in mathematics. By practicing problems like this, you reinforce your understanding of right triangle properties and the interconnections between different trigonometric functions. Keep exploring, practicing, and applying these concepts to become confident in solving advanced trigonometry questions.

Frequently Asked Questions

If $\cos A = 3/5$, what is the value of $\tan A$?

Since $\cos A = 3/5$, then $\sin A = 4/5$ (by Pythagoras' theorem). Therefore, $\tan A = \sin A / \cos A = (4/5) / (3/5) = 4/3$.

How do you find $\tan A$ given $\cos A = 3/5$?

First, find $\sin A$ using the Pythagorean identity: $\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - (3/5)^2} = \sqrt{1 - 9/25} = \sqrt{16/25} = 4/5$. Then, $\tan A = \sin A / \cos A = (4/5) / (3/5) = 4/3$.

What is the value of $9 + 9 \tan A$ if $\cos A = 3/5$?

Since $\tan A = 4/3$, then $9 + 9 \tan A = 9 + 9(4/3) = 9 + 12 = 21$.

Calculate $9 + 9 \tan A$ when $\cos A = 3/5$.

Using $\tan A = 4/3$, $9 + 9 \tan A = 9 + 9(4/3) = 9 + 12 = 21$.

What is the simplified result of $9 + 9 \tan A$ for $\cos A = 3/5$?

The simplified result is 21.

Is the value of $9 + 9 \tan A$ dependent on the value of $\cos A$?

Yes, because $\tan A$ depends on $\sin A$ and $\cos A$. Given $\cos A = 3/5$, $\tan A$ is $4/3$, leading to the total of 21.

Can you verify the value of $\tan A$ using the given cosine value?

Yes. Using $\cos A = 3/5$, $\sin A = 4/5$, so $\tan A = 4/3$, which confirms the calculation.

What is the importance of knowing $\cos A$ when calculating $\tan A$?

Knowing $\cos A$ allows you to find $\sin A$ using the Pythagorean identity, which then helps compute $\tan A$ as $\sin A / \cos A$.

How does the value of $\cos A = 3/5$ help in solving the problem?

It provides the value of $\cos A$ directly, allowing us to find $\sin A$ and then $\tan A$, simplifying the calculation of $9 + 9 \tan A$.

What is the final answer for $9 + 9 \tan A$ if $\cos A = 3/5$?

The final answer is 21.

Additional Resources

If $\cos A = 3/5$, find the Value of $9 + 9 \tan A$

Understanding and solving trigonometric problems is an essential part of mathematics, especially in fields such as engineering, physics, and geometry. The problem statement, "If $\cos A = 3/5$, find the value of $9 + 9 \tan A$," is a classic example of applying fundamental trigonometric identities and concepts to find unknown values based on given information. In this article, we will systematically analyze the problem, explore the relevant trigonometric functions, and derive the solution step by step.

Understanding the Problem

The problem provides the cosine of angle A, which is $\cos A = 3/5$, and asks for the value of $9 + 9 \tan A$. To approach this problem, we need to:

- Recall basic trigonometric identities.
- Find the value of $\tan A$ using the given $\cos A$.
- Substitute this value into the expression $9 + 9 \tan A$.

This process involves understanding the relationships between sine, cosine, and tangent functions and how to manipulate them to find unknowns.

Fundamental Concepts in Trigonometry

Before solving the problem, let's review some key trigonometric concepts and identities that will assist us:

Basic Definitions

- $\cos A = \text{Adjacent} / \text{Hypotenuse}$
- $\sin A = \text{Opposite} / \text{Hypotenuse}$
- $\tan A = \text{Opposite} / \text{Adjacent}$

Pythagorean Identity

$$- \sin^2 A + \cos^2 A = 1$$

Expressing Tan A in terms of Cos A

$$- \tan A = \sin A / \cos A$$

By knowing $\cos A$, we can find $\sin A$ using the Pythagorean identity, and subsequently find $\tan A$.

Step-by-Step Solution

Step 1: Identify the given data

$$- \cos A = 3/5$$

This indicates that in a right-angled triangle, the adjacent side to angle A is 3 units, and the hypotenuse is 5 units.

Step 2: Find Sin A

Using the Pythagorean theorem:

$$- \sin^2 A + \cos^2 A = 1$$

$$- \sin^2 A = 1 - \cos^2 A$$

$$- \sin^2 A = 1 - (3/5)^2$$

$$- \sin^2 A = 1 - 9/25$$

$$- \sin^2 A = 25/25 - 9/25 = 16/25$$

$$- \sin A = \sqrt{16/25} = 4/5$$

Assuming the angle A is in the first quadrant (where sine and cosine are positive), $\sin A = 4/5$.

Step 3: Find Tan A

$$- \tan A = \sin A / \cos A$$

$$- \tan A = (4/5) / (3/5) = (4/5) (5/3) = 4/3$$

Step 4: Calculate 9 + 9 Tan A

$$- 9 + 9 \tan A = 9 + 9 (4/3) = 9 + (9 \cdot 4 / 3) = 9 + (36 / 3) = 9 + 12 = 21$$

Final Answer

The value of $9 + 9 \tan A$ given $\cos A = \frac{3}{5}$ is 21.

Additional Insights and Applications

Understanding how to derive these values is fundamental in solving more complex trigonometric problems. This problem exemplifies how knowing a single trigonometric ratio can lead to the determination of others through identities and basic geometric principles.

Pros of Using Trigonometric Identities

- Simplifies complex problems
- Allows for quick computation once relationships are understood
- Provides deep insight into geometric relationships

Cons or Challenges

- Requires memorization of identities
- Can be confusing for beginners to visualize the relationships
- Mistakes in calculation or sign assumptions can lead to errors

Key Features and Tips for Solving Similar Problems

- Always draw a right-angled triangle when given trigonometric ratios to visualize the sides.
- Remember the Pythagorean identity to find missing sides.
- Be cautious with the signs of sine and cosine depending on the quadrant.
- Simplify step-by-step to avoid errors.
- Practice with different angles and ratios to strengthen understanding.

Conclusion

The problem "If $\cos A = \frac{3}{5}$, find the value of $9 + 9 \tan A$ " is a straightforward application

of basic trigonometric identities. By recognizing the given cosine ratio, applying the Pythagorean theorem, and calculating tangent, we arrived at the final answer of 21. Mastering these fundamental steps not only helps in solving similar problems efficiently but also deepens one's understanding of the interconnected nature of trigonometric functions. Continuous practice and visualization can make these concepts more intuitive, enabling learners to tackle more advanced mathematical challenges confidently.

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