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Understanding how to compute the product of two functions is a fundamental concept in algebra and calculus. When given specific functions such as $F(x) = 5x^3$ and $G(x) = x + 1$, finding the product $(fg)(x)$ involves applying the definition of function multiplication. This process not only enhances your algebraic skills but also prepares you for more complex calculus operations like derivatives and integrals of composite functions. In this comprehensive guide, we will explore the methodology to find $(fg)(x)$, analyze the steps involved, and discuss related concepts to deepen your understanding.

What Is the Product of Two Functions?

Before diving into the specific example, it is essential to understand what the product of two functions entails.

Definition of Function Multiplication

The product of two functions, $(fg)(x)$, is defined as the pointwise multiplication of the functions' outputs:

- Given functions F and G , $(fg)(x) = F(x) G(x)$
- This means for each value of x in the domain, you multiply the value of $F(x)$ by $G(x)$

Application in Mathematics

Function multiplication is an operation that appears frequently in various mathematical contexts, including:

- Algebra (manipulating polynomial expressions)
- Calculus (product rule for derivatives)
- Functional analysis (studying properties of function spaces)

Given Functions and Their Definitions

Let's analyze the functions provided:

Function $F(x) = 5x^3$

- This is a polynomial function, cubic in nature, scaled by 5
- For any real number x , $F(x)$ outputs 5 times the cube of x

Function $G(x) = x + 1$

- This is a linear function, representing a simple shift by 1
- For any real number x , $G(x)$ outputs the value of x increased by 1

Calculating $(fg)(x)$: The Step-by-Step Process

To find $(fg)(x)$, follow these steps:

Step 1: Write the Definition

- Recall that $(fg)(x) = F(x) G(x)$

Step 2: Substitute the Given Functions

- Replace $F(x)$ with $5x^3$
- Replace $G(x)$ with $x + 1$

Step 3: Multiply the Expressions

- Compute $(5x^3)(x + 1)$
- This involves distributing $5x^3$ over $(x + 1)$

Step 4: Expand and Simplify

- Multiply $5x^3$ by each term inside the parentheses:
- $5x^3 \cdot x = 5x^4$
- $5x^3 \cdot 1 = 5x^3$
- Combine the terms to get the final expression: $5x^4 + 5x^3$

Final Expression for $(fg)(x)$

After performing the multiplication and simplification, the product of the functions F and G is:

$$(fg)(x) = 5x^4 + 5x^3$$

Interpreting the Result

Understanding the structure of the resulting function provides insights into its behavior:

Polynomial Degree and Leading Coefficient

- The highest degree term is $5x^4$, making $(fg)(x)$ a quartic polynomial
- The leading coefficient is 5, indicating the end behavior of the polynomial as x approaches infinity or

negative infinity

Behavior of $(fg)(x)$

- For large $|x|$, the x^4 term dominates, and the function grows rapidly in magnitude
- The presence of the x^3 term influences the shape near zero and the overall curve's inflection points

Visualizing $(fg)(x)$: Graphical Representation

Graphing the function helps in understanding its properties:

Key Features to Observe

1. **Intercepts:** Find where $(fg)(x)$ crosses axes by setting $x=0$ and solving for y
2. **End Behavior:** As x approaches $\pm\infty$, $(fg)(x)$ tends to $\pm\infty$ depending on the sign of the leading coefficient
3. **Critical Points:** Derivatives can identify maxima, minima, and inflection points

Practical Tools for Graphing

- Graphing calculators
- Online graphing tools like Desmos or GeoGebra

Related Concepts and Extensions

Understanding the product of functions opens doors to various advanced topics:

Product Rule in Calculus

- When differentiating products, use the product rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$
- This emphasizes the importance of understanding product expressions

Function Composition vs. Product

- Differentiate between composing functions $(f(g(x)))$ and multiplying functions $(f(x) \cdot g(x))$
- Each operation has distinct properties and applications in mathematics

Additional Examples

- Suppose $F(x) = 2x^2$ and $G(x) = 3x + 4$, then $(fg)(x) = 2x^2 (3x + 4) = 6x^3 + 8x^2$
- Practice with different functions to strengthen understanding

Summary and Key Takeaways

- To find the product $(fg)(x)$, substitute the given functions into the definition and multiply them algebraically.
- For $F(x) = 5x^3$ and $G(x) = x + 1$, the product simplifies to $5x^4 + 5x^3$.
- The resulting polynomial's degree and leading coefficient influence its end behavior and shape.
- Visualizing the function helps in understanding its properties.
- Mastery of function multiplication is fundamental for progressing in algebra and calculus.

Conclusion

The process of calculating $(fg)(x)$ for the given functions $F(x) = 5x^3$ and $G(x) = x + 1$ is straightforward once you understand the underlying principles. Multiplying functions involves substituting their expressions and applying algebraic distribution. The resulting function, $5x^4 + 5x^3$, exemplifies how polynomial degrees and coefficients shape the function's behavior. Whether you're solving algebraic problems, preparing for calculus, or exploring advanced mathematics, mastering function multiplication is an essential skill. Keep practicing with various functions to enhance your proficiency and develop a deeper understanding of how functions interact in different mathematical contexts.

Frequently Asked Questions

What is the definition of $(fg)(x)$ in function notation?

$(fg)(x)$ represents the composition of functions f and g , evaluated at x , which is $f(g(x))$.

Given $F(x) = 5x^3$ and $G(x) = x + 1$, how do you compute $(fg)(x)$?

To find $(fg)(x)$, substitute $G(x)$ into F : $(fg)(x) = F(G(x)) = 5(G(x))^3$.

What is the step-by-step solution for $(fg)(x)$ with the provided functions?

First, find $G(x) = x + 1$. Then, compute $(fg)(x) = 5(x + 1)^3$ by substituting $G(x)$ into $F(x)$.

How do you expand $(x + 1)^3$?

$(x + 1)^3$ expands to $x^3 + 3x^2 + 3x + 1$ using the binomial theorem.

What is the final simplified expression for $(fg)(x)$?

The final expression is $5(x^3 + 3x^2 + 3x + 1) = 5x^3 + 15x^2 + 15x + 5$.

Why is function composition important in mathematics?

Function composition allows us to analyze how functions combine, which is essential in many areas like calculus, modeling, and problem-solving.

Can you verify the result of $(fg)(x)$ by substituting a specific value of x ?

Yes, for example, at $x=2$: $G(2)=3$, $F(3)= 5 \cdot 3^3= 5 \cdot 27=135$, which matches the composition result.

What is the key concept to remember when computing $(fg)(x)$?

The key is to evaluate $g(x)$ first and then substitute that result into $f(x)$ to find the composition.

Additional Resources

If $F(x) = 5x^3$ And $G(x) = x+1$, Find $(fg)(x)$.

Introduction

In the realm of mathematics, understanding how functions interact through operations like composition and multiplication is fundamental. For students and enthusiasts alike, grasping the process of combining functions provides a solid foundation for more advanced topics such as calculus, algebra, and mathematical modeling. Today, we delve into a specific problem involving two functions: $(F(x) = 5x^3)$ and $(G(x) = x + 1)$. The goal is to find $((fg)(x))$, which represents the product of these two functions at any given input (x) . This task not only reinforces core concepts of function operations but also illustrates how to approach such problems systematically.

Understanding the Problem

Before jumping into calculations, it's essential to clarify what is being asked. The notation $((fg)(x))$ denotes the product of two functions $(f(x))$ and $(g(x))$, evaluated at some input (x) . In other words:

$$\begin{aligned} & \backslash[\\ (fg)(x) &= f(x) \times g(x) \\ & \backslash] \end{aligned}$$

Given:

- $(F(x) = 5x^3)$
- $(G(x) = x + 1)$

Our task is to compute:

$$\begin{aligned} & \backslash[\\ (fg)(x) &= F(x) \times G(x) \\ & \backslash] \end{aligned}$$

This involves substituting the given functions into the product and simplifying the expression.

Step 1: Clarify the Functions

It is helpful to note the definitions explicitly:

- Function $(F(x))$: Multiplies (x) cubed by 5.

$$\begin{aligned} & \left[\right. \\ & F(x) = 5x^3 \\ & \left. \right] \end{aligned}$$

- Function $(G(x))$: Adds 1 to (x) .

$$\begin{aligned} & \left[\right. \\ & G(x) = x + 1 \\ & \left. \right] \end{aligned}$$

Understanding these functions individually will make the multiplication process straightforward.

Step 2: Write the Product Function

The product of the two functions $(F(x))$ and $(G(x))$ is:

$$\begin{aligned} & \left[\right. \\ & (fg)(x) = F(x) \times G(x) \\ & \left. \right] \end{aligned}$$

Substituting the given functions:

$$\begin{aligned} & \left[\right. \\ & (fg)(x) = (5x^3) \times (x + 1) \\ & \left. \right] \end{aligned}$$

This expression indicates that at any (x) , we multiply $(5x^3)$ by $(x + 1)$.

Step 3: Expand the Expression

To simplify $(fg)(x)$, expand the product:

$$(fg)(x) = 5x^3 \times (x + 1)$$

Distribute $(5x^3)$ over the sum:

$$(fg)(x) = 5x^3 \times x + 5x^3 \times 1$$

Calculate each term separately:

- $(5x^3 \times x = 5x^4)$ because $(x^3 \times x = x^{3+1} = x^4)$.
- $(5x^3 \times 1 = 5x^3)$.

Putting it all together gives:

$$\boxed{(fg)(x) = 5x^4 + 5x^3}$$

This is the simplified form of the product function.

Deep Dive: Why This Matters

Understanding the multiplication of functions is crucial because it appears frequently in many areas of mathematics and applied sciences. For example, in physics, the product of functions might represent combined forces or rates. In economics, it could model revenue as a function of quantity and price. Mastering these operations enables one to interpret and manipulate complex models effectively.

Visualizing the Function

Graphing the resulting function, $(fg)(x) = 5x^4 + 5x^3$, can offer valuable insights:

- Shape: The dominant term $(5x^4)$ determines the end behavior since it grows rapidly for large $(|x|)$.

- Roots: Finding where $((fg)(x) = 0)$ involves solving:

$$\begin{aligned} & \backslash[\\ & 5x^4 + 5x^3 = 0 \\ & \backslash] \end{aligned}$$

- Factoring: To find zeros:

$$\begin{aligned} & \backslash[\\ & 5x^3(x + 1) = 0 \\ & \backslash] \end{aligned}$$

Setting each factor to zero:

$$\begin{aligned} & - \backslash(5x^3 = 0 \rightarrow x = 0) \\ & - \backslash(x + 1 = 0 \rightarrow x = -1) \end{aligned}$$

Thus, the function crosses the x-axis at $(x = 0)$ and $(x = -1)$.

Extending the Concept: Related Operations

While this discussion centers on the product of two functions, it's beneficial to distinguish this from other operations:

- Function Composition $((f \circ g)(x))$: Applying (f) to the output of (g) :

$$\begin{aligned} & \backslash[\\ & (f \circ g)(x) = f(g(x)) \\ & \backslash] \end{aligned}$$

- Sum of functions $((f + g)(x))$: Adding the functions:

$$\begin{aligned} & \backslash[\\ & (f + g)(x) = f(x) + g(x) \\ & \backslash] \end{aligned}$$

- Difference and division: Subtracting or dividing functions follow similar patterns but require attention to domain restrictions.

Understanding these distinctions allows for precise calculations and avoids common misconceptions.

Practical Applications

The process of multiplying functions appears in various real-world contexts:

- Physics: Calculating total energy by combining different power functions.
- Economics: Modeling total revenue or cost functions.
- Engineering: Combining transfer functions in control systems.
- Biology: Modeling population growth with multiple influencing factors.

In each case, the ability to manipulate and simplify functions accurately is essential for meaningful analysis.

Summary and Key Takeaways

- The problem involves multiplying two functions $(F(x) = 5x^3)$ and $(G(x) = x + 1)$.
- The product function $((fg)(x))$ is obtained by substituting the given functions and expanding the product.
- The final simplified expression is:

$$\begin{aligned} & \left[\right. \\ (fg)(x) &= 5x^4 + 5x^3 \\ & \left. \right] \end{aligned}$$

- Understanding how to expand and factor functions enhances problem-solving skills in higher mathematics.
- Visualizing the resulting function helps interpret behavior and roots.

Final Thoughts

Mastering the operations involving functions, such as multiplication, is fundamental in mathematics. It builds a foundation for more complex topics like calculus, differential equations, and mathematical modeling. By practicing these techniques with concrete examples like the one discussed, students develop a deeper understanding of the structure and behavior of functions, empowering them to apply mathematical reasoning across diverse fields.

Whether you're a student preparing for exams or a professional applying math in real-world scenarios, the ability to manipulate functions confidently is invaluable. Keep practicing with different functions, explore their graphs, and understand their properties — this approach will unlock a richer appreciation and mastery

of mathematics.

If $F(x) = 5x - 3$ And $G(x) = x + 1$ Find $F \circ G(x)$

If $F(x) = 5x - 3$ And $G(x) = x + 1$ Find $F \circ G(x)$

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