If F(x)=cosxsinx Then F (4) Is Equal To ?

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Understanding the value of a function at a specific point is a fundamental aspect of calculus and mathematical analysis. In this article, we explore how to evaluate the function $F(x) = \cos x \sin x$ at $x = 4$. We will detail the steps involved, delve into the properties of the function, and provide some useful identities that simplify the calculation. Whether you're a student preparing for exams or a math enthusiast, this comprehensive guide aims to clarify the process and deepen your understanding.

Introduction to the Function $F(x) = \cos x \sin x$

Before calculating $F(4)$, it's important to understand what the function represents and its properties.

Basic Properties of $F(x) = \cos x \sin x$

- The function multiplies the cosine and sine of the variable x.
- It is periodic, with a period of π, since both sine and cosine are periodic functions with period 2π, but their product has a period of π.
- The function is symmetric, specifically, it is an odd function because $\cos(-x) = \cos x$ and $\sin(-x) = - \sin x$, so $F(-x) = - F(x)$.

Rewriting the Function Using Trigonometric Identities

One of the most effective ways to evaluate $F(4)$ is to express it using a known trigonometric identity.

Recall the double-angle identity:

$$
\sin 2x = 2 \sin x \cos x
$$

Rearranged, it gives:

$$
\sin x \cos x = \frac{1}{2} \sin 2x
$$

Thus, the function simplifies to:

$$
F(x) = \cos x \sin x = \frac{1}{2} \sin 2x
$$

This rewriting makes calculations straightforward.

Calculating $F(4)$ Using the Identity

Using the identity:

$$
F(x) = \frac{1}{2} \sin 2x
$$

we substitute $x = 4$:

$$
F(4) = \frac{1}{2} \sin 8
$$

Now, the task reduces to calculating $\sin 8$. Since the angle is in radians, and 8 radians are not one of the standard angles, we need to evaluate $\sin 8$ either numerically or using a calculator.

Numerical Approximation of $\sin 8$

Using a calculator:

$$
\sin 8 \approx 0.989358
$$

Therefore:

$$
F(4) \approx \frac{1}{2} \times 0.989358 \approx 0.494679
$$

So,

$$
\boxed{
F(4) \approx 0.494679
}
$$

This is the approximate value of the function at $x=4$.

Understanding the Significance of the Result

The value of $F(4)$ is roughly 0.495, indicating a positive value since $\sin 8$ is positive in this context.

When Is $F(x)$ Positive or Negative?

- Since $F(x) = \frac{1}{2} \sin 2x$, the sign of $F(x)$ depends entirely on $\sin 2x$.
- For $2x$ in the first and second quadrants ($0 < 2x < \pi$), $\sin 2x$ is positive, so $F(x)$ is positive.
- For $2x$ in the third and fourth quadrants ($\pi < 2x < 2\pi$), $\sin 2x$ is negative, so $F(x)$ is negative.

Applying this to $x=4$:

$$
2 \times 4 = 8
$$

Since 8 radians is approximately 458.37 degrees, which lies in the third quadrant (since π radians is 180 degrees, and 2π radians is 360 degrees), but more precisely, 8 radians ≈ 458.366 degrees:

- 360 degrees is a full rotation.
- 458.366 - 360 = 98.366 degrees, which is in the second quadrant.

In radians, 98.366 degrees is approximately 1.717 radians, which is in the second quadrant where sine is positive. Therefore, $\sin 8$ is positive, consistent with the numerical approximation.

Additional Methods to Calculate $F(4)$

While the identity approach is straightforward, other methods include:

1. Using Series Expansion

The sine function can be expanded as a Taylor series:

$$
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots
$$

At $x=8$:

\[

$$
\sin 8 \approx 8 - \frac{8^3}{6} + \frac{8^5}{120} - \cdots
$$

Calculating a few terms yields a close approximation, but this method is more complex and less practical without computational tools.

2. Using a Calculator or Software

The most efficient way in practice is to use a calculator or computational software to evaluate $\sin 8$ directly.

Summary of Steps to Find $F(4)$

To summarize, the process involves:

1. Recognizing the function's structure and properties.
2. Applying the double-angle identity: $\sin 2x = 2 \sin x \cos x$.
3. Rearranging to express $F(x)$ as $\frac{1}{2} \sin 2x$.
4. Substituting $x=4$ to evaluate $F(4) = \frac{1}{2} \sin 8$.
5. Calculating $\sin 8$ either numerically or using a calculator.
6. Multiplying by $\frac{1}{2}$ to find the approximate value.

Practical Applications of $F(x) = \cos x \sin x$

Understanding how to evaluate this function at various points has several practical implications:

Signal Processing

- The function appears in modulation techniques where phase and amplitude variations involve sine and cosine functions.
- Simplifying such functions using identities facilitates signal analysis.

Physics and Engineering

- In wave mechanics, the product of sine and cosine functions models interference patterns.
- Calculating specific values helps in predicting wave behavior at certain points.

Mathematics Education

- Serves as an example of how identities simplify complex trigonometric expressions.
- Demonstrates the importance of understanding function properties for evaluation.

Conclusion

Evaluating $F(4)$ for the function $F(x) = \cos x \sin x$ involves leveraging fundamental trigonometric identities to simplify the calculation. By recognizing that:

$$
F(x) = \frac{1}{2} \sin 2x
$$

we reduce the problem to calculating $\sin 8$. Approximating $\sin 8$ yields approximately 0.989358, leading to:

$$
F(4) \approx 0.494679
$$

This process exemplifies the power of identities in simplifying and solving seemingly complex problems. Understanding these methods enhances both mathematical intuition and computational efficiency, applicable across various scientific and engineering disciplines.

Key Takeaways:

- Use identities to simplify trigonometric functions.
- Recognize the periodicity and symmetry of functions.
- Employ calculators or software for precise numerical evaluation.
- Apply these techniques in practical contexts like physics, engineering, and signal processing.

By mastering these concepts, students and professionals can confidently evaluate functions like $F(x) = \cos x \sin x$ at any given point, deepening their understanding of trigonometric functions and their applications.

Frequently Asked Questions

What is the value of F(4) if F(x) = cos x sin x?

F(4) = cos 4 × sin 4.

How can I simplify F(x) = cos x sin x using a trigonometric identity?

Using the identity sin 2x = 2 sin x cos x, F(x) can be written as (1/2) sin 2x, so F(4) = (1/2) sin 8.

What is the exact value of F(4) when F(x) = cos x sin x?

F(4) = (1/2) sin 8 radians; the exact value depends on evaluating sin 8 radians.

Is there a formula to compute F(4) numerically if F(x) = cos x sin x?

Yes, F(4) = (1/2) sin 8. Numerically, sin 8 radians ≈ 0.9894, so F(4) ≈ 0.4947.

How does the identity F(x) = (1/2) sin 2x help in calculating F(4)?

It converts the product cos x sin x into a single sine function, making it easier to evaluate F(4) as (1/2) sin 8.

Additional Resources

If F(x)=cosxsinx Then F (4) Is Equal To ?

In the realm of calculus and mathematical analysis, functions form the backbone of understanding various phenomena, from physics to engineering. One intriguing function that often appears in trigonometric contexts is $F(x) = \cos x \sin x$. When asked to evaluate this function at a specific point, such as $x=4$, it offers a compelling opportunity to explore the principles of function evaluation, identities, and the nuances of exact versus approximate calculations. This article delves into the detailed process of determining $F(4)$ when $F(x) = \cos x \sin x$, providing an in-depth journey through the underlying mathematics, relevant identities, and practical computation methods.

Understanding the Function $F(x) = \cos x \sin x$

Before jumping into the evaluation at $x=4$, it's essential to understand the nature of the function itself. The expression $\cos x \sin x$ is a product of two fundamental trigonometric functions—cosine and sine—which are periodic and well-studied.

The Behavior of $\cos x \sin x$

- Periodicity: Both $\cos x$ and $\sin x$ have a period of 2π. Their product, $F(x)$, inherits a combined periodicity, leading to a period of π.
- Range: Since $\cos x$ and $\sin x$ vary between -1 and 1, their product varies between -1 and 1 as well.
- Symmetry: The function $\cos x \sin x$ exhibits symmetry properties that can be exploited for simplification.

Understanding these characteristics helps in both qualitative analysis and precise evaluation.

Simplifying $F(x)$ with Trigonometric Identities

A key step in evaluating $F(4)$ is recognizing that the product $\cos x \sin x$ can be expressed more simply using standard identities.

Double-Angle Identity for $\sin 2x$

The most useful identity here is:

$$
\sin 2x = 2 \sin x \cos x
$$

Rearranged, this gives:

$$
\sin x \cos x = \frac{1}{2} \sin 2x
$$

Applying this to $F(x)$:

$$
F(x) = \cos x \sin x = \frac{1}{2} \sin 2x
$$

This transformation reduces the problem of evaluating $F(4)$ to finding $\frac{1}{2} \sin 8$ because:

$$
F(4) = \frac{1}{2} \sin 2 \times 4 = \frac{1}{2} \sin 8
$$

where 8 is in radians.

Calculating $F(4)$: Step-by-Step Approach

With the identity in hand, the evaluation becomes straightforward:

$$
F(4) = \frac{1}{2} \sin 8
$$

The main task now is to compute $\sin 8$, where 8 is in radians.

Radians vs Degrees

- Radians: The standard unit in calculus; one full circle is $2\pi \approx 6.2832$ radians.
- Degrees: More common in everyday contexts; 360° corresponds to 2π radians.

Since the argument is in radians, direct computation involves the sine of 8 radians.

Numerical Approximation of $\sin 8$

Calculating $\sin 8$ exactly involves either a calculator, software, or known sine values.

Using a Scientific Calculator

- Input: $\sin 8$
- Result: approximately 0.989358

Thus,

$$
F(4) \approx \frac{1}{2} \times 0.989358 = 0.494679
$$

This approximation indicates that:

$$
\boxed{
F(4) \approx 0.4947
}
$$

for practical purposes, rounded to four decimal places.

Exact Expression and Its Significance

While the numerical approximation is useful for quick estimates, expressing $F(4)$ in its exact form is valuable for theoretical analysis:

$$
F(4) = \frac{1}{2} \sin 8
$$

This exact expression is vital when precise symbolic calculations are needed, such as in proofs or derivations, or when using software that handles symbolic mathematics.

Additional Insights and Related Concepts

Periodicity and Symmetry

Given that $F(x) = \frac{1}{2} \sin 2x$, the function's fundamental period is π:

- Period: π, since $\sin 2x$ repeats every π.
- Symmetry: $F(x)$ is an odd function because $\sin 2x$ is odd, and the coefficient $\frac{1}{2}$ is constant.

Graphical Interpretation

Graphing $F(x)$ reveals a wave oscillating between $-\frac{1}{2}$ and $\frac{1}{2}$, with peaks at points where $\sin 2x = 1$ (i.e., $2x = \frac{\pi}{2} + n\pi$).

Broader Applications and Related Problems

Understanding how to evaluate such functions is fundamental in multiple fields:

- Physics: Analyzing wave interference patterns where products of sine and cosine functions appear.
- Engineering: Signal processing often involves manipulating trigonometric functions and their identities.
- Mathematics: Solving integrals involving $\sin x \cos x$, or deriving related identities.

Moreover, similar techniques are used to evaluate functions at various points, explore maxima and minima, and analyze the behavior of complex trigonometric expressions.

Summary and Key Takeaways

- The function $F(x) = \cos x \sin x$ can be simplified using the double-angle identity to $\frac{1}{2} \sin 2x$.
- Evaluating $F(4)$ then reduces to calculating $\frac{1}{2} \sin 8$ radians.
- Numerical approximation yields $F(4) \approx 0.4947$.
- Exact expressions are essential for theoretical work, while approximations are sufficient for practical purposes.
- Understanding the properties of $F(x)$ provides insights into its behavior, symmetry, and applications across disciplines.

By mastering these concepts, students and professionals can confidently evaluate similar functions, appreciate the elegance of trigonometric identities, and apply these tools in diverse scientific and engineering contexts.

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