If F(x)=-x^2 And G(x) = -x^2+4x+5 What Is The Product

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Understanding the product of two functions involves a comprehensive examination of their individual expressions, their domains, and how they combine multiplicatively. In this article, we will explore in detail the functions $F(x) = -x^2$ and $G(x) = -x^2 + 4x + 5$, analyze their properties, and determine the resulting product function $P(x) = F(x) \times G(x)$. This exploration will include factorization, graphing insights, and the significance of the product in various mathematical contexts.

Analyzing the Given Functions

Function F(x) = -x^2

The function $F(x) = -x^2$ is a quadratic function with a few key properties:

- **Shape and Orientation:** Since the coefficient of x^2 is negative, the parabola opens downward.
- **Vertex:** The vertex occurs at the origin (0, 0), because there are no linear or constant terms shifting the parabola.
- **Domain and Range:** The domain is all real numbers ($-\infty, \infty$), while the range is $(-\infty, 0]$ because the parabola reaches a maximum at the vertex.
- **Symmetry:** The graph is symmetric about the y-axis, reflecting the even nature of the quadratic function.

Function G(x) = -x^2 + 4x + 5

The function $G(x) = -x^2 + 4x + 5$ is also quadratic, but with additional linear and constant terms:

- **Shape and Orientation:** Like $F(x)$, the parabola opens downward due to the negative leading coefficient.
- **Vertex:** The vertex can be found using the vertex formula $x = -\frac{b}{2a}$, where $a = -1$ and $b = 4$.
- **Calculating the Vertex:** $x = -\frac{4}{2 \times -1} = -\frac{4}{-2} = 2$. Substituting into $G(x)$:
    - $G(2) = -(2)^2 + 4 \times 2 + 5 = -4 + 8 + 5 = 9$
- **Vertex Coordinates:** (2, 9)

- **Domain and Range:** The domain is all real numbers; the range is $(-\infty, 9]$ with the maximum at the vertex.
- **Symmetry:** The parabola is symmetric about the vertical line $x=2$.

Understanding the Product Function $P(x) = F(x) \times G(x)$

The product of the two functions can be expressed as:

$$
P(x) = F(x) \times G(x) = (-x^2) \times (-x^2 + 4x + 5)
$$

Expanding this product allows us to analyze its algebraic form and properties.

Calculating $P(x)$

Let's perform the multiplication step-by-step:

$$
P(x) = -x^2 \times (-x^2 + 4x + 5)
$$

Distribute $-x^2$:

$$
P(x) = (-x^2) \times (-x^2) + (-x^2) \times 4x + (-x^2) \times 5
$$

Simplify each term:

- $(-x^2) \times (-x^2) = x^4$ (since negative times negative is positive)
- $(-x^2) \times 4x = -4x^3$
- $(-x^2) \times 5 = -5x^2$

Thus,

$$
P(x) = x^4 - 4x^3 - 5x^2
$$

This is a quartic polynomial, and understanding its characteristics requires analyzing its degree, roots, and graph.

Graphical Interpretation of $P(x) = x^4 - 4x^3 - 5x^2$

Visualizing the graph of $P(x)$ provides insights into its behavior:

Shape and End Behavior

- Since the leading term x^4 dominates for large $|x|$, the polynomial tends towards positive infinity as $x \to \pm \infty$.
- The polynomial has a degree of 4 (quartic), so it can have up to 3 turning points and up to 4 real roots.

Roots of $P(x)$

To find where $P(x) = 0$:

$$
x^4 - 4x^3 - 5x^2 = 0
$$

Factor out x^2:

$$
x^2 (x^2 - 4x - 5) = 0
$$

Set each factor equal to zero:

1. $x^2 = 0 \Rightarrow x = 0$

2. $x^2 - 4x - 5 = 0$

Solve the quadratic:

$$
x^2 - 4x - 5 = 0
$$

Using quadratic formula:

$$
x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times (-5)}}{2 \times 1} = \frac{4 \pm \sqrt{16 + 20}}{2} = \frac{4 \pm \sqrt{36}}{2}
$$

$$
x = \frac{4 \pm 6}{2}
$$

Calculate the roots:

- $x = \frac{4 + 6}{2} = \frac{10}{2} = 5$
- $x = \frac{4 - 6}{2} = \frac{-2}{2} = -1$

Roots of $P(x)$:

$$
x = 0, -1, 5
$$

Sign of $P(x)$ in Different Intervals

- For $x < -1$:

Choose $x = -2$:

$$
P(-2) = 16 - 4(-8) - 5(4) = 16 + 32 - 20 = 28 > 0
$$

- For $-1 < x < 0$:

Choose $x = -0.5$:

$$
P(-0.5) = 0.0625 - 4(-0.125) - 5(0.25) = 0.0625 + 0.5 - 1.25 = -0.6875 < 0
$$

- For $0 < x < 5$:

Choose $x=1$:

$$
P(1) = 1 - 4(1) - 5(1) = 1 - 4 - 5 = -8 < 0
$$

- For $x > 5$:

Choose $x=6$:

$$
P(6) = 1296 - 4(216) - 5(36) = 1296 - 864 - 180 = 252 > 0
$$

Summary of sign intervals:

Interval	Sign of $P(x)$	Roots at
$(-\infty, -1)$	Positive	Root at $x = -1$
$(-1, 0)$	Negative	Root at $x=0$
$(0, 5)$	Negative	Root at $x=5$
$(5, \infty)$	Positive	Root at $x=5$ (end of interval)

Implications of the Product Function

Understanding $P(x)$ extends beyond its algebraic form to its applications and implications:

Zeros of the Product Function

- The roots $x = -1, 0, 5$ indicate where the product equals zero.
- These points are intersections of the individual functions where either $f(x)$ or $g(x)$ or both are zero.

Relationship to Original Functions

- The zeros at \(x=0 \) reflect the zero of \(F(x) \) and \(G(x) \) at different points

Frequently Asked Questions

How do you find the product of the functions F(x) = -x^2 and G(x) = -x^2 + 4x + 5?

To find the product, multiply the two functions: F(x) G(x) = (-x^2) (-x^2 + 4x + 5).

What is the expression for the product of F(x) and G(x) given the functions F(x) = -x^2 and G(x) = -x^2 + 4x + 5?

The product is (-x^2) (-x^2 + 4x + 5), which simplifies to x^2 (x^2 - 4x - 5).

How can I simplify the product of F(x) = -x^2 and G(x) = -x^2 + 4x + 5?

First, rewrite the product as (-x^2) (-x^2 + 4x + 5). Distribute to get x^2 (x^2 - 4x - 5), then expand to x^4 - 4x^3 - 5x^2.

What is the expanded form of the product of the two functions F(x) = -x^2 and G(x) = -x^2 + 4x + 5?

The expanded form is x^4 - 4x^3 - 5x^2.

Can the product of F(x) = -x^2 and G(x) = -x^2 + 4x + 5 be factored further?

Yes, after expansion to x^4 - 4x^3 - 5x^2, you can factor out x^2: x^2(x^2 - 4x - 5). Further factoring yields x^2(x - 5)(x + 1).

What is the significance of finding the product of these two functions in algebra?

Finding the product helps in understanding combined behaviors of the functions, solving for zeros, and analyzing the resulting polynomial's properties.

Additional Resources

Understanding the Product of F(x) = -x^2 and G(x) = -x^2 + 4x + 5

When exploring mathematical functions, it's common to analyze how they interact through various operations such as addition, subtraction, and multiplication. One intriguing operation is finding the product of two functions. In this article, we will thoroughly examine what is the product of F(x) = -x^2 and G(x) = -x^2 + 4x + 5, breaking down the process step-by-step to help you understand the concept clearly. Whether you're a student, educator, or math enthusiast, this comprehensive guide will deepen your grasp of function multiplication and its implications.

Understanding the Functions

Before diving into the product, it's essential to understand the individual functions involved:

- $F(x) = -x^2$

This is a quadratic function representing a parabola opening downward, centered at the origin. Its graph is symmetric with respect to the y-axis, and it features the characteristic parabola shape with a vertex at (0, 0).

- $G(x) = -x^2 + 4x + 5$

This is also a quadratic function but shifted and stretched, with a different parabola opening downward. Its coefficients indicate a parabola that is shifted along the x-axis and y-axis, with the vertex located somewhere away from the origin.

Step 1: Recalling Function Multiplication

When we talk about the product of two functions, denoted as $(F \times G)(x)$, we are essentially multiplying the two functions point-by-point for each value of x:

$(F \times G)(x) = F(x)\, G(x)$

This operation results in a new function that combines the behavior of both original functions.

Step 2: Setting Up the Product

Given the functions:

- $F(x) = -x^2$
- $G(x) = -x^2 + 4x + 5$

The product function, let's call it $P(x)$, is:

$P(x) = F(x)\, G(x) = (-x^2)\, (-x^2 + 4x + 5)$

Our task is to expand and simplify this expression to understand its structure and properties.

Step 3: Expanding the Product

Applying the distributive property (also known as FOIL in binomials) for multiplication:

$P(x) = (-x^2)\, (-x^2 + 4x + 5)$

Distribute $-x^2$ across each term inside the parentheses:

- $(-x^2)\, (-x^2) = ?$

- $(-x^2)\, (4x) = ?$

- $(-x^2)\, (5) = ?$

Calculating each:

1. $(-x^2)(-x^2) = (-1)(-1)\, x^2\, x^2 = 1\, x^4 = x^4$

2. $(-x^2)(4x) = -1\, 4\, x^2\, x = -4\, x^3$

3. $(-x^2)(5) = -1\, 5\, x^2 = -5x^2$

Putting it all together:

$P(x) = x^4 - 4x^3 - 5x^2$

Step 4: Analyzing the Resulting Function

The final expanded form:

$P(x) = x^4 - 4x^3 - 5x^2$

This is a quartic polynomial, meaning it's a degree 4 polynomial with specific coefficients. Understanding its behavior involves analyzing its critical points, roots, and end behavior.

Step 5: Exploring the Properties of $P(x)$

a) Degree and Leading Coefficient

- Degree: 4 (since the highest exponent is 4)
- Leading coefficient: 1 (positive), indicating that as $x \square\ \pm\square$, $P(x) \square\ +\square$

b) Roots of $P(x)$

To find where the product function equals zero (roots), set $P(x) = 0$:

$x^4 - 4x^3 - 5x^2 = 0$

Factor out the greatest common factor, x^2:

$x^2 (x^2 - 4x - 5) = 0$

Set each factor equal to zero:

- $x^2 = 0 \square\ x = 0$

- $x^2 - 4x - 5 = 0$

Solve the quadratic:

$x^2 - 4x - 5 = 0$

Apply quadratic formula:

$x = [4 \pm \square\ (16 - 4\ 1\ (-5))] / (2\ 1)$

$x = [4 \pm \square\ (16 + 20)] / 2$

$x = [4 \pm \square\ 36] / 2$

$x = [4 \pm 6] / 2$

Calculate both solutions:

- $x = (4 + 6)/2 = 10/2 = 5$

- $x = (4 - 6)/2 = (-2)/2 = -1$

Roots of P(x):

- $x = 0$ (double root due to x^2 factor)

- $x = 5$

- $x = -1$

c) Critical Points and Behavior

To understand the shape of the graph, analyze the first and second derivatives to find local maxima and minima, inflection points, etc.

Practical Applications and Visual Insights

Knowing the product of these functions provides insight into their combined behavior:

- Growth and Decay: Since both functions are downward-opening quadratics, their product, a quartic, may exhibit complex behaviors such as multiple peaks and valleys.

- Intersection Points: The roots of the product correspond to points where either F(x) or G(x) (or both) are zero, indicating where the functions intersect the x-axis.

- Sign Analysis: By examining the signs of F(x) and G(x), we can determine where their product is positive or negative, which influences the shape of P(x).

Summary and Key Takeaways

- The product of $F(x) = -x^2$ and $G(x) = -x^2 + 4x + 5$ results in a quartic polynomial: $P(x) = x^4 - 4x^3 - 5x^2$

- This polynomial features roots at $x = -1, 0, 5$, with $x=0$ being a double root.

- The degree and leading coefficient suggest the function tends toward positive infinity as x approaches ± infinity.

- The behavior of the function's graph can be further analyzed through derivatives, but even without detailed calculus, understanding the roots and end behavior provides valuable insights.

Final Thoughts

The process of multiplying functions like F(x) and G(x) to produce a new, higher-degree polynomial opens the door to rich mathematical exploration. It illustrates how simple quadratic functions can combine to create complex behaviors, emphasizing the importance of algebraic manipulation and analysis in understanding functions' combined effects. Whether you're solving problems, graphing functions, or exploring calculus applications, mastering these fundamental operations enhances your overall mathematical toolkit.

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