

If $\sin A = 4/5$ Solve $\sin 2A$, $\cos 2A$ And $\tan 2A$

If $\sin A = 4/5$ Solve $\sin 2A$, $\cos 2A$ And $\tan 2A$

Understanding how to find the values of double angles such as $\sin 2A$, $\cos 2A$, and $\tan 2A$ is fundamental in trigonometry, especially when the value of $\sin A$ is known. In this article, we will explore the step-by-step process of solving for these double angle functions given that $\sin A$ equals $4/5$. This problem offers a great opportunity to review key trigonometric identities and apply them effectively. Whether you're a student preparing for exams or someone interested in enhancing your mathematical skills, this comprehensive guide will help you master these calculations.

Understanding the Given Data and Basic Concepts

Before diving into the calculations, it's essential to understand the given information and the foundational identities involved in solving for the double angles.

Given Data: $\sin A = 4/5$

- The value of $\sin A$ is $4/5$.
- Since sine is positive, A is in either the first or second quadrant.
- To proceed, we need to determine $\cos A$ and possibly $\tan A$.

Key Trigonometric Identities

- Double angle formulas:
 - $\sin 2A = 2 \sin A \cos A$
 - $\cos 2A = \cos^2 A - \sin^2 A$
 - $\tan 2A = 2 \tan A / (1 - \tan^2 A)$
- Pythagorean identity:
 - $\sin^2 A + \cos^2 A = 1$

Understanding these identities allows us to compute the necessary values step-by-step.

Step 1: Find $\cos A$

Since $\sin A$ is known, we can use the Pythagorean identity to find $\cos A$.

Calculating $\cos A$

Given:

$$- \sin A = 4/5$$

Using the Pythagorean identity:

$$- \cos A = \sqrt{1 - \sin^2 A}$$

Substitute the known value:

$$- \cos A = \sqrt{1 - (4/5)^2} = \sqrt{1 - 16/25}$$

Simplify:

$$- \cos A = \sqrt{(25/25 - 16/25)} = \sqrt{9/25}$$

Calculate:

$$- \cos A = \sqrt{9/25} = 3/5$$

Since $\sin A$ is positive and assuming A is in the first quadrant, $\cos A$ is also positive:

$$- \cos A = 3/5$$

Step 2: Calculate $\sin 2A$

Using the double angle formula:

$$- \sin 2A = 2 \sin A \cos A$$

Substitute the known values:

$$- \sin 2A = 2 (4/5) (3/5) = 2 (12/25) = 24/25$$

Result:

$$- \sin 2A = 24/25$$

Step 3: Calculate $\cos 2A$

Using the double angle formula:

$$- \cos 2A = \cos^2 A - \sin^2 A$$

Substitute the known values:

$$- \cos 2A = (3/5)^2 - (4/5)^2 = (9/25) - (16/25) = -7/25$$

Alternatively, $\cos 2A$ can also be calculated using the identity:

$$- \cos 2A = 2 \cos^2 A - 1$$

Check:

$$- 2 (3/5)^2 - 1 = 2 (9/25) - 1 = (18/25) - 1 = (18/25) - (25/25) = -7/25$$

Result:

$$- \cos 2A = -7/25$$

Step 4: Calculate $\tan 2A$

To find $\tan 2A$, first find $\tan A$:

$$- \tan A = \sin A / \cos A = (4/5) / (3/5) = 4/3$$

Now, use the double angle formula:

$$\text{Tan}2A = 2 \text{ Tan}A / (1 - \text{Tan}^2A)$$

Calculate Tan^2A :

$$\text{Tan}^2A = (4/3)^2 = 16/9$$

Substitute into the formula:

$$\text{Tan}2A = 2 (4/3) / (1 - 16/9)$$

Simplify numerator:

$$2 (4/3) = 8/3$$

Simplify denominator:

$$1 - 16/9 = (9/9) - (16/9) = -7/9$$

Now, divide numerator by denominator:

$$\text{Tan}2A = (8/3) / (-7/9) = (8/3) (-9/7) = (8 \cdot -9) / (3 \cdot 7) = (-72) / 21$$

Simplify:

$$\text{Tan}2A = -24/7$$

Result:

$$\text{Tan}2A = -24/7$$

Summary of Results

Double Angle Function	Value
Sin2A	24/25
Cos2A	-7/25
Tan2A	-24/7

Additional Insights and Applications

Understanding the calculations above is crucial for solving more complex trigonometric problems. The process demonstrates how foundational identities can be combined systematically to find double angle values. Here are some practical tips and insights for applying these methods:

Quadrant Considerations

- Since $\text{Sin}A = 4/5$ is positive and $\text{Cos}A = 3/5$ is positive, A is in the first quadrant.
- However, if $\text{Sin}A$ were positive and $\text{Cos}A$ negative, A would be in the second quadrant, which would affect the sign of $\text{Cos}A$ and $\text{Tan}A$.
- Always consider the quadrant when determining the signs of the trigonometric functions.

Using Identities Effectively

- The double angle formulas are powerful tools that simplify calculations.
- Remember multiple identities for $\cos 2A$, such as:
 - $\cos 2A = 2 \cos^2 A - 1$
 - $\cos 2A = 1 - 2 \sin^2 A$
- Choose the one that simplifies calculations based on known values.

Applications in Real-World Problems

- These calculations are useful in physics, engineering, and navigation where wave functions and oscillations are involved.
- They help in deriving angles and distances in practical scenarios.

Conclusion

By starting with the given $\sin A = 4/5$, we successfully calculated $\sin 2A$, $\cos 2A$, and $\tan 2A$, demonstrating the power of fundamental trigonometric identities. The key steps involved determining $\cos A$ using the Pythagorean theorem, applying the double angle formulas, and carefully considering the signs based on the quadrant. Mastering these techniques enables students and professionals to solve complex problems efficiently and accurately.

Remember, practice makes perfect. Try applying these identities to different values of A and explore how the results vary across quadrants. With consistent practice, you'll develop a strong intuition for trigonometric relationships and their applications in various fields.

Frequently Asked Questions

Given that $\sin A = 4/5$, how do you find $\cos A$?

Using Pythagoras theorem, $\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - (4/5)^2} = \sqrt{1 - 16/25} = \sqrt{9/25} = 3/5$.

What is the value of $\sin 2A$ if $\sin A = 4/5$?

$\sin 2A = 2 \sin A \cos A = 2 (4/5) (3/5) = 24/25$.

How do you calculate $\cos 2A$ given $\sin A = 4/5$?

$\cos 2A = \cos^2 A - \sin^2 A = (3/5)^2 - (4/5)^2 = 9/25 - 16/25 = -7/25$.

What is $\tan 2A$ when $\sin A = 4/5$?

$\tan 2A = (2 \tan A) / (1 - \tan^2 A)$. First, find $\tan A = \sin A / \cos A = (4/5) / (3/5) = 4/3$. Then, $\tan 2A = (2 \cdot 4/3) / (1 - (4/3)^2) = (8/3) / (1 - 16/9) = (8/3) / (-7/9) = (8/3) \cdot (-9/7) = -24/7$.

Are the values of $\sin 2A$, $\cos 2A$, and $\tan 2A$ consistent with the initial $\sin A$ value?

Yes, they are consistent. $\sin 2A = 24/25$, $\cos 2A = -7/25$, and $\tan 2A = -24/7$, derived from the initial $\sin A = 4/5$ and corresponding $\cos A$.

In what quadrant is angle A if $\sin A = 4/5$?

Since $\sin A$ is positive and $\cos A$ is positive ($3/5$), A is in the first quadrant.

Can you verify the values of $\sin 2A$, $\cos 2A$, and $\tan 2A$ with a calculator?

Yes, using the calculated values: $\sin 2A \approx 0.96$, $\cos 2A \approx -0.28$, $\tan 2A \approx -3.43$, which can be checked with calculator functions for accuracy.

Additional Resources

If $\sin A = 4/5$ Solve $\sin 2A$, $\cos 2A$ And $\tan 2A$

Understanding the relationships between trigonometric functions is fundamental to solving a wide array of problems in mathematics, physics, and engineering. When given a specific value of a sine function, such as $\sin A = 4/5$, it opens the door to calculating other related functions, particularly the double angle formulas. These formulas are essential because they allow us to express the sine, cosine, and tangent of twice an angle in terms of the original angle's sine and cosine.

In this article, we will explore the step-by-step process to solve for $\sin 2A$, $\cos 2A$, and $\tan 2A$ given that $\sin A = 4/5$. We will delve into the underlying principles, demonstrate the calculations thoroughly, and clarify any potential pitfalls or misconceptions along the way. Whether you are a student preparing for exams or a professional seeking to reinforce your understanding of trigonometry, this comprehensive guide aims to make the topic accessible and engaging.

Understanding the Given Data: $\sin A = 4/5$

The starting point for our problem is the given value:

- $\sin A = 4/5$

This suggests that the sine of angle A is a fraction, representing a ratio of the side opposite to angle A over the hypotenuse in a right-angled triangle. To proceed, it's crucial to visualize or determine the corresponding cosine value, which will be necessary for calculating the double angle functions.

Visualizing the Triangle

Since $\sin A = 4/5$, we can imagine a right triangle where:

- The side opposite angle A is 4 units.
- The hypotenuse is 5 units.

Using the Pythagorean theorem, the adjacent side (let's denote it as "b") can be calculated:

$$\begin{aligned} b &= \sqrt{(\text{hypotenuse})^2 - (\text{opposite})^2} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} \\ &= \sqrt{9} = 3 \end{aligned}$$

Thus, the triangle's sides are:

- Opposite side: 4
- Adjacent side: 3
- Hypotenuse: 5

This allows us to determine cosine and tangent values directly:

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{3}{5}$$

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{4}{3}$$

With these, the foundation for calculating double angles becomes straightforward.

Calculating Sin2A, Cos2A, and Tan2A

Double angle formulas are powerful tools in trigonometry, enabling us to find the sine, cosine, and tangent of twice an angle based on the original angle's sine and cosine. Let's explore each in detail.

1. Sin2A: The Double Angle Sine Formula

The formula for the sine of double an angle is:

$$\sin 2A = 2 \sin A \cos A$$

Using the values obtained:

$$\begin{aligned} \sin A &= \frac{4}{5} \\ \cos A &= \frac{3}{5} \end{aligned}$$

Plugging these into the formula:

$$\sin 2A = 2 \times \frac{4}{5} \times \frac{3}{5} = 2 \times \frac{12}{25} = \frac{24}{25}$$

\]

Result:

```
\[
\boxed{
\sin 2A = \frac{24}{25}
}
\]
```

This value signifies that the sine of twice angle A is 24/25, which is very close to 1, indicating a large value near the maximum of the sine function.

2. Cos2A: The Double Angle Cosine Formula

The double angle formula for cosine can be expressed in multiple ways:

```
\[
\cos 2A = \cos^2 A - \sin^2 A
\]
or equivalently,
\[
\cos 2A = 2 \cos^2 A - 1
\]
or
\[
\cos 2A = 1 - 2 \sin^2 A
\]
\]
```

Any of these forms can be used based on the known quantities. Using the first form:

```
\[
\cos 2A = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 =
\frac{9}{25} - \frac{16}{25} = -\frac{7}{25}
\]
```

Alternatively, using the second form:

```
\[
\cos 2A = 2 \times \left(\frac{3}{5}\right)^2 - 1 = 2 \times \frac{9}{25} - 1
= \frac{18}{25} - 1 = \frac{18}{25} - \frac{25}{25} = -\frac{7}{25}
\]
```

Both methods agree.

Result:

```
\[
\boxed{
\cos 2A = -\frac{7}{25}
}
\]
```

This negative value indicates that the double angle is in the second or third quadrant, where cosine can be negative.

3. Tan2A: The Double Angle Tangent Formula

The tangent of double an angle can be derived from the sine and cosine double angle results or directly from the original tangent:

$$\tan 2A = \frac{\sin 2A}{\cos 2A}$$

Plugging in the calculated values:

$$\tan 2A = \frac{\frac{24}{25}}{-\frac{7}{25}} = -\frac{24}{7}$$

Result:

$$\boxed{\tan 2A = -\frac{24}{7}}$$

The negative value indicates the double angle is in a quadrant where tangent is negative, consistent with the cosine value.

Summary and Interpretation of Results

Function	Value	Notes
Sin 2A	24/25	Very close to 1, indicating a large sine value for the double angle
Cos 2A	-7/25	Negative, implying the double angle resides in the second or third quadrant
Tan 2A	-24/7	Negative, consistent with cosine's sign

These results are consistent and align with the properties of the trigonometric functions in different quadrants. They exemplify how knowing just the sine value of an angle allows us to compute all the double angle functions with confidence.

Practical Applications and Further Insights

Knowing how to compute these functions is not purely academic; they have numerous applications:

- Wave analysis: In physics, understanding wave behavior often involves double angle formulas for phase shifts.
- Engineering: Signal processing and control systems utilize double angle identities to manipulate phase and amplitude.
- Navigation and Astronomy: Precise angle calculations are vital in celestial navigation and satellite positioning.

Furthermore, the methodology demonstrated here can be extended to solve for double angles when given other trigonometric ratios. The key steps are always:

1. Determine the basic sine and cosine values.
2. Apply the appropriate double angle formulas.
3. Simplify carefully, keeping track of signs and quadrant considerations.

Final Thoughts

The process of solving for $\sin 2A$, $\cos 2A$, and $\tan 2A$ when $\sin A = 4/5$ illustrates the elegance and power of trigonometric identities. With a clear understanding of the fundamental relationships and the Pythagorean theorem, complex-looking problems become manageable and intuitive.

By mastering these techniques, students and professionals alike can enhance their problem-solving toolkit, enabling them to tackle a wide range of mathematical and real-world challenges. Remember, the key lies in a systematic approach—visualize the problem, calculate the foundational values, and then apply the identities confidently.

Whether you're preparing for an exam or applying trigonometry in your field, the ability to manipulate and interpret these functions remains a valuable skill. The journey from a simple sine value to a comprehensive understanding of double angles embodies the beauty and utility of mathematics in deciphering the world around us.

If $\sin A = 4/5$ Solve $\sin 2A$ $\cos 2A$ And $\tan 2A$

If $\sin A = 4/5$ Solve $\sin 2A$ $\cos 2A$ And $\tan 2A$

Related Articles

- [Given The Figure, Find The Values Of X And Z.](#)
- [Which Cation Is Example Is A Foreign Policy Decision?](#)
- <https://www.oklahoma4.create.ou.edu/>

[Back to Home](#)