Inverse Laplace Of L^-1 {5s/s^2 + 3s - 4}

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Understanding the inverse Laplace transform is fundamental in solving differential equations, particularly in engineering and physics applications. The expression *Inverse Laplace Of L^-1 {5s/s^2 + 3s - 4}* is a common problem that involves transforming a complex algebraic function from the s-domain back into the time domain. This process allows engineers and mathematicians to analyze system behaviors, such as electrical circuits, mechanical systems, and control systems, in a more intuitive and tangible way.

In this comprehensive guide, we will explore the detailed process of computing the inverse Laplace transform of the given function. We will break down the problem step by step, covering essential concepts, techniques, and applications to ensure a thorough understanding.

Understanding the Laplace Transform and Its Inverse

Before diving into the specific problem, it is crucial to grasp the fundamentals of the Laplace transform and its inverse.

What is the Laplace Transform?

The Laplace transform is an integral transform that converts a time-domain function $f(t)$ into a complex frequency domain function $F(s)$:

$$
F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt
$$

This transformation simplifies differential equations into algebraic equations, making them easier to solve.

What is the Inverse Laplace Transform?

The inverse Laplace transform, denoted as $\mathcal{L}^{-1}$, reverses this process, converting a function $F(s)$ in the s-domain back into the original time-domain function $f(t)$:

$$
f(t) = \mathcal{L}^{-1}\{F(s)\}
$$

Calculating the inverse involves techniques such as partial fraction decomposition, the use of Laplace transform tables, and complex analysis methods.

Analyzing the Given Function: $\frac{5s}{s^2 + 3s - 4}$

The core of the problem is to compute:

$$
f(t) = \mathcal{L}^{-1} \left\{ \frac{5s}{s^2 + 3s - 4} \right\}
$$

The first step is to analyze the algebraic structure of the Laplace domain function.

Step 1: Factor the Denominator

Factor the quadratic denominator to identify its roots:

$$
s^2 + 3s - 4 = 0
$$

Using the quadratic formula:

$$
s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 - (-16)}}{2} = \frac{-3 \pm \sqrt{25}}{2}
$$

$$
s = \frac{-3 \pm 5}{2}
$$

Thus, the roots are:

$$
s = 1 \quad \text{and} \quad s = -4
$$

The factorization becomes:

$$
s^2 + 3s - 4 = (s - 1)(s + 4)
$$

Step 2: Rewrite the Function Using Partial Fractions

Express the original function as a sum of simpler fractions:

$$
\frac{5s}{(s - 1)(s + 4)} = \frac{A}{s - 1} + \frac{B}{s + 4}
$$

Multiply both sides by $(s - 1)(s + 4)$:

$$
5s = A(s + 4) + B(s - 1)
$$

Expand:

$$
5s = A s + 4A + B s - B
$$

Group like terms:

$$
5s = (A + B) s + (4A - B)
$$

Matching coefficients:

$$
\begin{cases}
A + B = 5 \\
4A - B = 0
\end{cases}
$$

Solve for A and B:

From the second equation:

$$
B = 4A
$$

Substitute into the first:

$$
A + 4A = 5 \Rightarrow 5A = 5 \Rightarrow A = 1
$$

Then:

$$
B = 4A = 4 \times 1 = 4
$$

The partial fraction expansion is:

$$
\frac{5s}{(s - 1)(s + 4)} = \frac{1}{s - 1} + \frac{4}{s + 4}
$$

Calculating the Inverse Laplace Transform

Now that the function is decomposed, the inverse Laplace transform can be found using standard tables or known transforms.

Standard Inverse Laplace Transforms

Recall:

$$
\mathcal{L}^{-1}\left\{\frac{1}{s - a}\right\} = e^{a t}
$$

$$
\mathcal{L}^{-1}\left\{\frac{1}{s + a}\right\} = e^{-a t}
$$

Applying these to each term:

$$
f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s - 1}\right\} + 4 \times
\mathcal{L}^{-1}\left\{\frac{1}{s + 4}\right\} = e^{t} + 4 e^{-4 t}
$$

Result:

$$
f(t) = e^{t} + 4 e^{-4 t}
$$

Summary of the Calculation

To summarize, the steps to compute the inverse Laplace of $\frac{5s}{s^2 + 3s - 4}$ are:

1. Factor the denominator: $(s - 1)(s + 4)$
2. Partial fraction decomposition: $\frac{A}{s - 1} + \frac{B}{s + 4}$
3. Solve for A and B: $A = 1$, $B = 4$
4. Apply inverse transforms: e^{t} and $4 e^{-4 t}$

Final Time-Domain Function:

$$
f(t) = e^{t} + 4 e^{-4 t}
$$

Applications of the Inverse Laplace Transform

Understanding how to compute inverse Laplace transforms like this one has numerous practical applications across various fields:

- Electrical Engineering: Analyzing the response of RLC circuits, filters, and control systems.
- Mechanical Engineering: Studying vibrations, damping, and system dynamics.
- Control Systems: Designing controllers and analyzing system stability.
- Physics: Solving differential equations related to heat conduction, wave propagation, and quantum mechanics.
- Mathematics: Solving linear differential equations with constant coefficients.

Advanced Techniques for More Complex Functions

While partial fraction decomposition works well for rational functions with simple roots, more complex functions may require advanced methods such as:

- Completing the square for quadratic denominators
- Using convolution theorem for products of transforms
- Applying complex analysis, including residues, for inverse transforms of

non-rational functions
- Utilizing Laplace transform tables for common transforms

Conclusion

The process of finding the inverse Laplace transform of $\frac{5s}{s^2 + 3s - 4}$ demonstrates fundamental techniques in operational calculus. By factoring the denominator, applying partial fraction decomposition, and leveraging standard inverse transforms, one can efficiently transition from the s-domain back to the time domain.

This methodology is essential for engineers and mathematicians working with differential equations, control systems, and signal processing. Mastery of inverse Laplace transforms enables the analysis and design of complex systems, providing insights into their dynamic behaviors in a clear and manageable form.

Additional Resources

- Laplace Transform Tables: A comprehensive list of common transforms and their inverses.
- Mathematical Software: Tools like MATLAB, Wolfram Alpha, and Python's SymPy library can automate inverse Laplace calculations.
- Textbooks: "Advanced Engineering Mathematics" by Erwin Kreyszig offers detailed explanations and numerous examples.
- Online Tutorials: Websites like Khan Academy and Paul's Online Math Notes provide accessible tutorials on Laplace transforms.

In summary, understanding and applying the inverse Laplace transform to functions like $\frac{5s}{s^2 + 3s - 4}$ is a vital skill in the mathematical toolkit for solving real-world problems involving differential equations and system analysis.

Frequently Asked Questions

What is the Laplace inverse of L^{-1} {5s / (s^2 +

3s - 4)}?

The inverse Laplace transform of L^{-1} {5s / (s^2 + 3s - 4)} is the function f(t) whose Laplace transform is 5s / (s^2 + 3s - 4).

How do you simplify the expression L^{-1} {5s / (s^2 + 3s - 4)}?

First, factor the denominator to find partial fractions, then apply inverse Laplace transforms to each term to find the time domain function.

What is the factorization of the denominator s^2 + 3s - 4?

The quadratic factors as (s + 4)(s - 1).

How can partial fractions be used to find the inverse Laplace transform in this case?

Express 5s / [(s + 4)(s - 1)] as A / (s + 4) + B / (s - 1), then solve for A and B to simplify the inverse transform.

What are the partial fraction coefficients for 5s / [(s + 4)(s - 1)]?

Solving for A and B gives A = 5/5 = 1 and B = 5/(-5) = -1, so the partial fractions are 1 / (s + 4) - 1 / (s - 1).

What is the inverse Laplace transform of 1 / (s + a)?

The inverse Laplace transform of 1 / (s + a) is e^{-a t}.

What is the final time domain expression for the given inverse Laplace transform?

The inverse Laplace transform of the original function is f(t) = e^{-4t} - e^{t}.

Are there any conditions or constraints for the inverse Laplace transform of this function?

Yes, the inverse transform is valid for t ≥ 0, assuming causal systems and standard Laplace transform conditions.

How is the inverse Laplace transform related to solving differential equations?

Inverse Laplace transforms convert algebraic functions in the s-domain back to the time domain, facilitating the solution of linear differential equations with initial conditions.

Additional Resources

Inverse Laplace Of L^-1 {5s / (s^2 + 3s - 4)} is a fascinating subject that bridges the theoretical aspects of complex analysis with practical applications in engineering and physics. Understanding how to invert Laplace transforms, especially when dealing with rational functions, is crucial for solving differential equations, analyzing systems, and modeling dynamic processes. This article aims to explore this specific inverse Laplace problem in detail, providing a comprehensive guide to its solution, interpretation, and applications.

Introduction to Laplace Transforms and Their Inverses

The Laplace transform is a powerful integral transform used to convert differential equations in the time domain into algebraic equations in the complex frequency domain (s-domain). Its primary advantage is simplifying the process of solving linear ordinary differential equations (ODEs), especially with initial conditions.

The Laplace transform of a function $f(t)$ is defined as:

$$
L\{f(t)\} = F(s) = \int_{0}^{\infty} e^{-st}f(t) dt
$$

The inverse Laplace transform, denoted as L^{-1}, takes a function $F(s)$ back into the original time domain $f(t)$.

Understanding how to perform inverse transforms, especially for rational functions like $\frac{5s}{s^2 + 3s - 4}$, is essential for analyzing system behaviors, response times, and stability.

Breaking Down the Problem: Inverse Laplace of $L^{-1} \left\{ \frac{5s}{s^2 + 3s - 4} \right\}$

The core challenge here is to find the inverse Laplace transform of the function:

$$
F(s) = \frac{5s}{s^2 + 3s - 4}
$$

This involves several steps:

1. Factorizing the denominator
2. Partial fraction decomposition
3. Matching the resulting terms to known inverse Laplace transforms
4. Interpreting the time-domain function

Let's explore each step in detail.

Step 1: Factorizing the Denominator

The denominator $s^2 + 3s - 4$ is a quadratic expression. To simplify the inverse transform process, it’s best to factor it into linear factors if possible.

Find roots of $s^2 + 3s - 4 = 0$:

$$
s = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times (-4)}}{2 \times 1} = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm \sqrt{25}}{2}
$$

$$
s = \frac{-3 \pm 5}{2}
$$

Thus, the roots are:

- $s = \frac{-3 + 5}{2} = 1$
- $s = \frac{-3 - 5}{2} = -4$

So the factorization is:

\[

$$
s^2 + 3s - 4 = (s - 1)(s + 4)
$$

Step 2: Partial Fraction Decomposition

Express $F(s)$ as partial fractions:

$$
\frac{5s}{(s - 1)(s + 4)} = \frac{A}{s - 1} + \frac{B}{s + 4}
$$

Multiply both sides by $(s - 1)(s + 4)$:

$$
5s = A(s + 4) + B(s - 1)
$$

Expand:

$$
5s = A s + 4A + B s - B
$$

Group like terms:

$$
5s = (A + B) s + (4A - B)
$$

Matching coefficients:

- For s: $A + B = 5$
- Constant term: $4A - B = 0$

Solve the system:

From the second:

$$
4A = B
$$

Substitute into the first:

$$
A + 4A = 5 \Rightarrow 5A = 5 \Rightarrow A = 1
$$

Then:

$$
B = 4A = 4 \times 1 = 4
$$

Thus,

$$
F(s) = \frac{1}{s - 1} + \frac{4}{s + 4}
$$

Step 3: Inverse Laplace Transform

Recall standard inverse Laplace transforms:

$$
L^{-1} \left\{ \frac{1}{s - a} \right\} = e^{a t}
$$

Applying this to each term:

$$
L^{-1} \left\{ \frac{1}{s - 1} \right\} = e^{t}
$$
$$
L^{-1} \left\{ \frac{4}{s + 4} \right\} = 4 e^{-4 t}
$$

Therefore, the inverse Laplace transform of $F(s)$ is:

$$
f(t) = e^{t} + 4 e^{-4 t}
$$

Final Result and Interpretation

The solution to the inverse Laplace transform of $\frac{5s}{s^2 + 3s - 4}$ is:

$$
\boxed{
f(t) = e^{t} + 4 e^{-4 t}

}
\]

This function describes the time-domain response corresponding to the original Laplace domain expression. It combines exponential growth and decay components, which are common in systems characterized by differential equations with real roots.

Features and Applications of the Result

Understanding this specific inverse Laplace transform has several notable features:

- Simple exponential terms: The solution involves straightforward exponential functions, making it easy to interpret in physical systems.
- System response analysis: Such functions are often used to describe system responses like electrical circuits, mechanical vibrations, or thermal processes.
- Stability considerations: The presence of positive and negative exponentials indicates potential transient behaviors, with the exponential decay term suggesting damping or stabilization over time.

Advantages and Limitations

Advantages:

- Clear analytical expression: The inverse transform yields a simple, interpretable function.
- Ease of computation: Factoring and partial fractions simplify the process significantly.
- Applicability: The method applies broadly to rational functions with quadratic denominators.

Limitations:

- Requires factorization: If the quadratic cannot be factored over real numbers, more advanced methods like completing the square or using complex roots are needed.
- Complex functions: For higher-order or more complicated functions, partial fractions become more involved.
- Assumption of causality: The inverse Laplace transform assumes the original function is causal (\(f(t) = 0\) for \(t < 0\)).

Extensions and Further Study

For those interested in exploring further, consider the following:

- Inverse transforms with complex roots: When roots are complex conjugates, the inverse involves sine and cosine functions.
- Inverse Laplace of more complex functions: Rational functions with higher-degree polynomials require more advanced partial fraction techniques.
- Numerical inversion techniques: For functions without closed-form inverses, numerical methods like the Talbot method or Stehfest algorithm can be employed.
- Application in control systems: Understanding how these inverse transforms relate to system stability, transient response, and frequency response.

Conclusion

The inverse Laplace transform of $\frac{5s}{s^2 + 3s - 4}$ exemplifies the power of algebraic manipulation and partial fraction decomposition in solving time-domain problems from the s-domain. By factoring the quadratic denominator, decomposing into simpler fractions, and applying well-known inverse transforms, we obtained an elegant and interpretable solution:

$$
f(t) = e^{t} + 4 e^{-4 t}
$$

This process highlights the importance of mastering fundamental techniques for analyzing and solving differential equations and dynamic systems, making inverse Laplace transforms an indispensable tool in engineering, physics, and applied mathematics.

Inverse Laplace Of L 1 5s S 2 3s 4

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