

Question 1 Find The Highest Common Factor Of 70 and 126

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Question 1: Find The Highest Common Factor Of 70 And 126 is a fundamental problem in number theory and mathematics education. It involves determining the greatest number that divides both 70 and 126 without leaving a remainder. Finding the Highest Common Factor (HCF), also known as the Greatest Common Divisor (GCD), is essential for simplifying fractions, solving problems involving ratios, and understanding the properties of numbers. This article provides a comprehensive guide to understanding, calculating, and applying the HCF of 70 and 126, along with detailed explanations, step-by-step methods, and practical tips.

Understanding the Highest Common Factor (HCF)

What is the Highest Common Factor?

The Highest Common Factor of two or more numbers is the largest positive integer that divides each of the numbers exactly, without leaving a remainder. For example, for the numbers 70 and 126, the HCF is the largest number that evenly divides both.

Importance of Finding the HCF

- Simplifies fractions by reducing numerator and denominator.
- Helps in solving ratio and proportion problems.
- Facilitates finding common divisors in algebraic expressions.
- Enhances understanding of factors and multiples in number theory.

Methods to Find the Highest Common Factor

There are several methods to determine the HCF of two numbers, including:

1. Listing Factors

- List all factors of each number.
- Identify the common factors.
- The largest common factor is the HCF.

2. Prime Factorization Method

- Find the prime factors of each number.
- Multiply the common prime factors with the lowest powers.
- The product is the HCF.

3. Division Method (Euclidean Algorithm)

- Divide the larger number by the smaller.
- Replace the larger number with the divisor and repeat.
- Continue until the remainder is zero.
- The last divisor is the HCF.

Calculating the HCF of 70 and 126

Let's explore each method in detail to find the HCF of 70 and 126.

Method 1: Listing Factors

First, list all the factors of each number:

- Factors of 70: 1, 2, 5, 7, 10, 14, 35, 70
- Factors of 126: 1, 2, 3, 6, 7, 9, 18, 21, 42, 63, 126

Common factors are: 1, 2, 7

The largest common factor is 7, so the HCF of 70 and 126 is 7.

Method 2: Prime Factorization

Prime factorization involves breaking each number into prime factors:

- $70 = 2 \times 5 \times 7$
- $126 = 2 \times 3^2 \times 7$

Identify the common prime factors with the lowest powers:

- Common prime factors: 2 and 7
- Lowest powers: 2^1 and 7^1

Multiply these common prime factors:

$$\text{HCF} = 2 \times 7 = 14$$

Method 3: Euclidean Algorithm

This method involves repeated division:

1. Divide 126 by 70:

$126 \div 70 = 1$ with a remainder of 56

2. Now, divide 70 by 56:

$70 \div 56 = 1$ with a remainder of 14

3. Next, divide 56 by 14:

$56 \div 14 = 4$ with a remainder of 0

Since the remainder is now zero, the last divisor, 14, is the HCF.

Reconciling the Results

- From listing factors, the HCF is 7.
- From prime factorization, the HCF is 14.
- From Euclidean Algorithm, the HCF is 14.

This apparent discrepancy arises because of an error in the listing factors method. Re-examining the factors confirms that 14 is indeed a common divisor:

Factors of 70: 1, 2, 5, 7, 10, 14, 35, 70

Factors of 126: 1, 2, 3, 6, 7, 9, 18, 21, 42, 63, 126

Common factors: 1, 2, 7

But 14 does not divide 126 evenly (since $126 \div 14 = 9$ with no remainder), so actually, 14 is a common divisor of 70 and 126. Let's verify:

- $70 \div 14 = 5$ (no remainder)
- $126 \div 14 = 9$ (no remainder)

Therefore, the HCF is 14.

Conclusion: The highest common factor of 70 and 126 is 14.

Practical Applications of Finding the HCF

Reducing Fractions

- To simplify fractions like $70/126$, divide numerator and denominator by their HCF:

$$70 \div 14 = 5$$

$$126 \div 14 = 9$$

Simplified fraction: $5/9$

Solving Ratios

- For example, if a recipe calls for a ratio of 70:126, simplifying to 5:9 helps in scaling the recipe.

Factorization in Algebra

- Finding common factors in algebraic expressions allows for factorization and solving equations more efficiently.

Additional Tips for Finding the HCF

- Always verify the common factors by division to avoid mistakes.
- Use prime factorization for larger numbers or when precision is necessary.
- Practice using the Euclidean Algorithm for quick calculations, especially with large numbers.
- Remember that the HCF always divides each of the numbers exactly.

Summary

Finding the Highest Common Factor of two numbers like 70 and 126 involves understanding the concept of common divisors and applying systematic methods such as listing factors, prime factorization, or the Euclidean Algorithm. In this case, the correct HCF is 14, which can be verified through multiple approaches. Mastering these methods enhances mathematical problem-solving skills and provides foundational knowledge useful in various areas of mathematics and real-world applications.

Final Thoughts

Whether you're a student tackling basic math problems or a professional dealing with ratios and fractions, knowing how to find the Highest Common Factor efficiently is invaluable. Practice with different pairs of numbers to become confident in selecting and applying the most suitable method. Remember, understanding the process is as important as the answer itself, as it deepens your grasp of number properties and mathematical reasoning.

Frequently Asked Questions

What is the highest common factor (HCF) of 70 and 126?

The HCF of 70 and 126 is 14.

How do you find the highest common factor (HCF) of 70 and 126?

To find the HCF of 70 and 126, prime factorize both numbers and multiply the common prime factors. For 70 ($2 \times 5 \times 7$) and 126 ($2 \times 3^2 \times 7$), the common factors are 2 and 7, so the HCF is 14.

Can you explain the prime factorization method to find the HCF of 70 and 126?

Yes. Prime factorize each number: $70 = 2 \times 5 \times 7$, $126 = 2 \times 3 \times 3 \times 7$. The common prime factors are 2 and 7. Multiply these to get the HCF: $2 \times 7 = 14$.

Is there a shortcut to find the HCF of 70 and 126 without prime factorization?

Yes. You can use the Euclidean algorithm, which involves repeated division and remainders to find the HCF efficiently.

What is the importance of finding the HCF of two numbers like 70 and 126?

Finding the HCF helps in simplifying fractions, solving problems involving ratios, and determining the largest common divisor for various mathematical applications.

Are 70 and 126 coprime?

No, 70 and 126 are not coprime because they have common factors (2 and 7). Their HCF is 14, which is greater than 1.

What are some real-life applications of finding the HCF of numbers like 70 and 126?

HCF is used in dividing items into equal parts, simplifying ratios in recipes, engineering measurements, and optimizing resource allocations.

Additional Resources

Question 1: Find The Highest Common Factor Of 70 and 126

Understanding the Highest Common Factor (HCF) of two numbers is a fundamental concept in mathematics that holds significance across various fields such as engineering, computer science, and everyday problem-solving. In this article, we will explore the process of finding the Highest Common Factor (HCF) of the numbers 70 and 126, breaking down the methodology into clear, comprehensible steps. Whether you're a student brushing up on your math skills or a curious reader interested in the mathematical underpinnings of common factors, this in-depth guide aims to shed light on the process with clarity and precision.

What Is the Highest Common Factor (HCF)?

Before delving into the specific problem involving 70 and 126, it's essential to understand what the Highest Common Factor (HCF) actually represents. The HCF of two or more numbers is the greatest positive integer that divides each of the numbers without leaving a remainder. It is also commonly known as the Greatest Common Divisor (GCD).

Key points to understand about HCF:

- Common Divisor: An integer that divides two or more numbers exactly, with no remainder.
- Greatest: Among all common divisors, the HCF is the largest.
- Application: HCF is useful for simplifying fractions, solving problems involving ratios, and reducing algebraic expressions.

Example:

For the numbers 8 and 12, the common divisors are 1, 2, and 4. Among these, 4 is the largest, so the HCF of 8 and 12 is 4.

With this understanding, let's turn our focus to the specific numbers in question: 70 and 126.

Methodologies to Find the HCF

There are several methods to compute the HCF of two numbers, each suitable for different contexts and preferences. The two most common methods are:

1. Listing Factors Method
2. Prime Factorization Method
3. Division Method (Euclidean Algorithm)

In this article, we'll primarily use the prime factorization method and the Euclidean Algorithm, as they are efficient and scalable for larger numbers.

Method 1: Listing Factors (Basic Approach)

Step-by-step process:

- List all factors of 70:
 - 1, 2, 5, 7, 10, 14, 35, 70
- List all factors of 126:
 - 1, 2, 3, 6, 7, 9, 18, 21, 42, 63, 126
- Identify common factors:
 - 1, 2, 7
- The greatest of these common factors:
 - 7

Result: The HCF of 70 and 126, based on listing factors, is 7.

While this approach works well for smaller numbers, it becomes cumbersome with larger integers. Therefore, more systematic methods like prime factorization or the Euclidean Algorithm are typically preferred.

Method 2: Prime Factorization

Prime factorization involves expressing each number as a product of prime numbers. This method provides a clear view of common prime factors, making it straightforward to identify the HCF.

Step 1: Prime Factorize 70

- $70 \div 2 = 35$
- $35 \div 5 = 7$
- 7 is prime

Therefore, prime factorization of 70:

$$70 = 2 \times 5 \times 7$$

Step 2: Prime Factorize 126

- $126 \div 2 = 63$
- $63 \div 3 = 21$
- $21 \div 3 = 7$
- 7 is prime

Prime factorization of 126:

$$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$$

Step 3: Identify Common Prime Factors

- Both numbers have prime factors 2 and 7.
- 70 has prime factors 2, 5, 7.
- 126 has prime factors 2, 3, 3, 7.

Step 4: Find the Common Factors and Multiply

- Common prime factors are 2 and 7.
- Multiply these common primes:

$$\text{HCF} = 2 \times 7 = 14$$

Result: The HCF of 70 and 126, based on prime factorization, is 14.

Note: This appears different from the previous listing factors method result (7), which indicates a need for careful analysis. Why the discrepancy?

Reconciling the Results: Which Is Correct?

The divergence between the two methods (7 vs. 14) warrants a detailed explanation. The listing factors method found 7 as the highest common factor, while prime factorization points to 14. To clarify, let's verify the divisibility:

- Does 14 divide both numbers?

- $70 \div 14 = 5$ (no remainder) ☐ Yes

- $126 \div 14 = 9$ (no remainder) ☐ Yes

- Does 7 divide both numbers?

- $70 \div 7 = 10$ ☐ Yes

- $126 \div 7 = 18$ ☐ Yes

- Is 14 the largest number dividing both?

- 14 is larger than 7.

- Both 14 and 7 are common divisors.

So, which is the correct HCF?

Since both 14 and 7 divide both numbers, and 14 is larger, the correct Highest Common Factor (HCF) of 70 and 126 is 14.

Therefore, the initial listing of factors was incomplete — listing factors explicitly is prone to oversight, especially with larger numbers. The prime factorization method provides a more reliable approach for larger numbers and confirms that 14 is the true HCF.

Method 3: Euclidean Algorithm (Efficient Calculation)

The Euclidean Algorithm is a systematic process based on successive division to find the HCF efficiently.

Step 1: Divide the larger number by the smaller number:

- $126 \div 70 = 1$ with a remainder of 56

Step 2: Replace the larger number with the smaller number, and the smaller with the remainder:

- Now, find HCF of 70 and 56

Step 3: Divide 70 by 56:

- $70 \div 56 = 1$ with a remainder of 14

Step 4: Repeat with 56 and 14:

- $56 \div 14 = 4$ with a remainder of 0

Since the remainder is zero, the divisor at this step, 14, is the HCF.

Result: The HCF of 70 and 126 is 14.

This method confirms our previous result, highlighting its efficiency and reliability.

Summary and Final Conclusion

Through multiple methods—listing factors, prime factorization, and the Euclidean Algorithm—we have determined that the Highest Common Factor (HCF) of 70 and 126 is 14.

Key takeaways:

- The HCF of two numbers is the largest number that divides both without remainder.
- Prime factorization is a robust method, especially when combined with the Euclidean Algorithm.
- For 70 and 126, the HCF is 14, as it is the largest number dividing both evenly.

Practical Applications of HCF

Understanding how to find the HCF is not merely an academic exercise; it has real-world applications:

- Simplifying Fractions: Dividing numerator and denominator by their HCF simplifies the fraction to its lowest terms.
- Scheduling and Dividing Resources: HCF helps in equally distributing items or tasks.
- Algebraic Simplification: Reducing algebraic expressions to their simplest form often involves calculating HCF.
- Cryptography and Computer Algorithms: Efficient algorithms for calculating GCD/HCF underpin encryption and data processing.

Final Thoughts

Finding the Highest Common Factor of 70 and 126 illustrates the beauty and utility of fundamental mathematical concepts. Whether utilizing prime factorization, the Euclidean Algorithm, or systematic listing, the goal remains the same: to identify the largest integer that evenly divides both numbers. Mastery of these techniques enhances problem-solving skills and deepens understanding of number theory, which forms the backbone of many advanced mathematical and computational disciplines.

In practice, the Euclidean Algorithm stands out for its efficiency, especially as numbers grow larger. It exemplifies how a simple recursive process can yield precise and reliable results—a testament to the elegance of mathematics.

By grasping these methods, learners and professionals alike can confidently tackle similar problems, fostering a deeper appreciation of the interconnectedness of mathematical principles and their applications in everyday life.

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