Question 5 Evaluate The Limit: Lim A3 A^3-27/a-3=______

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Understanding how to evaluate limits is a fundamental concept in calculus, especially when dealing with indeterminate forms. The problem presented—"Lim_{a→ 3} (a^3 - 27) / (a - 3)"—is a classic example that involves indeterminate forms such as 0/0. This article provides a comprehensive guide on how to evaluate this specific limit, exploring various methods including algebraic manipulation, factoring, and L'Hôpital's Rule, along with the underlying mathematical principles.

Introduction to Limits in Calculus

Limits describe the behavior of a function as the input approaches a specific value. They are essential for understanding derivatives, integrals, and the continuity of functions.

Definition of a Limit:

- The limit of a function f(a) as a approaches a point c is the value that f(a) gets closer to as a gets closer to c.
- Notation: lim_{a→ c} f(a) = L

Indeterminate Forms:

- Occur when direct substitution results in expressions like 0/0 or ∞/∞.
- Require algebraic manipulation or advanced techniques to evaluate.

Understanding the Given Limit Problem

The specific problem to evaluate is:

$$\lim_{a \to 3} \frac{a^3 - 27}{a - 3}$$

At a glance, direct substitution:

$$\frac{3^3 - 27}{3 - 3} = \frac{27 - 27}{0} = \frac{0}{0}$$

which is an indeterminate form. Therefore, we need to apply other methods to find the limit.

Mathematical Techniques to Evaluate the Limit

Several strategies can be employed to evaluate limits that result in indeterminate forms:

1. Algebraic Factoring

Factoring the numerator can often simplify the expression, especially if it resembles a difference of cubes.

Difference of Cubes Formula:

$$ a^3 - b^3 = (a - b)(a^2 + ab + b^2) $$

In this case:

$$ a^3 - 27 = a^3 - 3^3 $$

Applying the difference of cubes:

$$ a^3 - 3^3 = (a - 3)(a^2 + 3a + 9) $$

Now, the original expression becomes:

$$ \frac{(a - 3)(a^2 + 3a + 9)}{a - 3} $$

Provided $a \neq 3$, we can cancel the common factor:

$$ a^2 + 3a + 9 $$

Evaluating the Limit:

As $a \to 3$:

$$ a^2 + 3a + 9 \to 3^2 + 3 \times 3 + 9 = 9 + 9 + 9 = 27 $$

Conclusion:

$$ \boxed{27} $$

2. L'Hôpital's Rule

L'Hôpital's Rule states that if the limit results in an indeterminate form 0/0 or ∞/∞, then:

$$ \lim_{a \to c} \frac{f(a)}{g(a)} = \lim_{a \to c} \frac{f'(a)}{g'(a)} $$

provided the latter limit exists.

Applying L'Hôpital's Rule:

- Numerator derivative:

$$ \frac{d}{da}(a^3 - 27) = 3a^2 $$

- Denominator derivative:

$$ \frac{d}{da}(a - 3) = 1 $$

Thus, the limit becomes:

$$ \lim_{a \to 3} \frac{3a^2}{1} = 3 \times 3^2 = 3 \times 9 = 27 $$

Result:

Again, the limit evaluates to 27.

Step-by-Step Solution Summary

Step	Method	Description	Result
1	Direct substitution	Substitute a=3	0/0 (indeterminate)
2	Algebraic factoring	Factor numerator as a difference of cubes	Simplifies to $(a^2 + 3a + 9)$
3	Limit after factoring	Evaluate at a=3	27
4	L'Hôpital's Rule	Derivatives of numerator and denominator	27

Both methods lead to the same conclusion: the limit is 27.

Implications and Applications of the Limit

The evaluation of this limit is not just an academic exercise; it demonstrates key ideas:

- Understanding of polynomial behavior: Recognizing difference of cubes simplifies complex expressions.
- Application of calculus rules: Using L'Hôpital's Rule provides a straightforward alternative.
- Foundation for derivatives: This particular limit is related to the derivative of the function $f(a) = a^3$ at $a=3$.

Connection to Derivatives:

Recall that:

$$ f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} $$

For $f(a) = a^3$, the derivative at $a=3$:

$$ f'(3) = \lim_{h \to 0} \frac{(3+h)^3 - 3^3}{h} $$

which resembles our original limit, indicating that:

$$ \lim_{a \to 3} \frac{a^3 - 27}{a - 3} = f'(3) = 27 $$

This illustrates how limits are fundamental to understanding derivatives.

Summary of Key Concepts

- Indeterminate forms like 0/0 require special techniques such as factoring or L'Hôpital's Rule.
- Factoring differences of cubes simplifies the numerator, making the limit straightforward.
- L'Hôpital's Rule involves derivatives and is effective for limits resulting in indeterminate forms.
- The limit in question corresponds to the derivative of a^3 at $a=3$, reinforcing the connection between limits and derivatives.

Additional Practice Questions

To reinforce understanding, consider practicing with similar problems:

- Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$.
- Find $\lim_{x \to 0} \frac{\sin x}{x}$.
- Determine $\lim_{x \to \infty} \frac{3x^2 + 2}{x^2 + 1}$.

These exercises help solidify skills in limit evaluation, algebraic manipulation, and application of L'H� pital's Rule.

Conclusion

The limit:

\[\lim_{a \to 3} \frac{a^3 - 27}{a - 3} \]

can be efficiently evaluated using algebraic factoring or L'H� pital's Rule, both yielding the value 27. Recognizing the structure of the numerator as a difference of cubes is crucial, and understanding the connection to derivatives enhances comprehension of calculus fundamentals. Mastery of such limit evaluations is essential for advanced mathematical topics, including differential calculus, where they underpin the formal definition of derivatives.

Remember: When faced with indeterminate forms, always consider algebraic simplification first, then apply L'H� pital's Rule if necessary. These techniques are powerful tools for solving a wide range of limit problems in calculus.

Frequently Asked Questions

What is the limit of (a^3 - 27)/(a - 3) as a approaches 3?

The limit is 27.

How can I evaluate the limit lim a� 3 (a^3 - 27)/(a - 3)?

By recognizing that the expression is a difference quotient for a^3 at a=3, so applying direct substitution or factoring helps. The limit equals 3^3 = 27.

Is there a shortcut to evaluate lim a� 3 (a^3 - 27)/(a - 3)?

Yes, using the difference of cubes formula: a^3 - 27 = (a - 3)(a^2 + 3a + 9). Cancelling (a - 3), the limit becomes lim a� 3 (a^2 + 3a + 9) = 3^2 + 33 + 9 = 9 + 9 + 9 = 27.

What mathematical concept is used to evaluate lim a� 3 (a^3 - 27)/(a - 3)?

The difference of cubes factorization and direct substitution are used to evaluate this limit.

Can L'H� pital's Rule be applied to compute lim a� 3 (a^3 - 27)/(a - 3)?

Yes. Derive numerator and denominator separately: derivative of a^3 - 27 is 3a^2, and derivative of a - 3 is 1. Plugging in a=3 gives 3(3)^2 = 39=27.

Why does the limit of (a^3 - 27)/(a - 3) as a approaches 3 equal 27?

Because (a^3 - 27) factors as (a - 3)(a^2 + 3a + 9), and cancelling (a - 3) allows direct substitution, resulting in 3^2 + 33 + 9 = 27.

Is the limit finite or infinite for Lim a→ 3 (a^3 - 27)/(a - 3)?

The limit is finite and equals 27.

What is the value of the limit Lim a→ 3 (a^3 - 27)/(a - 3)?

The value of the limit is 27.

How does factoring help in evaluating the limit Lim a→ 3 (a^3 - 27)/(a - 3)?

Factoring the numerator as a difference of cubes allows cancelling the (a - 3) term, simplifying the expression and enabling direct substitution.

Could you verify the limit Lim a→ 3 (a^3 - 27)/(a - 3) using substitution after factoring?

Yes. After factoring, the expression simplifies to a^2 + 3a + 9. Substituting a=3 gives 9 + 9 + 9 = 27, confirming the limit.

Additional Resources

Evaluate the Limit: Lima→ 3(a3 - 27) / (a - 3)

When faced with the question of evaluating a limit such as Lima→ 3(a3 - 27) / (a - 3), students and mathematicians alike often encounter the challenge of dealing with indeterminate forms. This particular limit is a classic example in calculus that involves polynomial expressions and indeterminate forms like 0/0, which require careful analysis and application of limit laws. Understanding how to approach such problems is fundamental for mastering calculus concepts, especially derivatives and polynomial limits.

Understanding the Limit Expression

Before diving into the solution, it's essential to interpret the given expression:

Lima→ 3(a3 - 27) / (a - 3)

Here, "a" is the variable approaching the value 3. The numerator is a3 - 27, and the denominator is a - 3. Notice that plugging in a=3 directly results in 0/0, an indeterminate form. Such forms indicate that we need to manipulate the expression to find the limit or apply a suitable theorem.

Key observations:

- The numerator resembles a difference of cubes, since 27 is 33.
- Direct substitution leads to 0/0, which suggests factoring or other techniques are necessary.

Methods for Evaluating the Limit

There are several approaches to evaluate limits like this:

1. Direct Substitution

- Plugging in A=3 yields 0/0, an indeterminate form.
- Therefore, direct substitution is not sufficient here.

2. Factoring and Simplification

- Recognize the numerator as a difference of cubes:

A3 - 33 = (A - 3)(A2 + 3A + 9)
- Factor numerator:

A3 - 27 = (A - 3)(A2 + 3A + 9)
- Rewrite the limit:

Lima→3 [(A - 3)(A2 + 3A + 9)] / (A - 3)

- Cancel (A - 3):

Lima→3 A2 + 3A + 9

- Now, substitute A=3:

32 + 3×3 + 9 = 9 + 9 + 9 = 27

Result: The limit evaluates to 27.

Advantages:
- Straightforward and relies on algebraic factoring.
- Avoids advanced calculus theorems.

Limitations:
- Requires recognizing the difference of cubes.
- Not always applicable if the numerator isn't factorable easily.

3. Using the Derivative (L'H□pital's Rule)

- Since the direct substitution yields 0/0, L'H□pital's rule states:

Lima→3 (A3 - 27) / (A - 3) = Lima→3 d/dA [A3 - 27] / d/dA [A - 3]

- Compute derivatives:

d/dA [A3 - 27] = 3A2

d/dA [A - 3] = 1

- Now, evaluate the limit of the derivatives:

Lima→3 3A2 = 3×(3)2 = 3×9 = 27

Advantages:
- Powerful for indeterminate forms.
- Generalizable to more complex functions.

Limitations:
- Requires the functions to be differentiable at the point.
- Slightly more advanced.

Step-by-Step Solution Summary

1. Identify the form of the limit: Direct substitution yields 0/0, an indeterminate form.
2. Recognize the numerator as a difference of cubes: A3 - 27 = (A - 3)(A2 + 3A + 9).
3. Factor numerator and cancel common factors: Simplifies to A2 + 3A + 9.
4. Substitute the approaching value: A=3 yields 27.
5. Conclude the limit: The value of the original expression as A approaches 3 is 27.

Features, Pros, and Cons of Different Methods

Method	Features	Pros	Cons
Direct Substitution	Simple, initial step	Quick when applicable	Fails with indeterminate forms
Factoring	Algebraic manipulation	Clear, straightforward	Requires recognizing patterns (difference of cubes)
L'Hôpital's Rule	Calculus-based, derivative approach	Effective for complex indeterminate forms	Requires differentiability, more complex

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Real-world Applications and Significance

Understanding how to evaluate limits is crucial in many mathematical and engineering applications:

- Derivatives: Limits form the foundation of derivative definitions.
- Continuity Analysis: Limits determine the behavior of functions at specific points.
- Physics: Calculating instantaneous rates of change.
- Economics: Marginal analysis often involves evaluating limits of functions representing cost or revenue.

In particular, this problem demonstrates the importance of algebraic manipulation as an accessible approach, and L'Hôpital's Rule as a powerful calculus tool.

Conclusion

The limit Lima→3 (A3 - 27) / (A - 3) evaluates elegantly to 27. The key steps involve recognizing the numerator as a difference of cubes, factoring, simplifying, and then substituting the approaching value. Alternatively, applying L'H�pital's Rule confirms this result efficiently. Both methods showcase fundamental calculus techniques, emphasizing the importance of algebraic insight and derivative applications in limit evaluation.

This problem exemplifies core principles in calculus—dealing with indeterminate forms and employing algebraic or calculus-based strategies—making it an essential example for students seeking to master limit calculations. Whether through factoring or derivatives, understanding these methods enhances mathematical intuition and problem-solving skills, paving the way for more complex calculus challenges.

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