Sketch The Graph Of The Transformed Function Y=1(x-2)^2

Sketch The Graph Of The Transformed Function Y=1(x-2)^2 is a fundamental topic in algebra and coordinate geometry that helps students understand how functions are affected by transformations. Visualizing the graph of a quadratic function like this one allows learners to grasp key concepts such as shifts, stretching, and reflections. In this article, we will explore step-by-step how to sketch the graph of the transformed function Y=1(x-2)^2, analyze its features, and understand the transformations involved.

Understanding the Basic Quadratic Function Y=x^2

Before delving into the transformed function, it's essential to understand the basic quadratic graph.

The Standard Parabola

- The original quadratic function Y=x^2 is a parabola opening upward.
- The vertex of this graph is at the origin (0, 0).
- It is symmetric about the y-axis.
- The graph passes through points like (1,1), (-1,1), (2,4), and (-2,4).

Key Features of Y=x^2

- **Vertex:** (0, 0)
- **Axis of Symmetry:** x=0
- **Domain:** all real numbers (-∞, ∞)
- **Range:** y ≥ 0

Understanding these basics provides a foundation for analyzing transformations applied to the quadratic function.

Analyzing the Transformed Function Y=1(x-2)^2

The given function, Y=1(x-2)^2, is a quadratic function with specific transformations applied to the parent function Y=x^2.

Breaking Down the Function

- **Coefficient (a=1):** The coefficient 1 indicates that the parabola maintains its original size and shape, with no vertical stretching or compression.
- **Horizontal Shift (x-2):** The (x-2) term inside the squared function shifts the graph horizontally.

Understanding Horizontal Shifts

- The general form for horizontal shifts is Y=(x-h)^2, which shifts the graph h units right if h > 0, and h units left if h < 0.
- In our case, h=2, so the graph shifts 2 units to the right.

Impact of the Coefficient (a=1)

- Since a=1, the parabola's width and shape remain unchanged from the basic Y=x^2.
- If a were greater than 1, the parabola would be narrower (vertical stretch).
- If a were between 0 and 1, the parabola would be wider (vertical compression).
- If a were negative, the parabola would open downward (reflection across the x-axis).

Step-by-Step Guide to Sketching the Graph

Creating an accurate graph involves identifying key features affected by the transformation.

Step 1: Find the Vertex

- For Y=1(x-2)^2, the vertex form reveals the vertex directly.
- Since the form is Y=a(x-h)^2 + k, and here k=0, the vertex is at (h, k) = (2, 0).

Step 2: Determine the Axis of Symmetry

- The axis of symmetry is the vertical line x=h, which in this case is x=2.

Step 3: Plot the Vertex

- Plot the point (2, 0) on the coordinate plane.

Step 4: Find Additional Points

- Choose x-values around the vertex, e.g., 1 and 3, to find corresponding y-values:
- For x=1: Y=1(1-2)^2 = 1(-1)^2 = 1(1)=1 → point (1,1)
- For x=3: Y=1(3-2)^2= 1(1)^2=1 → point (3,1)
- Similarly, for x=0: Y=1(0-2)^2=4 → (0,4)
- And for x=4: Y=1(4-2)^2=4 → (4,4)

Step 5: Draw the Parabola

- Sketch a smooth, symmetric parabola passing through the plotted points, ensuring it opens upward.
- Make sure the parabola is symmetric about the axis $x=2$.

Key Features of the Transformed Graph

Understanding the features of the graph helps in interpreting and analyzing the function.

Vertex

- At (2, 0), the lowest point of the parabola.

Axis of Symmetry

- The vertical line $x=2$, about which the parabola is symmetric.

Intercepts

- **Y-intercept:** When $x=0$, $Y=4$ → point (0,4).
- **X-intercepts:** When $Y=0$, $(x-2)^2=0$ → $x=2$, so the only x-intercept is at (2,0).

Opening Direction

- The parabola opens upward because the coefficient $a=1$ is positive.

Transformations Summary

Summarizing the transformations applied to the basic quadratic:

- **Horizontal Shift:** 2 units to the right.
- **Vertical Stretch/Compression:** None, since $a=1$.
- **Reflection:** None, since $a > 0$.

Additional Tips for Sketching Parabolas

To enhance accuracy and understanding, keep these tips in mind:

Use Symmetry

- Since parabolas are symmetric, plotting points on one side of the axis of symmetry and reflecting them across it helps in drawing a precise curve.

Plot Multiple Points

- Plotting several points around the vertex ensures a smooth and accurate parabola.

Check the Vertex and Intercepts

- Confirm the vertex and intercepts to verify the correctness of your sketch.

Applications of Graph Transformations

Understanding how to sketch transformed functions like Y=1(x-2)^2 is crucial in various fields:

Real-World Problem Solving

- Modeling physical phenomena such as projectile paths or optimization problems often involves transformations of quadratic functions.

Graphical Analysis

- Transformations help in analyzing how changes in equations affect graphs, which is essential in calculus and advanced algebra.

Conclusion

Sketching the graph of the transformed function Y=1(x-2)^2 involves understanding the basic properties of quadratics and recognizing how transformations like shifts, stretches, and reflections affect the graph. By identifying the vertex at (2, 0), plotting key points, and utilizing symmetry, you can accurately draw the parabola. Remember that the coefficient 1 maintains the parabola's shape, and the horizontal shift moves the graph to the right by 2 units. Mastery of these concepts not only aids in graphing but also deepens comprehension of the behavior of quadratic functions in various mathematical contexts.

Whether you're a student preparing for exams or a teacher explaining function transformations, practicing sketching transformed parabolas like Y=1(x-2)^2 will enhance your understanding and visualization skills in algebra.

Frequently Asked Questions

What is the base function in the equation y = 1(x - 2)^2?

The base function is y = x^2, which is a standard parabola opening upwards.

What transformation does the equation y = 1(x - 2)^2 represent?

It represents a horizontal shift of the parabola y = x^2 to the right by 2 units.

What is the vertex of the transformed parabola y = 1(x - 2)^2?

The vertex is at the point (2, 0).

Does the coefficient 1 affect the shape of the graph of y = 1(x - 2)^2?

No, since the coefficient is 1, the parabola retains its standard width and shape, opening upwards.

How does the graph of y = 1(x - 2)^2 compare to y = x^2?

It is the same shape as y = x^2 but shifted 2 units to the right.

What is the axis of symmetry for y = 1(x - 2)^2?

The axis of symmetry is the vertical line x = 2.

What are some key points to plot for y = 1(x - 2)^2?

Key points include (2, 0) (the vertex), (1, 1), (3, 1), (0, 4), and (4, 4).

How does the graph change if the coefficient were changed to a value other than 1?

Changing the coefficient would stretch or compress the parabola vertically, making it wider or narrower.

Is the parabola y = 1(x - 2)^2 symmetric? If so, about which line?

Yes, it is symmetric about the vertical line x = 2.

What is the domain and range of y = 1(x - 2)^2?

The domain is all real numbers (-∞, ∞), and the range is y ≥ 0.

Additional Resources

Graphing Transformed Quadratic Functions: An Expert Guide to Sketching \(y = (x - 2)^2 \)

Understanding how to accurately sketch the graph of a transformed quadratic function is a fundamental skill in algebra and calculus. Today, we delve into the specifics of the function $y = (x - 2)^2$, a classic example of a parabola undergoing horizontal translation. This comprehensive guide aims to equip students, educators, and math enthusiasts with a detailed, step-by-step approach to visualizing and sketching this important function, emphasizing the principles of transformations, domain, range, and key features.

Decoding the Basic Function: The Standard Parabola $y = x^2$

Before exploring transformations, it's essential to understand the foundation: the basic parabola $y = x^2$.

Characteristics of $y = x^2$:

- Vertex: The minimum point at the origin $(0,0)$.
- Axis of Symmetry: The vertical line $x = 0$.
- Opening: Upward.
- Domain: All real numbers $(-\infty, \infty)$.
- Range: $[0, \infty)$.

This basic shape serves as the starting point for understanding all transformations.

Understanding the Transformation: From $y = x^2$ to $y = (x - 2)^2$

The function $y = (x - 2)^2$ is a transformed version of the standard parabola. Recognizing this transformation involves understanding the effect of the "shift" inside the squared term.

Horizontal Shift: The Role of $(x - h)$

In the general form $y = (x - h)^2$, the parameter h dictates the horizontal translation of the parabola.

- When $h > 0$: The parabola shifts to the right by h units.
- When $h < 0$: The parabola shifts to the left by $|h|$ units.

Applying to $y = (x - 2)^2$

Given $h = 2$, the parabola is shifted 2 units to the right compared to the standard $y = x^2$.

Step-by-Step Approach to Sketching $y = (x - 2)^2$

To accurately graph the transformed parabola, follow a systematic process:

1. Identify the Vertex

Since the transformation involves $(x - 2)^2$, the vertex of the parabola is at the point where the expression inside the parentheses is zero:

$$
x - 2 = 0 \Rightarrow x = 2
$$

Substitute $x = 2$ into the function:

$$
y = (2 - 2)^2 = 0
$$

Vertex: $(2, 0)$

This point is the minimum of the parabola, as it opens upward.

2. Determine the Axis of Symmetry

The axis of symmetry passes through the vertex:

$$
x = 2
$$

This vertical line divides the parabola into two mirror-image halves.

3. Plot the Vertex

Begin your sketch by plotting the vertex at $(2, 0)$. This is the pivotal point around which the parabola is symmetric.

4. Find Additional Points

To understand the shape, select several x-values around the vertex:

x	Calculation $(x - 2)^2$	y	Coordinates
0	$(0 - 2)^2 = 4$	4	$(0, 4)$
1	$(1 - 2)^2 = 1$	1	$(1, 1)$
3	$(3 - 2)^2 = 1$	1	$(3, 1)$
4	$(4 - 2)^2 = 4$	4	$(4, 4)$

Plot these points to get a sense of the parabola's curvature.

5. Sketch the Parabola

Using the plotted points, draw a smooth, symmetric curve passing through the vertex and the other points. Remember:

- The parabola opens upward.
- Symmetry about the line $x=2$.

6. Label Key Features

Ensure the graph includes:

- The vertex at $(2, 0)$.
- The axis of symmetry $x = 2$.
- Additional points for clarity.

Analyzing the Graph: Key Features and Properties

Understanding the characteristics of $y = (x - 2)^2$ enhances your ability to interpret and leverage its properties:

1. Domain and Range

- Domain: All real numbers $(-\infty, \infty)$, since you can input any real x.
- Range: $[0, \infty)$, because the parabola opens upward and its minimum value is 0 at $x=2$.

2. Symmetry

- The graph is symmetric about the vertical line $x = 2$.
- For every point (a, y), there is a corresponding point $(4 - a, y)$.

3. Vertex and Minimum Point

- The vertex at $(2, 0)$ is the lowest point on the graph.
- Since the parabola opens upward, the vertex represents the minimum.

4. Intercepts

- Y-intercept: When $x = 0$,

$$
y = (0 - 2)^2 = 4
$$

- X-intercepts: The parabola intersects the x-axis where $y=0$:

$$
(x - 2)^2 = 0 \Rightarrow x = 2
$$

This is a single root, indicating the parabola touches the x-axis at the vertex.

Transformations and Their Impact on the Graph

Understanding how transformations affect the basic parabola is key to mastering graph sketching.

Horizontal Translations

- The form $y = (x - h)^2$ shifts the parabola h units to the right if $h > 0$, and h units to the left if $h < 0$.
- In our case, $h = 2$.

Reflection and Vertical Shifts

- If the function were $y = - (x - 2)^2$, the parabola would reflect across the x-axis.

- Vertical shifts are represented as $y = (x - 2)^2 + k$, shifting the graph up or down by k.

Stretching and Shrinking

- The coefficient outside the squared term affects the parabola's width:

$$
y = a(x - h)^2
$$

- $|a| > 1$: parabola becomes narrower.
- $0 < |a| < 1$: parabola becomes wider.

While not present in $y = (x - 2)^2$, these concepts are vital for understanding more complex transformations.

Visualizing the Graph: From Theory to Practice

To cement your understanding, consider these practical tips for sketching:

- Draw the axes with appropriate scale.
- Mark the vertex precisely at $(2, 0)$.
- Plot the additional points $(0, 4)$, $(1, 1)$, $(3, 1)$, and $(4, 4)$.
- Use the symmetry about $x=2$ to reflect points on either side.
- Draw a smooth parabola passing through all points, ensuring symmetry and proper curvature.
- Label the key points and axes for clarity.

Real-World Applications and Significance

The function $y = (x - 2)^2$ is not just an abstract mathematical construct; it models many real-world phenomena:

- Physics: Describing projectile motion or the shape of a satellite dish.
- Engineering: Designing parabolic reflectors.
- Economics: Modeling cost functions with minimum points.

- Biology: Representing growth rates with optimal conditions.

Mastering the graphing of such functions enhances problem-solving skills across diverse fields.

Conclusion: The Art and Science of Graphing Transformed Parabolas

Sketching the graph of \(y = (x - 2)^2 \) exemplifies the powerful interplay between algebraic expressions and geometric visualization. By systematically identifying key features—vertex, axis of symmetry, intercepts—and understanding the effects of transformations, you can confidently depict complex quadratic functions with precision.

This process underscores the importance of foundational concepts like symmetry, translation, and curvature, which serve as the building blocks for more advanced mathematical analysis. Whether for academic purposes, professional applications, or personal curiosity, mastering these techniques transforms your approach to quadratic functions from rote memorization

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