Solve 2x2 6x + 10 = 0 By Completing The Square.

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Solving quadratic equations is a fundamental skill in algebra, and one of the most elegant methods to find roots is by completing the square. This technique not only helps in solving equations but also deepens understanding of the structure of quadratic expressions. In this article, we will explore how to solve the quadratic equation $2x^2 + 6x + 10 = 0$ by completing the square, step by step. Whether you're a student preparing for exams or someone looking to reinforce your algebra skills, mastering this method is essential.

Understanding the Quadratic Equation

Before diving into the completing the square method, it's important to understand the general form of a quadratic equation.

Standard Form of a Quadratic Equation

A quadratic equation is typically written as:
$$ ax^2 + bx + c = 0 $$
where:
- a is the coefficient of x^2 (and $a \neq 0$)
- b is the coefficient of x
- c is the constant term

In our specific case:
$$ 2x^2 + 6x + 10 = 0 $$
Here, $a = 2$, $b = 6$, and $c = 10$.

Step-by-Step Guide to Completing the Square

Completing the square involves manipulating the quadratic equation to express it as a perfect square trinomial. Let's walk through the process carefully.

Step 1: Normalize the Coefficient of x^2

Since the coefficient of the quadratic term is 2 (not 1), the first step is to divide the entire equation by 2 to make the coefficient of x^2 equal to 1.

$$ \frac{2x^2 + 6x + 10}{2} = 0 $$
$$ x^2 + 3x + 5 = 0 $$

This simplifies our work. Now, the equation to solve is:
$$ x^2 + 3x + 5 = 0 $$

Step 2: Move the Constant Term to the Other Side

To prepare for completing the square, subtract 5 from both sides:

$$ x^2 + 3x = -5 $$

Step 3: Complete the Square

The next step is to add a specific value to both sides of the equation to turn the left side into a perfect square trinomial.

- Take half of the coefficient of x:
$$ \frac{3}{2} $$

- Square this value:
$$ \left(\frac{3}{2}\right)^2 = \frac{9}{4} $$

- Add $\frac{9}{4}$ to both sides:

$$ x^2 + 3x + \frac{9}{4} = -5 + \frac{9}{4} $$

To combine the right side, convert -5 to a fraction with denominator 4:
$$ -5 = -\frac{20}{4} $$

So,
$$ -\frac{20}{4} + \frac{9}{4} = -\frac{11}{4} $$

The equation now becomes:
$$ \left(x + \frac{3}{2}\right)^2 = -\frac{11}{4} $$

Note that the left side is a perfect square trinomial:
$$ (x + \frac{3}{2})^2 $$

Solving for x

Having completed the square, the next step is to solve for x by taking the square root of both sides.

Step 4: Take the Square Root of Both Sides

Remember to consider both positive and negative roots:

$$ x + \frac{3}{2} = \pm \sqrt{-\frac{11}{4}} $$

Since the right side involves the square root of a negative number, the solutions will be complex (imaginary).

Express the square root:

$$ \sqrt{-\frac{11}{4}} = \sqrt{-1} \times \sqrt{\frac{11}{4}} = i \times \frac{\sqrt{11}}{2} $$

Thus, the solutions are:

$$ x + \frac{3}{2} = \pm i \frac{\sqrt{11}}{2} $$

Step 5: Isolate x

Subtract $\frac{3}{2}$ from both sides:

$$ x = -\frac{3}{2} \pm i \frac{\sqrt{11}}{2} $$

This gives us the two complex solutions:

$$ x = -\frac{3}{2} + i \frac{\sqrt{11}}{2} \quad \text{and} \quad x = -\frac{3}{2} - i \frac{\sqrt{11}}{2} $$

Summary of the Solution

The solutions to the quadratic equation $2x^2 + 6x + 10 = 0$ by completing the square are:

$$ x = -\frac{3}{2} \pm i \frac{\sqrt{11}}{2} $$

These are complex conjugates, indicating that the quadratic does not intersect the x-axis.

Additional Tips and Insights

To deepen your understanding, here are some useful tips and insights related to solving quadratics by completing the square:

- **Always normalize the quadratic coefficient to 1** before completing the square for simplicity.
- **Adding the square of half the linear coefficient** ensures the left side becomes a perfect square trinomial.
- **Remember to consider the discriminant (b^2 - 4ac)** to determine whether solutions are real or complex.
- **Complex solutions occur when the discriminant is negative.** In our case, the negative discriminant resulted in imaginary solutions.

Alternative Methods for Solving Quadratics

While completing the square is a powerful method, it's also beneficial to understand other approaches:

Quadratic Formula

The quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

provides a quick way to find solutions, including complex ones, without completing the square manually.

Factoring

If the quadratic factors easily, factoring can be a straightforward method. However, in this case, the coefficients do not lend themselves to simple factoring.

Practice Problems

To reinforce your understanding, try solving these equations by completing the square:

1. $x^2 + 4x + 1 = 0$
2. $3x^2 - 12x + 7 = 0$
3. $x^2 + 2x + 3 = 0$

Completing the square enhances your algebraic toolkit and prepares you for more advanced mathematics topics.

Conclusion

Solving quadratic equations by completing the square is a fundamental technique that provides insight into the structure of quadratic expressions and their roots. In the specific case of $2x^2 + 6x + 10 = 0$, we've seen how dividing through, completing the square, and taking square roots (including complex roots) leads to the solutions. Mastery of this method deepens your algebraic understanding and prepares you for tackling more complex problems in mathematics. Practice regularly, and you'll find completing the square becomes an intuitive and valuable skill.

Frequently Asked Questions

How do I start solving the quadratic equation $2x^2 + 6x + 10 = 0$ by completing the square?

First, divide the entire equation by 2 to simplify: $x^2 + 3x + 5 = 0$. Then, move the constant term to the other side: $x^2 + 3x = -5$. This sets up for completing the square.

What is the next step after isolating the x^2 and x terms in 2x^2 + 6x + 10 = 0?

Identify the coefficient of x, which is 3, divide it by 2 to get 3/2, and then square it: $(3/2)^2 = 9/4$. Add this value to both sides to complete the square.

How do I write the equation after adding the completing the square term?

The equation becomes: $x^2 + 3x + 9/4 = -5 + 9/4$. Simplify the right side: $-5 + 9/4 = -20/4 + 9/4 = -11/4$. So, the equation is $(x + 3/2)^2 = -11/4$.

What do I do when the right side of the equation is negative after completing the square?

Since $(x + 3/2)^2 = -11/4$, and the square of a real number cannot be negative, this indicates that the solutions are complex numbers.

How do I find the solutions to 2x^2 + 6x + 10 = 0 using the completed square method?

Starting from $(x + 3/2)^2 = -11/4$, take the square root of both sides: $x + 3/2 = \pm\sqrt{-11/4}$. This simplifies to $x + 3/2 = \pm(\sqrt{11}/2)i$, leading to $x = -3/2 \pm (\sqrt{11}/2)i$.

What is the final solution for x in 2x^2 + 6x + 10 = 0?

The solutions are $x = -3/2 + (\sqrt{11}/2)i$ and $x = -3/2 - (\sqrt{11}/2)i$, which are complex conjugates.

Can I solve 2x^2 + 6x + 10 = 0 by completing the square without dividing the entire equation first?

It's possible but more complicated. Dividing through by 2 first simplifies the coefficients, making completing the square easier and more straightforward.

Additional Resources

Solve $2x^2 + 6x + 10 = 0$ By Completing The Square

Understanding how to solve quadratic equations is fundamental in algebra, and among various methods, completing the square offers a systematic approach that not only provides solutions but also deepens comprehension of quadratic structure. This article explores in detail how to solve the quadratic equation $2x^2 + 6x + 10 = 0$ using the completing the square method, providing insights, step-by-step procedures, and mathematical explanations suitable for students, educators, and enthusiasts interested in advanced algebraic techniques.

Introduction to Completing the Square Method

Completing the square is a technique used to solve quadratic equations by rewriting the quadratic in the form of a perfect square trinomial plus a constant. This method is particularly useful because it transforms the quadratic equation into a form that makes solutions more transparent, especially when factoring is challenging.

The general quadratic form is:

$$ ax^2 + bx + c = 0 $$

The goal of completing the square is to manipulate this into:

$$ a(x + d)^2 + e = 0 $$

where d and e are constants derived from the coefficients a, b, and c.

Step-by-Step Solution of $2x^2 + 6x + 10 = 0$

Applying the completing the square method to the specific quadratic $2x^2 + 6x + 10 = 0$ involves a systematic process. Let's dissect each step thoroughly.

Step 1: Normalize the Quadratic Equation

Since the coefficient of x^2 is 2, not 1, the first step is to divide the entire equation by 2 to simplify calculations:

$$
\frac{2x^2}{2} + \frac{6x}{2} + \frac{10}{2} = 0
$$

which simplifies to:

$$
x^2 + 3x + 5 = 0
$$

This normalization makes it easier to complete the square.

Step 2: Isolate the Constant Term

Rearranged as:

$$
x^2 + 3x = -5
$$

The goal now is to turn the left side into a perfect square trinomial.

Step 3: Complete the Square on the Left Side

To complete the square, take half of the coefficient of x (which is 3), square it, and add it to both sides:

- Half of 3: $\frac{3}{2}$
- Square of half: $\left(\frac{3}{2} \right)^2 = \frac{9}{4}$

Add $\frac{9}{4}$ to both sides:

$$
x^2 + 3x + \frac{9}{4} = -5 + \frac{9}{4}
$$

The left side now becomes a perfect square trinomial:

$$
\left(x + \frac{3}{2} \right)^2 = -5 + \frac{9}{4}
$$

Step 4: Simplify the Right Side

Express -5 as a fraction with denominator 4:

$$
-5 = -\frac{20}{4}
$$

Adding:

$$
-\frac{20}{4} + \frac{9}{4} = -\frac{11}{4}
$$

Thus, the equation becomes:

$$
\left(x + \frac{3}{2} \right)^2 = -\frac{11}{4}
$$

Step 5: Solve for x by Taking Square Roots

Take the square root of both sides, remembering to consider both positive and negative roots:

$$
x + \frac{3}{2} = \pm \sqrt{ -\frac{11}{4} }
$$

The square root of a negative number introduces complex solutions. Rewrite as:

$$
x + \frac{3}{2} = \pm \sqrt{ -1 \times \frac{11}{4} } = \pm i \sqrt{ \frac{11}{4} } = \pm i \frac{\sqrt{11}}{2}
$$

where i is the imaginary unit.

Step 6: Isolate x and Write the Final Solution

Subtract $\frac{3}{2}$ from both sides:

$$
x = -\frac{3}{2} \pm i \frac{\sqrt{11}}{2}
$$

Alternatively, write as:

$$
x = -\frac{3}{2} \pm \frac{i \sqrt{11}}{2}
$$

This is the set of solutions in complex form.

Summary of Solutions

The quadratic $2x^2 + 6x + 10 = 0$ has complex solutions given by:

$$
x = -\frac{3}{2} \pm \frac{i \sqrt{11}}{2}

\]

which can also be expressed as:

\[
x = -\frac{3}{2} \pm i \frac{\sqrt{11}}{2}
\]

This demonstrates how completing the square reveals the nature of solutions (real or complex) based on the discriminant.

Understanding the Discriminant and Its Role

The discriminant \(\Delta \) in a quadratic \(ax^2 + bx + c = 0 \) is:

\[
\Delta = b^2 - 4ac
\]

- If \(\Delta > 0 \), solutions are real and distinct.
- If \(\Delta = 0 \), solutions are real and equal.
- If \(\Delta < 0 \), solutions are complex conjugates.

For our original equation:

\[
2x^2 + 6x + 10 = 0
\]

Calculate the discriminant:

\[
\Delta = (6)^2 - 4 \times 2 \times 10 = 36 - 80 = -44 < 0
\]

which confirms the complex solutions obtained via completing the square.

Implications and Applications of Completing the Square

Completing the square is more than a solution technique; it provides geometric insights and is foundational in deriving the quadratic formula, analyzing conic sections, and solving differential equations.

Applications include:

- Deriving the quadratic formula by completing the square on the general quadratic.
- Analyzing the vertex form of parabolas for graphing.
- Solving quadratic inequalities.
- Integrating quadratic expressions in calculus.
- Modeling physical phenomena where quadratic relationships appear.

Conclusion

The process of solving $2x^2 + 6x + 10 = 0$ via completing the square showcases the power of algebraic manipulation and the importance of understanding quadratic structures. By normalizing the quadratic, completing the square, and interpreting the discriminant, we find that the solutions are complex conjugates:

$$
x = -\frac{3}{2} \pm i \frac{\sqrt{11}}{2}
$$

This method not only yields solutions but also enhances conceptual understanding, offering a pathway to explore more advanced topics in algebra and beyond. Mastery of completing the square is essential for anyone seeking a deeper grasp of quadratic equations and their applications in mathematics and sciences.

References:

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Larson, R., & Edwards, B. H. (2017). Precalculus with Limits. Cengage Learning.
- Blitzer, R. (2014). Algebra and Trigonometry. Pearson.

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