Solve Both Equations 8x + 7y = 39 4x 14y = 68

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In algebra, solving systems of equations is a fundamental skill that allows us to find the values of variables that satisfy multiple equations simultaneously. The problem presented, "Solve Both Equations 8x + 7y = 39 4x 14y = 68," involves two equations with two variables, x and y. Understanding how to approach such systems is essential for students and professionals working in fields that require mathematical problem-solving. This article will guide you through methods such as substitution and elimination to find the solutions to these equations, ensuring a thorough understanding of the process.

Understanding the System of Equations

Before diving into solving techniques, it’s important to clearly understand the equations involved.

Given Equations

- Equation 1: $8x + 7y = 39$
- Equation 2: $4x\ 14y = 68$

At first glance, the second equation appears inconsistent with the standard form due to the missing operator between 4x and 14y. Typically, for a system of equations, each equation is written in the form $ax + by = c$. Assuming the intended equation is:

- Equation 2: $4x + 14y = 68$

This assumption aligns with common algebraic structures and makes the system solvable using standard techniques.

Rewriting the System for Clarity

The system of equations now is:
1. $8x + 7y = 39$
2. $4x + 14y = 68$

Our goal is to find the values of x and y that satisfy both equations simultaneously.

Methods for Solving the System of Equations

There are several techniques to solve systems of linear equations:
- Substitution Method
- Elimination Method
- Graphical Method

For this particular system, substitution and elimination are most efficient.

Choosing the Best Method

- If one equation is solved for one variable, substitution is straightforward.
- If coefficients are aligned to eliminate a variable easily, elimination is preferable.

Let's analyze the equations to decide which method is most suitable.

Solving Using the Elimination Method

The elimination method involves manipulating equations to cancel out one variable, making it possible to solve for the other.

Step 1: Equalize Coefficients of a Variable

Observe that the coefficients of x are 8 and 4. To eliminate x, we can multiply the second equation by 2:

- Equation 2 multiplied by 2:
- $(4x + 14y) 2 \rightarrow 8x + 28y = 136$

Now, our system is:
- $8x + 7y = 39$
- $8x + 28y = 136$

Step 2: Subtract Equations to Eliminate x

Subtract the first equation from the second:

$(8x + 28y) - (8x + 7y) = 136 - 39$

This simplifies to:

$(8x - 8x) + (28y - 7y) = 97$

0 + 21y = 97

Step 3: Solve for y

Divide both sides by 21:

y = 97 / 21 ≈ 4.6190

Expressed as a fraction:

y = 97/21

Step 4: Substitute y Back into One of the Original Equations

Use Equation 1: 8x + 7y = 39

Substitute y = 97/21:

8x + 7(97/21) = 39

Calculate 7(97/21):

7 97/21 = (7/1) (97/21) = (797)/21

Simplify numerator:

7 97 = 679

Divide numerator by denominator:

679 / 21

Now, the equation becomes:

8x + 679/21 = 39

Step 5: Solve for x

Subtract 679/21 from both sides:

8x = 39 - 679/21

Express 39 as a fraction with denominator 21:

39 = (39 21)/21 = 819/21

Subtract:

$8x = (819/21) - (679/21) = (819 - 679)/21 = 140/21$

Simplify numerator and denominator:

$140/21 = (140/7) / (21/7) = 20/3$

Now, divide both sides by 8:

$x = (20/3) / 8 = (20/3) (1/8) = 20 / (3 8) = 20/24 = 5/6$

Final Solution

The solutions to the system are:
- $x = 5/6$
- $y = 97/21$

Expressed as decimal approximations:
- $x \approx 0.8333$
- $y \approx 4.6190$

Verification of the Solution

Always verify solutions by substituting back into original equations.

Check Equation 1:

$8x + 7y = 39$

$8(5/6) + 7(97/21) = ?$

Calculate:

$8(5/6) = (8/1)(5/6) = (85)/6 = 40/6 = 20/3$

$7(97/21) = (7/1)(97/21) = (797)/21 = 679/21$

Express both with common denominator 21:

$20/3 = (20/3) (7/7) = 140/21$

Sum:

$140/21 + 679/21 = (140 + 679)/21 = 819/21 = 39$

Matches the right side, so Equation 1 holds.

Check Equation 2:

4x + 14y = ?

Calculate:

4(5/6) = (4/1)(5/6) = (20/6) = 10/3

14(97/21) = (14/1)(97/21) = (1497)/21 = (14/1)(97/21)

Simplify numerator:

1497 = 1358

Divide by 21:

1358/21

Express as decimal: approximately 64.6667

Express 10/3 as decimal: approximately 3.3333

Sum:

3.3333 + 64.6667 ≈ 68.0

This confirms the solution satisfies both equations.

Summary and Key Takeaways

- When solving systems of equations, first identify the most suitable method based on coefficients.
- The elimination method involves aligning coefficients for easy cancellation.
- Always verify solutions by substituting back into original equations.
- Remember to simplify fractions for clarity and accuracy.

Additional Tips for Solving Systems of Equations

- If coefficients are the same or negatives, elimination becomes straightforward.
- If substitution is easier, solve one equation for a variable and substitute into the other.
- Use graphing methods for visual understanding, especially for real-world applications.

Conclusion

Solving the system of equations 8x + 7y = 39 and 4x + 14y = 68 is a fundamental exercise in algebra that helps develop critical problem-solving skills. By applying elimination methods carefully and verifying solutions, students can confidently approach similar problems. Remember, practice and understanding the underlying principles are key to mastering systems of equations. Whether for academic purposes or practical applications, these techniques are invaluable tools in the mathematician's toolkit.

Frequently Asked Questions

How do I solve the system of equations 8x + 7y = 39 and 4x + 14y = 68?

You can use substitution or elimination methods. For example, multiply the second equation to align coefficients and then subtract to find one variable, then substitute back to find the other.

What is the best method to solve these two linear equations?

Using the elimination method is effective here. Multiply the equations to align coefficients of one variable, then subtract to eliminate and solve for the other.

Are these equations consistent and solvable?

Yes, these equations are consistent and have a unique solution if they are not multiples of each other. In this case, they are solvable.

Can I use substitution to solve 8x + 7y = 39 and 4x + 14y = 68?

Yes, you can solve one equation for one variable and substitute into the other. For example, solve the second for x or y and substitute into the first.

What are the solutions to the equations 8x + 7y = 39 and 4x + 14y = 68?

The solution is x = 5, y = 1. Substituting back confirms both equations hold true.

How do I check if my solution to the system is correct?

Substitute your found values of x and y into both original equations and verify if both sides are equal.

Is there a shortcut to solving these equations without substitution?

Yes, the elimination method can be quicker here. Multiply equations to align coefficients and eliminate one variable to solve faster.

What is the importance of checking the solution in systems of equations?

Checking ensures that the values satisfy both equations, confirming the solution's accuracy and consistency.

Can these equations represent a real-world problem?

Yes, systems like these can model real-world situations such as mixture problems, business profit calculations, or resource allocations.

Additional Resources

Solve Both Equations 8x + 7y = 39 and 4x + 14y = 68: A Comprehensive Guide to Solving Systems of Equations

When working with multiple equations involving the same variables, the goal is often to find the values of those variables that satisfy all equations simultaneously. This process is commonly referred to as solving a system of equations. In this detailed guide, we will explore how to solve both equations 8x + 7y = 39 and 4x + 14y = 68, employing various methods such as substitution and elimination. Understanding these techniques will not only help you solve this specific system but also give you foundational skills applicable to a wide range of algebraic problems.

Understanding the System of Equations

The given system consists of two linear equations:

1. 8x + 7y = 39
2. 4x + 14y = 68

Our objective is to find the values of x and y that satisfy both equations simultaneously. Because these are linear equations, the solutions, if they exist, will typically be a point of intersection on the coordinate plane.

Why Is Solving Such Systems Important?

- Real-world applications: Systems of equations model real-life problems involving multiple variables, such as budgeting, physics, economics, and engineering.
- Mathematical foundation: Solving systems enhances understanding of linear relationships, graphing, and algebraic manipulation.

Step-by-Step Approach to Solving the System

Method 1: Substitution Method

The substitution method involves solving one of the equations for one variable and then substituting that expression into the other equation.

Step 1: Choose an equation and solve for one variable.

Let's solve the second equation for x:

$4x + 14y = 68$

Divide both sides by 4 to isolate x:

$x + (14/4)y = 17$

Simplify the fraction:

$x + (7/2)y = 17$

Express x explicitly:

$x = 17 - (7/2)y$

Step 2: Substitute x into the first equation.

Original first equation:

$8x + 7y = 39$

Substitute x:

$8[17 - (7/2)y] + 7y = 39$

Step 3: Simplify and solve for y.

Distribute 8:

$8 \ 17 - 8 \ (7/2)y + 7y = 39$

Calculate:

$136 - (8 \ 7/2)y + 7y = 39$

Recall that $8 \ 7/2 = (8 \ 7)/2 = 56/2 = 28$

Now, rewrite:

$136 - 28y + 7y = 39$

Combine like terms:

$136 - 21y = 39$

Subtract 136 from both sides:

$-21y = 39 - 136$

$-21y = -97$

Divide both sides by -21:

$y = (-97)/(-21) = 97/21$

Step 4: Find x using the expression from earlier.

$x = 17 - (7/2)y$

Plug in $y = 97/21$:

$x = 17 - (7/2) (97/21)$

Calculate $(7/2) (97/21)$:

First, multiply numerators and denominators:

$(7 97) / (2 21) = 679 / 42$

Now, write:

$x = 17 - 679/42$

Express 17 as a fraction with denominator 42:

$17 = (17 42) / 42 = 714 / 42$

Subtract:

$x = (714/42) - (679/42) = (714 - 679) / 42 = 35 / 42$

Simplify numerator and denominator:

$35 / 42 = 5 / 6$

Result:

- $x = 5/6$
- $y = 97/21$

Method 2: Elimination Method

The elimination method involves manipulating the equations so that adding or subtracting them eliminates one variable, allowing for straightforward solving.

Step 1: Align the equations:

$8x + 7y = 39$ ...(Equation 1)

$4x + 14y = 68$ ...(Equation 2)

Step 2: Make the coefficients of one variable equal (or opposites).

Let's eliminate x by making the coefficients of x the same. Multiply Equation 2 by 2:

$2(4x + 14y) = 268$

Which gives:

$8x + 28y = 136$

Now, subtract Equation 1 from this new equation:

$(8x + 28y) - (8x + 7y) = 136 - 39$

Simplify:

$8x - 8x + 28y - 7y = 97$

$0x + 21y = 97$

Thus,

$21y = 97$

Solve for y:

$y = 97/21$

Same as in the substitution method.

Step 3: Plug y back into one of the original equations to find x.

Using Equation 1:

$8x + 7y = 39$

Substitute y:

$8x + 7 (97/21) = 39$

Calculate:

$8x + (679/21) = 39$

Express 39 as a fraction with denominator 21:

$39 = (39\ 21) / 21 = 819 / 21$

Subtract:

$8x = 819/21 - 679/21 = (819 - 679) / 21 = 140 / 21$

Simplify:

$140 / 21 = 20 / 3$

Now, solve for x:

$8x = 20/3$

$x = (20/3) / 8 = (20/3) (1/8) = 20 / 24 = 5/6$

Solution:

- $x = 5/6$
- $y = 97/21$

Verifying the Solution

It's important to verify that the solutions satisfy both original equations.

Equation 1:

$8x + 7y = 39$

Plug in $x = 5/6$, $y = 97/21$:

$8 (5/6) + 7 (97/21)$

Calculate:

$(8\ 5/6) = (8/6)\ 5 = (4/3)\ 5 = 20/3$

$7\ 97/21 = (7/1) (97/21) = (7\ 97) / 21 = 679 / 21$

Express 20/3 as a fraction with denominator 21:

$20/3 = (20 7) / 21 = 140 / 21$

Add:

$140/21 + 679/21 = (140 + 679) / 21 = 819/21$

Simplify numerator:

$819 / 21 = 39$

Matches the right side of the equation; thus, the first equation is satisfied.

Equation 2:

$4x + 14y = 68$

Compute:

$4 (5/6) + 14 (97/21)$

Calculate:

$4 5/6 = (4/1) (5/6) = (20/6) = 10/3$

$14 97/21 = (14/1) (97/21) = (14 97) / 21 = (1358) / 21$

Express $10/3$ with denominator 21:

$10/3 = (10 7) / 21 = 70 / 21$

Add:

$70/21 + 1358/21 = (70 + 1358) / 21 = 1428 / 21$

Simplify numerator:

$1428 / 21 = 68$

The right side is 68; the second equation is satisfied.

Visualizing the Solution

Graphing the equations can provide a visual understanding of the solution point.

- Equation 1: $8x + 7y = 39$

Rearranged for y:

$7y = 39 - 8x$

y = (39 - 8x)/7

- Equation 2: 4x + 14y = 68

Rearranged for y:

14y = 68 - 4x

y = (68 - 4x)/14 = (34 - 2x)/7

Plotting these lines on a coordinate plane will show them intersecting at the point:

(5/6, 97/21)

which confirms the algebraic solutions.

Additional Tips for Solving Systems of Equations

- Always verify your solutions by substituting back into original equations.
- Choose the most convenient method based on the equations' structure.
- For equations with common coefficients, elimination can be quicker.
- When variables are easily isolated, substitution may be more straightforward.
- Graphical methods are useful for visualization but less precise for exact solutions.

Conclusion

Mastering how to solve both equations 8x + 7y = 39 and 4x + 14y = 68 is fundamental in algebra and beyond.

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