Solve S 1 8x-x Dx Through Trigonometric Substitution.

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When faced with integrals involving expressions like $8x - x$, especially within a definite integral such as $\int_{1}^{8} (8x - x) \, dx$, or more complex variants, it’s crucial to understand the techniques that can simplify the process. One of the most powerful methods for solving integrals involving radicals and quadratic expressions is trigonometric substitution. This method transforms challenging algebraic expressions into trigonometric functions, leveraging identities to make the integral more manageable. In this article, we will explore how to evaluate the integral of $8x - x$ through trigonometric substitution, providing a comprehensive guide, step-by-step procedures, and useful tips to master this technique.

Understanding the Integral: What Are We Solving?

Before diving into the solution, let's clarify the integral at hand. The expression "Solve S 1 8x - x Dx" appears to refer to a definite integral, possibly:

$$
\int_{1}^{8} (8x - x) \, dx
$$

which simplifies to:

$$
\int_{1}^{8} 7x \, dx
$$

or perhaps a different, more complex integral involving radicals or quadratic expressions. For the purpose of this tutorial, we will assume the integral involves a quadratic expression that benefits from trigonometric substitution, such as:

$$
I = \int \frac{dx}{\sqrt{a^2 - x^2}}
$$

or

$$
I = \int \frac{dx}{\sqrt{x^2 - a^2}}
$$

or

$$
I = \int \frac{dx}{x^2 + a^2}
$$

which are classic cases suitable for trigonometric substitution.

Why Use Trigonometric Substitution?

Trigonometric substitution is particularly useful for integrating functions involving:

- $\sqrt{a^2 - x^2}$
- $\sqrt{x^2 - a^2}$
- $\sqrt{a^2 + x^2}$

The substitution leverages well-known trigonometric identities such as:

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\tan^2 \theta + 1 = \sec^2 \theta$
- $1 + \cot^2 \theta = \csc^2 \theta$

By substituting x with a trigonometric function, these identities simplify radicals, transforming the integral into a form that can be integrated using basic trigonometric formulas.

Step-by-Step Guide to Solving Integrals Using Trigonometric Substitution

Let's walk through the general steps involved in applying trigonometric substitution to an integral involving radicals.

Step 1: Identify the Form of the Integral

Determine whether the integral contains:

- $\sqrt{a^2 - x^2}$: Use $x = a \sin \theta$
- $\sqrt{x^2 - a^2}$: Use $x = a \sec \theta$
- $\sqrt{a^2 + x^2}$: Use $x = a \tan \theta$

This identification guides the choice of substitution.

Step 2: Make the Substitution

Replace x with the appropriate trigonometric function, and compute dx accordingly.

Step 3: Simplify the Integral

Use the Pythagorean identities to simplify the radical expressions. The substitution should eliminate radicals, reducing the integral to a basic trigonometric integral.

Step 4: Integrate with Respect to θ

Perform the integration using standard trigonometric integral formulas.

Step 5: Back-Substitute to x

After integrating, convert back to the variable x using the inverse trigonometric functions.

Step 6: Apply Limits (for definite integrals)

If the integral is definite, change the limits of integration to the θ domain corresponding to the original x limits, or evaluate the indefinite integral first and then substitute the limits.

Example: Solving $\int \frac{dx}{\sqrt{a^2 - x^2}}$ Using Trigonometric Substitution

Suppose we want to evaluate:

$$
I = \int \frac{dx}{\sqrt{a^2 - x^2}}
$$

Step 1: Recognize the form $\sqrt{a^2 - x^2}$. Use the substitution:

$$
x = a \sin \theta
$$

Step 2: Compute dx:

$$
dx = a \cos \theta \, d\theta
$$

Step 3: Simplify the radical:

$$
\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sin^2 \theta} = a \cos \theta
$$

Step 4: Rewrite the integral:

$$
I = \int \frac{a \cos \theta \, d\theta}{a \cos \theta} = \int d\theta = \theta + C
$$

Step 5: Back-substitute:

$$
\theta = \arcsin \left(\frac{x}{a}\right)
$$

Final answer:

$$
I = \arcsin \left(\frac{x}{a}\right) + C
$$

This classic example demonstrates how trigonometric substitution simplifies radicals involving quadratic expressions.

Applying Trigonometric Substitution to the Specific Integral in Question

If the integral you are referring to involves a radical such as $\sqrt{8x - x^2}$, which resembles the form $\sqrt{a^2 - (x - h)^2}$, it’s a candidate for trigonometric substitution.

Suppose the integral is:

$$
I = \int \frac{dx}{\sqrt{8x - x^2}}
$$

First, rewrite the radicand:

\[

$$
8x - x^2 = -(x^2 - 8x) = -(x^2 - 8x + 16 - 16) = -((x - 4)^2 - 16)
$$

Thus,

$$
\sqrt{8x - x^2} = \sqrt{16 - (x - 4)^2}
$$

The integral becomes:

$$
I = \int \frac{dx}{\sqrt{16 - (x - 4)^2}}
$$

Now, apply the substitution:

$$
x - 4 = 4 \sin \theta
$$

which implies:

$$
dx = 4 \cos \theta \, d\theta
$$

and the radical simplifies to:

$$
\sqrt{16 - 16 \sin^2 \theta} = 4 \cos \theta
$$

The integral transforms into:

$$
I = \int \frac{4 \cos \theta \, d\theta}{4 \cos \theta} = \int d\theta = \theta + C
$$

Back-substitute:

$$
\theta = \arcsin \left(\frac{x - 4}{4}\right)
$$

Result:

$$
I = \arcsin \left(\frac{x - 4}{4}\right) + C
$$

This example illustrates how to handle radicals of quadratic expressions, leading to straightforward solutions via trigonometric substitution.

Key Tips for Effective Trigonometric Substitution

- Always identify the form of the integrand carefully to choose the appropriate substitution.
- Simplify the radicand before substituting to recognize perfect squares or differences of squares.
- Remember the Pythagorean identities to simplify radicals.
- Be cautious with the limits when performing definite integrals; convert them into the θ domain.
- Always back-substitute to express the final answer in terms of the original variable x.

Conclusion: Mastering the Technique for Complex Integrals

Trigonometric substitution is a vital tool in the calculus toolkit, especially for integrals involving radicals and quadratic expressions. By understanding the different forms and how to choose the correct substitution, you can tackle a wide array of challenging integrals with confidence. Whether you're working through textbook problems or applying these techniques in advanced applications, mastering trigonometric substitution will significantly enhance your problem-solving efficiency and mathematical understanding.

If you're dealing with an integral like $\int_{1}^{8} (8x - x) \, dx$, or more complex radicals, applying the principles outlined above will help you simplify and evaluate the integral accurately. Practice is key—so try different problems and ensure you understand each step of the process.

Remember: The power of trigonometric substitution lies in converting complicated radicals into manageable trigonometric integrals, making your calculus journey smoother and more intuitive.

Frequently Asked Questions

What is the main goal when solving the integral ∫ (1 / (8x - x^2)) dx using trigonometric substitution?

The main goal is to rewrite the quadratic expression in the denominator into a form that allows the use of standard trigonometric identities, simplifying the integral for easier evaluation.

Which trigonometric substitution is best suited for the integral ∫ 1 / (8x - x^2) dx?

A common substitution is completing the square to express the quadratic in the form (a - x)^2, then using a substitution like x = a - b sin θ or x = a ± b cos θ depending on the form, to facilitate integration.

How do you complete the square for the expression 8x - x^2 in the integral?

Rewrite 8x - x^2 as -(x^2 - 8x) and complete the square inside the parentheses: x^2 - 8x + 16 - 16, resulting in -((x - 4)^2 - 16), simplifying to 16 - (x - 4)^2.

What is the step-by-step process to evaluate ∫ 1 / (8x - x^2) dx using trigonometric substitution?

First, rewrite the denominator as a completed square: 16 - (x - 4)^2. Then, set (x - 4) = 4 sin θ, which transforms the integral into a form involving sqrt(16 - 16 sin^2 θ). Simplify using identities and substitute dx accordingly, then integrate and back-substitute to find the solution.

What trigonometric identities are useful in simplifying the integral after substitution?

Identities such as sin^2 θ + cos^2 θ = 1 and the Pythagorean identity for radicals, like sqrt(a^2 - b^2 sin^2 θ) = a cos θ, are particularly useful for simplifying the integral after substitution.

What is the final form of the integral after applying trigonometric substitution and how do you interpret the result?

After substitution, the integral reduces to a standard form involving trigonometric functions, which can be integrated directly. The result typically involves inverse trigonometric functions (e.g., arcsin), and back-substituting θ in terms of x yields the final solution expressed in terms of x.

Additional Resources

Solve \(\int \frac{1}{8x - x^2} \, dx \) through Trigonometric Substitution

When faced with integrals involving quadratic expressions in the denominator, such as \(\int \frac{1}{8x - x^2} \, dx \), trigonometric substitution often provides an elegant and effective solution pathway. This technique leverages the relationships between algebraic expressions and trigonometric identities to simplify complex integrals into more manageable forms. In this detailed guide, we'll explore how to solve \(\int \frac{1}{8x - x^2} \, dx \) through trigonometric substitution, breaking down each step for clarity and comprehension.

Understanding the Integral and Its Challenges

The integral in question,

$$
\int \frac{1}{8x - x^2} \, dx,
$$

features a quadratic in the denominator. Quadratic expressions like $8x - x^2$ can be tricky to integrate directly due to their nonlinear nature. Standard methods such as partial fractions require the quadratic to factor nicely or be reducible, but often, when the quadratic doesn't factor conveniently or is not a perfect square, substitution techniques—particularly trigonometric substitution—are favored.

Step 1: Rewrite the Denominator in a Standard Form

The first step is to rewrite the quadratic in a form suitable for substitution. Let's manipulate $8x - x^2$:

$$
8x - x^2 = - (x^2 - 8x).
$$

Complete the square inside the parentheses:

$$
x^2 - 8x = x^2 - 8x + 16 - 16 = (x - 4)^2 - 16.
$$

Substituting back:

$$
8x - x^2 = - [(x - 4)^2 - 16] = 16 - (x - 4)^2.
$$

Thus, the integral becomes:

$$
\int \frac{1}{16 - (x - 4)^2} \, dx.
$$

This form resembles the standard integral involving quadratic expressions suitable for trigonometric substitution, notably of the form:

$$
\int \frac{1}{a^2 - u^2} \, du,
$$

which has a well-known solution involving tangent functions.

Step 2: Substitution Strategy

Given the expression:

$$
\int \frac{1}{a^2 - u^2} \, du,
$$

the substitution $u = a \sin \theta$ simplifies it because:

$$
a^2 - u^2 = a^2 - a^2 \sin^2 \theta = a^2 (1 - \sin^2 \theta) = a^2 \cos^2 \theta.
$$

This leads to a straightforward integral in terms of θ.

In our case, set:

$$
u = x - 4,
$$

so the integral becomes:

$$
\int \frac{1}{16 - u^2} \, dx.
$$

Since $u = x - 4$, $du = dx$. The integral now is:

$$
\int \frac{1}{16 - u^2} \, du.
$$

This matches the standard form with $a^2 = 16$, i.e., $a = 4$.

Step 3: Applying Trigonometric Substitution

Using the substitution:

$$
u = 4 \sin \theta,
$$

then:

\[

$$
du = 4 \cos \theta \, d\theta,
$$

and the denominator:

$$
16 - u^2 = 16 - 16 \sin^2 \theta = 16 (1 - \sin^2 \theta) = 16 \cos^2 \theta.
$$

Rewriting the integral:

$$
\int \frac{1}{16 \cos^2 \theta} \times 4 \cos \theta \, d\theta = \int \frac{4 \cos \theta}{16 \cos^2 \theta} \, d\theta.
$$

Simplify:

$$
\int \frac{4 \cos \theta}{16 \cos^2 \theta} \, d\theta = \int \frac{4}{16 \cos \theta} \, d\theta = \int \frac{1}{4 \cos \theta} \, d\theta.
$$

Recall that:

$$
\frac{1}{\cos \theta} = \sec \theta,
$$

so the integral becomes:

$$
\frac{1}{4} \int \sec \theta \, d\theta.
$$

Step 4: Integrating $\sec \theta$

The integral of $\sec \theta$ is a standard result:

$$
\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C.
$$

Therefore,

$$
\frac{1}{4} \int \sec \theta \, d\theta = \frac{1}{4} \ln |\sec \theta + \tan \theta| + C.
$$

Step 5: Back-Substitution to x

Recall the substitutions:

$$
u = 4 \sin \theta \Rightarrow \sin \theta = \frac{u}{4} = \frac{x - 4}{4}.
$$

From the right triangle perspective:

- Opposite side: $u = x - 4$,
- Hypotenuse: 4,

so:

$$
\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left(\frac{x - 4}{4}\right)^2} = \frac{\sqrt{16 - (x - 4)^2}}{4}.
$$

Similarly,

$$
\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{x - 4}{4}}{\frac{\sqrt{16 - (x - 4)^2}}{4}} = \frac{x - 4}{\sqrt{16 - (x - 4)^2}}.
$$

And,

$$
\sec \theta = \frac{1}{\cos \theta} = \frac{4}{\sqrt{16 - (x - 4)^2}}.
$$

Putting it all together:

$$
\boxed{
\int \frac{1}{8x - x^2} \, dx = \frac{1}{4} \ln \left| \frac{4}{\sqrt{16 - (x - 4)^2}} + \frac{x - 4}{\sqrt{16 - (x - 4)^2}} \right| + C.
}
$$

Simplify the logarithm:

$$
= \frac{1}{4} \ln \left| \frac{4 + (x - 4)}{\sqrt{16 - (x - 4)^2}} \right| + C,
$$

or:

$$
\boxed{
\int \frac{1}{8x - x^2} \, dx = \frac{1}{4} \ln \left| \frac{x}{\sqrt{16 - (x - 4)^2}} \right| + C.
}
$$

Summary and Key Takeaways

- Quadratic in denominator can be rewritten in a completed square form to facilitate substitution.
- Trigonometric substitution is particularly effective for integrals of the form $\frac{1}{a^2 - u^2}$.
- Standard identities like $\sec \theta$ and $\tan \theta$ are instrumental in simplifying the integral.
- Always back-substitute carefully to express the final answer in terms of the original variable x.
- Recognize that the integral's structure resembles classic forms, making substitution a systematic approach.

Additional Tips for Solving Similar Integrals

1. Identify the form of quadratic: Complete the square to recognize standard forms.
2. Choose the appropriate substitution:
- $u = a \sin \theta$ for $a^2 - u^2$,
- $u = a \tan \theta$ for $u^2 + a^2$,
- $u = a \sec \theta$ for $u^2 - a^2$.
3. Use identities to simplify the integral after substitution.
4. Always verify the domain restrictions when dealing with inverse trigonometric functions.

Final Thoughts

Mastering the technique of solving integrals through trigonometric substitution enhances your problem-solving toolkit for calculus. It provides a powerful method for tackling integrals involving quadratic expressions—especially when other methods like partial fractions are cumbersome or inapplicable. With practice, recognizing the form and deploying the right substitution becomes intuitive, enabling you to navigate complex integrals with confidence and precision.

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