Solve The Inequality 5(2h + 8) < 60

Solve The Inequality 5(2h + 8) < 60 is a fundamental algebraic problem that introduces students and learners to the process of solving inequalities involving algebraic expressions. Inequalities are essential in mathematics because they help us understand the range or set of values that satisfy certain conditions. Whether you're tackling algebra homework, preparing for exams, or just brushing up on your math skills, mastering how to solve inequalities is a critical step. In this article, we will explore the systematic approach to solving the inequality 5(2h + 8) < 60, break down each step, and review related concepts to deepen your understanding.

Understanding the Inequality 5(2h + 8) < 60

Before jumping into solving, it’s important to understand what the inequality represents. The expression 5(2h + 8) < 60 states that five times the quantity (2h + 8) is less than 60. Our goal is to find all possible values of h that satisfy this inequality. Solving such inequalities involves similar steps to solving equations but requires extra care with the inequality sign, especially if multiplication or division by negative numbers is involved.

Step-by-Step Solution of the Inequality

Let's go through the steps to solve 5(2h + 8) < 60 systematically.

Step 1: Expand the expression

The first step is to distribute the 5 across the terms inside the parentheses:

5 2h + 5 8 < 60

which simplifies to:

10h + 40 < 60

Step 2: Isolate the variable term

Subtract 40 from both sides of the inequality to get all variable terms on one side:

$10h + 40 - 40 < 60 - 40$

which simplifies to:

$10h < 20$

Step 3: Solve for h

Divide both sides of the inequality by 10 to isolate h:

$(10h)/10 < 20/10$

which simplifies to:

$h < 2$

Interpreting the Solution

The solution $h < 2$ means that any value of h less than 2 satisfies the original inequality. It’s important to note that h cannot be equal to 2 because the inequality is strict (<), not inclusive (≤). Therefore, the solution set includes all real numbers less than 2.

Expressing the Solution Set

The solution set can be written in various ways:

- Interval notation: *(-∞, 2)*
- Set builder notation: *{h | h < 2}*

This notation indicates all real numbers less than 2.

Graphical Representation of the Solution

Graphing the solution on a number line provides a visual understanding:

- Draw a number line.
- Mark the point at h = 2.
- Since the inequality is strict, use an open circle at 2 to show that it is

not included.
- Shade the region to the left of 2 to illustrate all values less than 2.

Important Concepts in Solving Inequalities

Understanding inequalities requires familiarity with several key concepts:

1. Distributive Property

Used to expand expressions like 5(2h + 8), which simplifies calculations and prepares the inequality for isolating the variable.

2. Maintaining the Inequality Sign

When multiplying or dividing both sides of an inequality by a positive number, the inequality sign remains unchanged. However, if multiplying or dividing both sides by a negative number, the inequality sign must be reversed.

3. Solving for the Variable

The goal is to isolate the variable (h) to determine its possible values.

Handling Inequalities with Negative Coefficients

Suppose the coefficient of the variable was negative, for example, if the inequality was -5(2h + 8) < 60. The steps would include:

- Distribute: -5 2h + (-5) 8 = -10h - 40
- Isolate the variable: Add 40 to both sides: -10h < 100
- Divide both sides by -10: h > -10

Notice that the inequality sign flips when dividing by a negative number. Always remember this rule to avoid errors.

Common Mistakes to Avoid

When solving inequalities, learners often make several common mistakes:

- Forgetting to reverse the inequality sign when dividing or multiplying both sides by a negative number.
- Not distributing correctly, leading to incorrect expansion.
- Mixing up the solution set notation.
- Ignoring the strictness of the inequality sign (less than vs. less than or equal to).

Being cautious and double-checking each step can prevent these errors.

Practice Problems for Mastery

To reinforce understanding, try solving these similar inequalities:

1. Solve 3(4h - 5) < 27
2. Solve -2(3h + 7) < 14
3. Solve 5h + 10 > 2h + 20
4. Solve (h - 3)/2 < 4

Work through each problem step-by-step, applying the same principles discussed.

Real-Life Applications of Inequalities

Inequalities are not just theoretical; they have practical uses in various fields:

- Budgeting: Ensuring expenses stay below a certain limit.
- Physics: Describing speed limits or constraints.
- Economics: Setting bounds for profit, cost, or resource allocation.
- Engineering: Designing systems within safety margins.

Understanding how to solve inequalities equips you with tools to approach these real-world scenarios effectively.

Conclusion

Mastering the process of solving inequalities like $5(2h + 8) < 60$ is fundamental to advancing in algebra and higher mathematics. By systematically expanding, isolating the variable, and respecting the rules concerning the inequality sign, you can confidently find the solution set. Remember to practice with various problems to solidify your understanding and recognize the importance of inequalities across different contexts. With practice, solving inequalities becomes an intuitive and powerful skill that opens doors to more complex mathematical concepts and real-world problem-solving.

Frequently Asked Questions

How do I start solving the inequality $5(2h + 8) < 60$?

First, distribute the 5 across the parentheses: $5 \times 2h + 5 \times 8$, which simplifies to $10h + 40 < 60$. Then, proceed to isolate h.

What is the next step after expanding $5(2h + 8) < 60$?

Subtract 40 from both sides to isolate the term with h: $10h + 40 - 40 < 60 - 40$, simplifying to $10h < 20$.

How do I solve for h in the inequality $10h < 20$?

Divide both sides by 10: $h < 20 \div 10$, which simplifies to $h < 2$.

What is the solution to the inequality $5(2h + 8) < 60$?

The solution is $h < 2$.

Are there any restrictions or special cases to consider for this inequality?

No, since dividing by 10 is valid and there are no variables in the denominator, there are no restrictions. The solution is simply $h < 2$.

How can I check if my solution h < 2 is correct?

Choose a value less than 2, such as h = 1, and substitute back into the original inequality: 5(2×1 + 8) = 5(2 + 8) = 5×10 = 50, which is less than 60. So, it satisfies the inequality.

What if I want to express the solution in interval notation?

The solution in interval notation is (-∞, 2).

Can this inequality be solved graphically?

Yes, you can graph the related equation 10h = 20 as a vertical line at h = 2 on a number line and shade the region to the left of it, representing h < 2.

Why is it important to distribute before solving the inequality?

Distributing ensures all terms are simplified and the inequality is in a standard form, making it easier to isolate the variable and solve accurately.

Additional Resources

Solve The Inequality 5(2h + 8) < 60

Introduction

Mathematics, often regarded as the language of science, plays a crucial role in various domains ranging from engineering to economics. Among the foundational skills in mathematics is the ability to solve inequalities—a process essential for understanding relationships between quantities and making informed decisions based on those relationships. In this article, we examine the inequality 5(2h + 8) < 60 in meticulous detail, providing a comprehensive guide to solving this inequality and interpreting its implications.

Understanding the Inequality

Before delving into the solution process, it is vital to comprehend what the inequality 5(2h + 8) < 60 represents and how it fits within the broader context of algebraic problem-solving.

The Components of the Inequality

- Coefficient and expression: The number 5 is multiplied by the binomial expression $(2h + 8)$.
- Variable: The variable h is the unknown quantity we aim to solve for.
- Comparison: The entire expression is less than 60, indicating that the product must be strictly smaller than 60.

Significance

Solving this inequality determines the set of values for h that satisfy the condition, which can be crucial in optimization problems, constraints in real-world scenarios, or mathematical modeling.

Step-by-Step Solution Process

To solve $5(2h + 8) < 60$, a systematic approach is necessary. Here, we outline each step in detail, ensuring clarity for learners and practitioners alike.

Step 1: Expand the Expression

Apply the distributive property to eliminate parentheses:

$$
5 \times 2h + 5 \times 8 < 60
$$
$$
10h + 40 < 60
$$

Step 2: Isolate the Variable Term

Subtract 40 from both sides to move constants to the right side:

$$
10h + 40 - 40 < 60 - 40
$$
$$
10h < 20
$$

Step 3: Solve for h

Divide both sides by 10 to isolate h:

$$
\frac{10h}{10} < \frac{20}{10}
$$
$$
h < 2
$$

Final Solution

The inequality simplifies to $h < 2$, which indicates that any value of h less than 2 satisfies the original inequality.

Interpreting the Solution

Set Notation and Number Line Representation

The solution $h < 2$ can be expressed in set notation as:

$$
\boxed{\{ h \in \mathbb{R} \mid h < 2 \}}
$$

Graphically, on a number line:

- A shaded region extending to the left of 2
- An open circle at 2 indicating that 2 itself is not included

Real-World Implications

Understanding the solution's implications depends on the context where the inequality arises. For example:

- If h represents a quantity like hours, resources, or measurements, the solution constrains h to be less than 2.
- In business scenarios, it might denote that a certain process should be completed in less than 2 units of time or cost.

Common Challenges and Misconceptions

While the steps seem straightforward, learners often encounter pitfalls. Here are some common issues:

1. Forgetting to Distribute Properly

Neglecting to expand $5(2h + 8)$ correctly can lead to errors, especially in more complex inequalities.

2. Sign Errors During Division

When dividing both sides by a negative number (not applicable here but common in other inequalities), the inequality sign must be flipped. Always check the sign of the divisor.

3. Misinterpreting Inequality Symbols

Remember that $<$ means strictly less than, excluding the boundary point. For $\leq$, the boundary would be included.

4. Overlooking the Domain

Consider whether restrictions on h exist based on the context (e.g., h must be positive), which might influence the interpretation of the solution.

Extending the Analysis: Inequalities with Variables on Both Sides

While the current inequality is straightforward, more complex inequalities often involve variables on both sides. For example:

$$
5(2h + 8) < 60 + 3h
$$

Solving such inequalities involves additional steps:

- Expand both sides
- Collect like terms
- Isolate the variable

This process emphasizes the importance of algebraic manipulation skills and careful attention to detail.

Practical Applications of Solving Inequalities

Understanding inequalities like $5(2h + 8) < 60$ has practical significance, including:

- Resource Allocation: Ensuring resources or time do not exceed certain limits.
- Quality Control: Maintaining measurements within specified bounds.
- Economics: Establishing constraints for profit or cost functions.
- Engineering: Designing systems with parameters within safe or optimal ranges.

Mastery of inequality solving equips individuals to model real-world constraints effectively.

Summary

The inequality $5(2h + 8) < 60$ was solved through systematic algebraic procedures:

1. Expansion: Applied distributive property to eliminate parentheses.
2. Isolation: Subtracted constants to isolate the term with h.
3. Division: Divided both sides to solve for h.

The final solution $h < 2$ provides a clear criterion for the values that satisfy the inequality. Recognizing the importance of careful algebraic manipulation and the interpretation of solutions is essential for success in mathematics and its applications.

Final Thoughts

Mastering the art of solving inequalities is fundamental for advancing in mathematics and applying it meaningfully in various fields. The process exemplifies critical thinking, attention to detail, and the ability to translate abstract algebraic expressions into practical insights. Whether in academic pursuits or real-world problem-solving, such skills are invaluable and form the cornerstone of analytical reasoning.

By thoroughly analyzing and solving the inequality $5(2h + 8) < 60$, this article aims to reinforce foundational algebraic concepts and provide a robust framework for tackling similar problems in diverse contexts.

Solve The Inequality 5 2h 8 Lt 60

Solve The Inequality 5 2h 8 Lt 60

Related Articles

- List The Factor Pairs Of The Number 16
- Identify The Domain Of The Function Shown In The Graph.
- Exponential Growth: Solve For Te^(2t - 3) = 300

Back to Home