Solve The System Of Equations 5x+3y=5 5x+7y=25

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Understanding how to solve systems of equations is a fundamental skill in algebra that enables students and professionals alike to find the values of variables that satisfy multiple conditions simultaneously. In this article, we will explore the methods to solve the specific system of equations:

- $5x + 3y = 5$
- $5x + 7y = 25$

This system involves two linear equations with two variables, x and y. Solving such systems allows us to find the point(s) where the lines represented by these equations intersect. The solutions can be unique, infinite, or nonexistent, depending on the nature of the equations.

In the following sections, we will discuss various methods to solve this system, including substitution, elimination, and graphical approaches. Additionally, we'll analyze the solutions to understand their significance and application in real-world scenarios.

Understanding Systems of Equations

Before delving into solving the specific system, it is important to understand what a system of equations entails.

Definition of a System of Equations

A system of equations consists of two or more equations with the same variables. The solutions to the system are the values of variables that satisfy all equations simultaneously.

Types of Solutions

- Unique Solution: The equations intersect at a single point (distinct lines).
- Infinite Solutions: The equations represent the same line (coincident lines).
- No Solution: The lines are parallel and do not intersect.

In our case, the system appears to have potentially a unique solution, but we will verify this through calculation.

Methods to Solve the System of Equations

There are several methods to solve systems of linear equations. The most common ones are substitution, elimination, and graphical methods. Each has its advantages depending on the system's structure.

1. Substitution Method

This method involves solving one equation for one variable and substituting that into the other equation.

2. Elimination Method

This technique eliminates one variable by combining the equations, making it easier to solve for the remaining variable.

3. Graphical Method

Plotting both equations on a coordinate plane and locating their intersection point visually.

For our specific system, we'll primarily use the elimination method due to the structure of the equations.

Step-by-Step Solution Using the Elimination Method

Let's apply the elimination method to solve the system:

- $5x + 3y = 5$ ...(Equation 1)
- $5x + 7y = 25$ ...(Equation 2)

Step 1: Subtract Equation 1 from Equation 2

Subtract the left sides and right sides:

$(5x + 7y) - (5x + 3y) = 25 - 5$

Simplify:

$5x + 7y - 5x - 3y = 20$

This simplifies to:

$(7y - 3y) = 20$

$2y = 20$

Step 2: Solve for y

Divide both sides by 2:

$y = 20 / 2$

$y = 10$

Step 3: Substitute y back into one of the original equations

Use Equation 1:

$5x + 3y = 5$

Plug in $y = 10$:

$5x + 3(10) = 5$

$5x + 30 = 5$

Subtract 30 from both sides:

$5x = 5 - 30$

$5x = -25$

Divide both sides by 5:

$x = -25 / 5$

$x = -5$

Result:

The solution to the system is:

x = -5, y = 10

This point (-5, 10) is the intersection point of the two lines, representing the unique solution to the system.

Verifying the Solution

Verification is crucial to ensure the accuracy of the solution.

Substitute into Equation 2:

5x + 7y = 25

5(-5) + 7(10) = 25

-25 + 70 = 25

45 ≠ 25

This indicates a discrepancy. Let's double-check the calculations.

Wait, earlier, we found y = 10. Let's re-verify by substituting x = -5 into Equation 2:

5x + 7y = 25

5(-5) + 7(10) = 25

-25 + 70 = 45

This does not satisfy the original second equation, which states the sum should be 25, but our calculation yields 45. Therefore, there's an inconsistency indicating an error in the initial elimination step.

Let's revisit the elimination process carefully.

Re-evaluating the Solution

Original equations:

1. $5x + 3y = 5$
2. $5x + 7y = 25$

Subtract Equation 1 from Equation 2:

$(5x + 7y) - (5x + 3y) = 25 - 5$

Simplify:

$5x + 7y - 5x - 3y = 20$

$(7y - 3y) = 20$

$4y = 20$

Now, dividing both sides by 4:

$y = 20 / 4 = 5$

Now, substitute $y = 5$ into Equation 1:

$5x + 3(5) = 5$

$5x + 15 = 5$

Subtract 15 from both sides:

$5x = 5 - 15 = -10$

Divide both sides by 5:

$x = -10 / 5 = -2$

Solution:

$x = -2, y = 5$

Verification:

Equation 1:

$5(-2) + 3(5) = -10 + 15 = 5$ ✓

Equation 2:

$5(-2) + 7(5) = -10 + 35 = 25$ ✓

The corrected solution is $x = -2$, $y = 5$.

Graphical Representation of the System

Visualizing the system's equations can provide intuition about the solution.

Plotting the Equations

- Equation 1: $5x + 3y = 5$

- Intercept form:

$y = (5 - 5x) / 3$

- When $x = 0$, $y = 5/3 \approx 1.67$

- When $y = 0$, $x = 1$

- Equation 2: $5x + 7y = 25$

- $y = (25 - 5x) / 7$

- When $x = 0$, $y = 25/7 \approx 3.57$

- When $y = 0$, $x = 5$

Plotting these lines on a graph will show they intersect at the point (-2, 5), confirming our algebraic solution.

Applications of Solving Systems of Equations

Understanding how to solve systems of equations is essential in various fields:

- Economics: Determining supply and demand equilibrium points.
- Engineering: Calculating parameters that satisfy multiple design constraints.
- Physics: Solving for variables in kinematic equations.
- Business: Optimizing profit and cost functions simultaneously.
- Computer Science: Algorithm design involving linear constraints.

Common Mistakes and Tips for Solving Systems

When solving systems of equations, be aware of common pitfalls:

- Mixing up signs during subtraction or addition.
- Forgetting to verify solutions in all original equations.
- Misidentifying the method best suited for the system.
- Overlooking special cases like parallel or coincident lines.

Tips:

- Always double-check calculations.
- Use substitution when one variable is easily isolated.
- Use elimination when coefficients are the same or additive inverses.
- Graphically verify solutions for visual confirmation.

Conclusion: Mastering the Solution of the System

The system of equations:

- $5x + 3y = 5$
- $5x + 7y = 25$

has a unique solution at $x = -2$, $y = 5$. By applying the elimination method carefully, you can efficiently find the intersection point of two lines, which is fundamental in solving real-world problems involving multiple constraints. Remember, verifying your solutions and understanding the nature of the equations ensures accuracy and confidence in your results.

Whether you're a student learning algebra or a professional applying these concepts, mastering methods like substitution, elimination, and graphing will enhance your problem-solving toolkit. Keep practicing with different systems to build your skills and understanding of linear equations and their applications.

SEO Keywords:
- Solve system of equations
- Linear equations solution
- Elimination method
- Substitution method
- Graphical solution of equations
- System of two equations
- Algebraic methods
- Linear system solutions
- Equation intersection point
- Solving equations step-by-step

Frequently Asked Questions

What are the steps to solve the system of equations 5x + 3y = 5 and 5x + 7y = 25?

To solve the system, you can use the elimination method. Subtract the first equation from the second: $(5x + 7y) - (5x + 3y) = 25 - 5$, which simplifies to $4y = 20$. Then, solve for y: $y = 5$. Substitute $y = 5$ into one of the original equations (e.g., $5x + 3(5) = 5$) to find x: $5x + 15 = 5$, so $5x = -10$, and $x = -2$.

What is the solution to the system of equations 5x + 3y = 5 and 5x + 7y = 25?

The solution is $x = -2$ and $y = 5$.

Can the system of equations 5x + 3y = 5 and 5x + 7y = 25 be solved using substitution?

Yes. You can solve for x or y from one equation and substitute into the other. For example, from the first equation: $5x = 5 - 3y$, so $x = (5 - 3y)/5$. Substitute into the second equation and solve for y.

Are the equations 5x + 3y = 5 and 5x + 7y = 25 consistent?

Yes. They have a unique solution at $x = -2$ and $y = 5$, so the system is consistent and independent.

What method is most efficient for solving the system 5x + 3y = 5 and 5x + 7y = 25?

The elimination method is efficient here because the coefficients of x are the same in both equations. Subtracting the equations eliminates x, making it straightforward to find y.

What is the significance of the coefficients in the system 5x + 3y = 5 and 5x + 7y = 25?

The coefficients determine the slope and intercept of the lines. Since both equations have the same coefficient for x, the lines are parallel if y coefficients differ, but here they intersect at a point, indicating a unique solution.

Can the system 5x + 3y = 5 and 5x + 7y = 25 be inconsistent?

No, because the system has a consistent solution at $x = -2$ and $y = 5$. If the equations were contradictory (e.g., leading to a false statement), then they would be inconsistent.

Additional Resources

Solve The System Of Equations 5x + 3y = 5 and 5x + 7y = 25: An In-Depth Exploration

When tackling systems of equations, understanding the underlying principles and methods is essential for solving them accurately and efficiently. The system of equations:

$$
\begin{cases}
5x + 3y = 5 \quad \text{(Equation 1)} \\
5x + 7y = 25 \quad \text{(Equation 2)} \\
\end{cases}
$$

serves as a classic example used in algebra to illustrate various solution strategies such as substitution, elimination, and graphical interpretation. This detailed review aims to dissect every aspect of this system, providing comprehensive insight into methods of solution, the nature of solutions, and real-world applications.

Understanding the Nature of the System

Before delving into solution strategies, it is crucial to understand the types of systems of equations and what their solutions imply.

Types of Systems of Equations

- Consistent and Independent: The system has exactly one solution. The lines intersect at a single point.
- Consistent and Dependent: The system has infinitely many solutions. The lines coincide.
- Inconsistent: The system has no solution. The lines are parallel and do not intersect.

Given the equations:

$$5x + 3y = 5$$

$$5x + 7y = 25$$

we anticipate this system to be consistent with a unique solution because the coefficients of x are the same but the coefficients of y differ, suggesting the lines may intersect at a single point.

Graphical Interpretation

Visualizing the equations on the Cartesian plane offers an intuitive understanding of their solution.

Plotting the Equations

1. Equation 1: $5x + 3y = 5$

- To plot, find intercepts:
- When $x = 0$,

$$3y = 5 \Rightarrow y = \frac{5}{3} \approx 1.666...$$

- When $y = 0$,

$$
5x = 5 \Rightarrow x = 1
$$

- Plot points: $(0, 1.666...)$ and $(1, 0)$

2. Equation 2: $5x + 7y = 25$

- Find intercepts:
- When $x = 0$,

$$
7y = 25 \Rightarrow y = \frac{25}{7} \approx 3.571
$$

- When $y = 0$,

$$
5x = 25 \Rightarrow x = 5
$$

- Plot points: $(0, 3.571)$ and $(5, 0)$

3. Intersection Point:
- The lines will intersect at a point that satisfies both equations. Graphically, the intersection point is the solution to the system.

Methods to Solve the System

Multiple algebraic techniques can be employed to find the exact solution to the system. The most common methods include substitution, elimination, and graphing. Each method has advantages depending on the structure of the system.

Method 1: Substitution

This method involves solving one of the equations for one variable and substituting into the other.

Step-by-step process:
1. Solve Equation 1 for x:

$$
5x + 3y = 5 \Rightarrow 5x = 5 - 3y \Rightarrow x = \frac{5 - 3y}{5}
$$

2. Substitute x into Equation 2:

\[
5 \left(\frac{5 - 3y}{5}\right) + 7y = 25
\]
3. Simplify:
\[
(5 - 3y) + 7y = 25
\]
\[
5 + 4y = 25
\]
4. Solve for \(y\):
\[
4y = 20 \Rightarrow y = 5
\]
5. Substitute \(y = 5\) back into the expression for \(x\):
\[
x = \frac{5 - 3(5)}{5} = \frac{5 - 15}{5} = \frac{-10}{5} = -2
\]

Solution:
\[
\boxed{
x = -2,\quad y = 5
}
\]

Verification:
- Check in Equation 1:
\[
5(-2) + 3(5) = -10 + 15 = 5 \quad \checkmark
\]
- Check in Equation 2:
\[
5(-2) + 7(5) = -10 + 35 = 25 \quad \checkmark
\]

The solution satisfies both equations, confirming its correctness.

Method 2: Elimination

Elimination involves aligning coefficients to cancel one variable.

Step-by-step process:

1. Observe that the coefficients of x in both equations are identical (5). To eliminate x, subtract one equation from the other:

$$
(5x + 3y) - (5x + 7y) = 5 - 25
$$
$$
5x - 5x + 3y - 7y = -20
$$
$$
-4y = -20
$$
$$
y = 5
$$

2. Substitute $y = 5$ into either original equation, say Equation 1:
$$
5x + 3(5) = 5
$$
$$
5x + 15 = 5
$$
$$
5x = -10
$$
$$
x = -2
$$

Solution:
$$
x = -2,\quad y = 5
$$

Verification:
- Same as above, the solution is consistent with both equations.

Method 3: Graphical Solution

Plotting the two lines as described earlier confirms the intersection point at $(-2, 5)$. While graphical methods lack algebraic precision, they are useful for visual understanding and approximate solutions.

Analyzing the Solution

The solution $(x, y) = (-2, 5)$ indicates a unique intersection point, confirming that the system is consistent and independent.

Key observations:
- The lines are neither parallel nor coincident.
- The equations have different slopes when written in slope-intercept form.

Expressing the Equations in Slope-Intercept Form

Transforming the given equations into $y = mx + b$ forms facilitates understanding their slopes and intercepts.

1. Equation 1:
$$
5x + 3y = 5 \Rightarrow 3y = -5x + 5 \Rightarrow y = -\frac{5}{3}x + \frac{5}{3}
$$
- Slope $m_1 = -\frac{5}{3}$
- Y-intercept $b_1 = \frac{5}{3}$

2. Equation 2:
$$
5x + 7y = 25 \Rightarrow 7y = -5x + 25 \Rightarrow y = -\frac{5}{7}x + \frac{25}{7}
$$
- Slope $m_2 = -\frac{5}{7}$
- Y-intercept $b_2 = \frac{25}{7}$

Since $m_1 \neq m_2$, the lines are not parallel and will intersect at a single point, aligning with the algebraic solution.

Applications of Solving Such Systems

Systems of linear equations like this appear extensively in various fields:

- Economics: To find equilibrium points where supply equals demand.
- Engineering: For circuit analysis where multiple equations describe different constraints.
- Physics: To solve for multiple variables in kinematic problems.
- Computer Science: In algorithms involving linear programming and optimization.
- Statistics: For regression analysis involving multiple variables.

In real-world scenarios, the ability to solve systems accurately and efficiently is critical for decision-making and modeling.

Advanced Considerations and Variations

While the current system has a unique solution, systems can present different challenges and structures.

Systems with No Solution

- When the equations represent parallel lines, the system has no solution.
- For example:

$$
2x + 3y = 4
$$

$$
4x + 6y = 10
$$

These are parallel and inconsistent.

Systems with Infinitely Many Solutions

- When the two equations represent the same line.
- For example:

$$
3x + 2y = 6
$$

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