Solve (x + 3)2 5 = 0 Using The Quadratic Formula.

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Mathematics often presents students and professionals with complex equations that require precise methods to solve. One such fundamental technique is solving quadratic equations, which frequently appear across various fields such as physics, engineering, economics, and everyday problem-solving. Among the methods used to find solutions to quadratic equations, the quadratic formula stands out for its reliability and universal applicability. In this article, we explore how to solve the equation (x + 3)^2 5 = 0 using the quadratic formula, providing step-by-step guidance, insights into the process, and tips for mastering this essential skill.

Understanding the Equation: (x + 3)^2 5 = 0

Before diving into the solution process, it’s crucial to interpret and understand the given equation properly.

Decoding the Equation

The equation is written as:

(x + 3)^2 5 = 0

At first glance, the notation might seem ambiguous. Typically, in algebra, when an equation involves a squared term, it appears as (x + 3)^2. The number 5 placed immediately after the squared term could suggest multiple interpretations:

1. (x + 3)^2 5 = 0: This indicates that the squared binomial is multiplied by 5.
2. (x + 3)^2 + 5 = 0: An addition operation; however, this is less likely given the notation.
3. A typographical error or formatting issue, intending to write (x + 3)^2 = 5.

Given the context and common algebraic structure, the most plausible interpretation is:

(x + 3)^2 5 = 0

which simplifies to:

5 (x + 3)^2 = 0

Rewriting the Equation for Clarity

If the original equation is:

$5 (x + 3)^2 = 0$

then, dividing both sides of the equation by 5 (since $5 \neq 0$):

$(x + 3)^2 = 0$

This simplifies the problem significantly, reducing it to solving a perfect square equal to zero.

Solving the Equation Step-by-Step

Let's explore how to solve $(x + 3)^2 = 0$ using the quadratic formula, even though this specific equation can be solved more straightforwardly by taking square roots. Applying the quadratic formula provides a comprehensive understanding and reinforces problem-solving skills.

Step 1: Expand the Equation into Standard Quadratic Form

The equation:

$(x + 3)^2 = 0$

can be expanded as:

$(x + 3)^2 = x^2 + 6x + 9 = 0$

Now, the quadratic equation is in the standard form:

$ax^2 + bx + c = 0$

where:

- $a = 1$
- $b = 6$
- $c = 9$

Step 2: Recall the Quadratic Formula

The quadratic formula is:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

This formula provides solutions to any quadratic equation, regardless of whether it factors easily.

Step 3: Calculate the Discriminant

The discriminant, D, determines the nature of the roots:

$$
D = b^2 - 4ac
$$

Plugging in the values:

$$
D = (6)^2 - 4 \times 1 \times 9 = 36 - 36 = 0
$$

A discriminant of zero indicates that the quadratic has exactly one real root (a repeated root).

Step 4: Compute the Root Using the Quadratic Formula

Since D = 0, the quadratic formula simplifies to:

$$
x = \frac{-b}{2a}
$$

Calculating:

$$
x = \frac{-6}{2 \times 1} = \frac{-6}{2} = -3
$$

This indicates a single real solution: x = -3.

Verifying the Solution

Substitute $x = -3$ back into the original expanded equation:

$$
(x + 3)^2 = 0
$$

$$
(-3 + 3)^2 = 0
$$

$$
0^2 = 0
$$

which confirms that $x = -3$ is the correct and only solution.

Addressing the Original Equation: $(x + 3)^2 5 = 0$

Recall that earlier, the original problem involved multiplying by 5:

$$
5 \times (x + 3)^2 = 0
$$

Dividing both sides by 5:

$$
(x + 3)^2 = 0
$$

which leads us directly to the solution $x = -3$.

Additional Insights: When to Use the Quadratic Formula

While the quadratic formula is versatile, it’s beneficial to recognize scenarios where it is particularly useful:

- When the quadratic equation does not factor easily.
- When the coefficients are complex or involve irrational numbers.
- When the discriminant is negative, indicating complex roots.
- To verify solutions obtained through factoring or completing the square.

Alternative Methods for Solving Quadratic Equations

Although the quadratic formula is a universal tool, other methods exist:

Factoring

- Suitable when the quadratic factors neatly into binomials.
- For example, if the quadratic is x^2 + 6x + 9, it factors as (x + 3)^2.

Completing the Square

- A method to convert the quadratic into a perfect square trinomial.
- Useful for deriving the quadratic formula in some contexts or solving equations where factoring is difficult.

Graphical Method

- Plotting the quadratic function and identifying the roots visually.
- Useful for understanding the nature of roots and their approximate values.

Practical Applications of Solving Quadratic Equations

Quadratic equations appear in numerous real-world situations, such as:

- Calculating projectile motion in physics.
- Optimizing business profits and costs.
- Engineering design problems involving parabolic structures.
- Economics models involving quadratic cost or revenue functions.
- Biological models describing population growth patterns.

Mastering the quadratic formula enables solving these problems efficiently and accurately.

Tips for Mastering the Quadratic Formula

To become proficient in solving quadratic equations using the quadratic formula, consider the following tips:

1. Always write the quadratic in standard form: ax^2 + bx + c = 0.
2. Identify the coefficients: a, b, and c.
3. Calculate the discriminant carefully: D = b^2 - 4ac.
4. Determine the nature of roots:
- D > 0: Two real roots.
- D = 0: One real root.
- D < 0: Complex roots.
5. Simplify radicals when computing the square root of the discriminant.
6. Check solutions by substitution to verify their correctness.

Conclusion

In summary, solving the equation (x + 3)^2 5 = 0 using the quadratic formula involves recognizing the structure of the quadratic, expanding it, calculating the discriminant, and applying the formula to find the roots. The key steps include transforming the equation into standard form, computing the discriminant, and simplifying the solution. In this case, the solution is x = -3, which is a repeated root due to the discriminant being zero.

Understanding how to effectively utilize the quadratic formula not only enhances problem-solving skills but also equips learners with a powerful tool applicable across diverse mathematical and real-world problems. Practice with different quadratic equations, explore alternate methods, and verify solutions to gain confidence and mastery in solving quadratic equations efficiently.

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Frequently Asked Questions

How do you solve the quadratic equation $(x + 3)^2 - 5 = 0$ using the quadratic formula?

First, expand and rewrite the equation in standard form: $(x + 3)^2 - 5 = 0$ becomes $x^2 + 6x + 9 - 5 = 0$, which simplifies to $x^2 + 6x + 4 = 0$. Then, identify a=1, b=6, and c=4, and apply the quadratic formula $x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$ to find the solutions.

What are the steps to solve $(x + 3)^2 - 5 = 0$ using the quadratic formula?

Step 1: Expand the equation to $x^2 + 6x + 9 - 5 = 0$. Step 2: Simplify to $x^2 + 6x + 4 = 0$. Step 3: Identify coefficients a=1, b=6, c=4. Step 4: Calculate the discriminant $D = b^2 - 4ac = 36 - 16 = 20$. Step 5: Use the quadratic formula to find $x = [-6 \pm \sqrt{20}] / 2$, which simplifies to $x = [-6 \pm 2\sqrt{5}] / 2$, resulting in $x = -3 \pm \sqrt{5}$.

Why do we use the quadratic formula to solve $(x + 3)^2 - 5 = 0$?

Because the equation is a quadratic in standard form after expansion, and the quadratic formula provides a systematic way to find exact solutions even when factoring is difficult or not straightforward.

Can $(x + 3)^2 - 5 = 0$ be solved by factoring instead of using the quadratic formula?

Yes, since $(x + 3)^2 - 5 = 0$ can be written as $(x + 3)^2 = 5$, then taking the square root of both sides gives $x + 3 = \pm\sqrt{5}$, leading to solutions $x = -3 \pm \sqrt{5}$. However, if the quadratic cannot be easily factored, the quadratic formula is a reliable method.

What are the solutions to $(x + 3)^2 - 5 = 0$?

The solutions are $x = -3 + \sqrt{5}$ and $x = -3 - \sqrt{5}$.

Additional Resources

Solving the Quadratic Equation $(x + 3)^2 - 5 = 0$ Using The Quadratic Formula

Understanding how to solve quadratic equations is a fundamental skill in algebra, and the quadratic formula is one of the most powerful tools in a mathematician’s toolkit. This detailed review explores the process of solving the specific quadratic equation $(x + 3)^2 - 5 = 0$ using the quadratic formula. We will delve into the equation's structure, the steps involved in transforming it into the standard quadratic form, and the application of the quadratic formula itself. Additionally, we will discuss the interpretation of solutions, potential pitfalls, and the broader context of solving quadratic equations.

Understanding the Equation: $(x + 3)^2 - 5 = 0$

Before jumping into solving, it’s essential to understand the structure of the given equation:

- Equation Form: $(x + 3)^2 - 5 = 0$
- Type: Quadratic equation (since the highest power of the variable x is 2 when expanded)
- Goal: Find the value(s) of x that satisfy the equation

This equation involves a perfect square $(x + 3)^2$ minus 5. Recognizing the structure facilitates the process of solving, especially when using the quadratic formula, which requires the quadratic in standard form $ax^2 + bx + c = 0$.

Transforming the Equation into Standard Quadratic Form

The quadratic formula applies directly to equations in standard form, so the first step is to expand and simplify the given equation:

Step 1: Expand the squared term

$$
(x + 3)^2 = x^2 + 6x + 9
$$

Step 2: Rewrite the original equation

$$
x^2 + 6x + 9 - 5 = 0
$$

Step 3: Simplify the constant terms

$$
x^2 + 6x + 4 = 0
$$

Now, we have the quadratic equation in standard form:

$$
ax^2 + bx + c = 0
$$
where:
- $a = 1$
- $b = 6$
- $c = 4$

Applying the Quadratic Formula

The quadratic formula is given by:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

This formula allows us to find the roots of any quadratic equation, provided that the discriminant $b^2 - 4ac$ is non-negative.

Step 1: Calculate the discriminant

$$
D = b^2 - 4ac
$$

Substitute the known values:

$$
D = (6)^2 - 4 \times 1 \times 4 = 36 - 16 = 20
$$

Since $D = 20 > 0$, the quadratic equation has two real and distinct solutions.

Step 2: Compute the square root of the discriminant

$$
\sqrt{D} = \sqrt{20} = \sqrt{4 \times 5} = 2 \sqrt{5}
$$

Step 3: Plug into the quadratic formula

$$
x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-6 \pm 2\sqrt{5}}{2 \times 1} = \frac{-6 \pm 2\sqrt{5}}{2}
$$

Step 4: Simplify the solutions

Divide numerator and denominator by 2:

$$
x = \frac{-6}{2} \pm \frac{2\sqrt{5}}{2} = -3 \pm \sqrt{5}
$$

Final Solutions and Interpretation

The solutions to the original equation are:

$$
\boxed{
x = -3 + \sqrt{5} \quad \text{and} \quad x = -3 - \sqrt{5}
}
$$

These solutions are exact, involving irrational numbers, which is typical when the discriminant is not a perfect square.

Numerical Approximations:

- $\sqrt{5} \approx 2.236$

Thus,

- $x \approx -3 + 2.236 = -0.764$
- $x \approx -3 - 2.236 = -5.236$

These approximate solutions can be useful for practical applications or graphing purposes.

Understanding the Solutions and Their Significance

1. Nature of the Roots

- Since the discriminant was positive, the quadratic has two real roots.
- These roots are irrational because $\sqrt{5}$ is irrational.
- The roots are symmetric about the axis of symmetry of the parabola.

2. Graphical Interpretation

- The quadratic $x^2 + 6x + 4$ is a parabola opening upwards (since $a=1 > 0$).
- The roots correspond to the points where the parabola intersects the x-axis.
- The solutions $x = -3 \pm \sqrt{5}$ mark these intersection points.

3. Broader Context

- Recognizing the structure of the original equation helps in quicker solutions.
- The quadratic formula is a universal method applicable to all quadratic equations, whether factorable or not.
- Understanding the discriminant guides expectations about the number and type of roots, saving time in problem-solving.

Potential Pitfalls and Common Mistakes

While solving quadratic equations using the quadratic formula is straightforward, some common errors include:

1. Incorrect expansion

- Failing to correctly expand $(x + 3)^2$ as $x^2 + 6x + 9$.
- Overlooking the need to simplify constants after expansion.

2. Errors in substitution

- Mixing up the signs or coefficients when substituting into the quadratic formula.
- Forgetting to include the $\pm$ in the formula, leading to only one solution.

3. Miscalculating the discriminant

- Square roots of negative numbers require complex solutions; misinterpreting the discriminant can lead to incorrect conclusions about the nature of roots.

4. Simplification mistakes

- Simplifying the quadratic formula, especially when dividing numerator and denominator, can lead to errors in the final answer.

5. Not checking solutions

- Substituting solutions back into the original equation to verify correctness is a good practice, especially in more complex problems.

Extensions and Broader Applications

1. Connection to Factoring

- The solutions $x = -3 \pm \sqrt{5}$ can sometimes be expressed via factoring methods, but irrational roots often make factoring less practical.

2. Completing the Square

- Alternatively, the original equation $(x + 3)^2 - 5 = 0$ can be solved by taking square roots directly:

$$
(x + 3)^2 = 5
$$

$$
x + 3 = \pm \sqrt{5}
$$

$$
x = -3 \pm \sqrt{5}
$$

- This demonstrates the equivalence of different solution methods.

3. Graphing and Visualization

- Plotting $y = (x + 3)^2 - 5$ helps visualize where the parabola crosses the x-axis, reinforcing the solutions.

4. Real-world Applications

- Quadratic equations and the quadratic formula appear in physics (projectile motion), engineering, economics, and various sciences where relationships are quadratic.

Conclusion: Mastering the Quadratic Formula for $(x + 3)^2 - 5 = 0$

Solving the equation $(x + 3)^2 - 5 = 0$ via the quadratic formula exemplifies the power and flexibility of this method. By carefully transforming the original expression into standard form, calculating the discriminant, and applying the formula, solutions emerge clearly and precisely. The process underscores the importance of understanding the structure of quadratic equations, the significance of the discriminant in predicting the nature of roots, and the value of systematic problem-solving approaches.

Whether for academic purposes, practical applications, or to deepen mathematical understanding, mastering the quadratic formula is essential. It not only provides solutions to specific equations but also enhances problem-solving skills applicable across diverse mathematical contexts. Remember, attention to detail during each step ensures accuracy and confidence in deriving solutions. With practice, solving quadratics like $(x + 3)^2 - 5 = 0$ becomes a routine yet powerful skill in the mathematician’s toolkit.

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