Sum Up The Series 2/3+5/9+8/27+11/81+..... To N Terms

Sum Up The Series 2/3+5/9+8/27+11/81+..... To N Terms is a problem that often appears in algebra and mathematical series topics, especially in the context of arithmetic and geometric progressions. Understanding how to evaluate the sum of such a series requires recognizing the pattern in the numerators and denominators, and then applying appropriate formulas or techniques to find the sum to a specified number of terms, N. This article provides a comprehensive guide to summing this particular series, exploring the pattern, deriving the general term, and then calculating the sum for any number of terms.

Understanding the Series Pattern

Examining the Series Terms

The series under consideration is:

2/3 + 5/9 + 8/27 + 11/81 + ... to N terms.

Let's analyze the numerators and denominators separately:

- Numerators: 2, 5, 8, 11, ...
- Denominators: 3, 9, 27, 81, ...

Notice that:

- The numerators form an arithmetic sequence starting at 2 and increasing by 3 each time.
- The denominators form a geometric sequence starting at 3 and multiplying by 3 each time.

Pattern in Numerators

The sequence of numerators:

2, 5, 8, 11, ...

can be expressed as:

Numerator $a_n = 2 + (n-1) \times 3$

which simplifies to:

$$a_n = 3n - 1$$

for $n \geq 1$.

Pattern in Denominators

The denominators:

3, 9, 27, 81, ...

are powers of 3:

$$ 3^1, 3^2, 3^3, 3^4, ... $$

so the denominator for the nth term:

$$ d_n = 3^n $$

General Term of the Series

Combining these observations, the n-th term of the series, T_n, can be written as:

$$ T_n = \frac{a_n}{d_n} = \frac{3n - 1}{3^n} $$

This formula allows us to compute any term directly, which is essential for summing the series.

Sum of the Series to N Terms

The sum of the first N terms, S_N, is:

$$ S_N = \sum_{n=1}^{N} T_n = \sum_{n=1}^{N} \frac{3n - 1}{3^n} $$

This sum can be separated into two sums:

$$ S_N = 3 \sum_{n=1}^{N} \frac{n}{3^n} - \sum_{n=1}^{N} \frac{1}{3^n} $$

which simplifies the problem into calculating two known types of series.

Calculating the Series Components

Sum of Geometric Series

The second sum:

$$ \sum_{n=1}^{N} \frac{1}{3^n} $$

is a finite geometric series with first term $\frac{1}{3}$ and common ratio $\frac{1}{3}$. Its sum is:

$$ \sum_{n=1}^{N} r^n = \frac{r(1 - r^N)}{1 - r} $$

Applying this formula:

$$ \sum_{n=1}^{N} \frac{1}{3^n} = \frac{\frac{1}{3}(1 - (1/3)^N)}{1 - \frac{1}{3}} = \frac{\frac{1}{3}(1 - (1/3)^N)}{\frac{2}{3}} = \frac{1 - (1/3)^N}{2} $$

Sum of n/3^n Series

The first sum:

$$ \sum_{n=1}^{N} \frac{n}{3^n} $$

is a bit more involved, but there is a known formula for the sum of $n r^n$ for $|r| < 1$:

$$ \sum_{n=1}^{\infty} n r^n = \frac{r}{(1 - r)^2} $$

For the finite sum up to N, the sum is:

$$ \sum_{n=1}^{N} n r^n = \frac{r(1 - (N+1)r^N + N r^{N+1})}{(1 - r)^2} $$

Applying this with $r = \frac{1}{3}$:

$$ \sum_{n=1}^{N} n \left(\frac{1}{3}\right)^n = \frac{\frac{1}{3} \left[1 - (N+1)\left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^{N+1} \right]}{\left(1 - \frac{1}{3}\right)^2} $$

Since $1 - \frac{1}{3} = \frac{2}{3}$, then:

$$ (1 - r)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9} $$

Thus, the sum simplifies to:

$$ \sum_{n=1}^{N} n \left(\frac{1}{3}\right)^n = \frac{\frac{1}{3} \left[1 - (N+1)\left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^{N+1} \right]}{\frac{4}{9}} $$

Multiplying numerator and denominator:

$$ = \left(\frac{\frac{1}{3}}{\frac{4}{9}} \right) \left[1 - (N+1)\left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^{N+1} \right] $$

$$ = \left(\frac{\frac{1}{3} \times 9}{4} \right) \left[1 - (N+1)\left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^{N+1} \right] $$

$$ = \left(\frac{3}{4} \right) \left[1 - (N+1)\left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^{N+1} \right] $$

Note that:

$$ \left(\frac{1}{3}\right)^{N+1} = \left(\frac{1}{3}\right)^N \times \frac{1}{3} $$

So the sum becomes:

$$ \sum_{n=1}^{N} n \left(\frac{1}{3}\right)^n = \frac{3}{4} \left[1 - (N+1)\left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^N \times \frac{1}{3} \right] $$

\[= \frac{3}{4} \left[1 - (N+1)\left(\frac{1}{3}\right)^N + \frac{N}{3}

\left(\frac{1}{3}\right)^N \right] \]

\[= \frac{3}{4} \left[1 - (N+1)\left(\frac{1}{3}\right)^N + \frac{N}{3} \left(\frac{1}{3}\right)^N \right] \]

Simplify the terms inside brackets:

\[- (N+1)\left(\frac{1}{3}\right)^N + \frac{N}{3} \left(\frac{1}{3}\right)^N = \left[- (N+1) + \frac{N}{3} \right] \left(\frac{1}{3}\right)^N \]

Expressing both over a common denominator 3:

\[- \frac{3(N+1)}{3} + \frac{N}{3} = \frac{-3(N+1) + N}{3} = \frac{-3N - 3 + N}{3} = \frac{-2N - 3}{3} \]

Therefore, the sum:

\[\sum_{n=1}^{N} n \left(\frac{1}{3}\right)^n = \frac{3}{4} \left[1 + \frac{-2N - 3}{3} \left(\frac{1}{3}\right)^N \right] \]

which simplifies to:

\[\frac{3}{4} \left[1 - \frac{2N + 3}{3} \left(\frac{1}{3}\right)^N \right] \]

or,

\[

Frequently Asked Questions

What is the general term of the series 2/3 + 5/9 + 8/27 + 11/81 + ... to N terms?

The nth term Tn can be written as Tn = (3n - 1) / 3^n.

How can I find the sum of the series 2/3 + 5/9 + 8/27 + ... to N terms?

You can express the series as the sum of two separate series and use known formulas for geometric series to find the total sum.

Is the series 2/3 + 5/9 + 8/27 + ... convergent, and does it have a finite sum as N approaches infinity?

Yes, since the denominator grows exponentially, the series converges, and its sum approaches a finite limit as N approaches infinity.

What is the sum of the series as N approaches infinity?

The sum converges to 2, i.e., lim_{N→∞} S_N = 2.

Can the sum of the series be calculated using the formula for the sum of a series with a general term?

Yes, by expressing the series as a combination of geometric series, the sum can be derived using standard summation formulas.

How do I derive the sum formula for this series step-by-step?

First, rewrite the series as the sum of two separate series involving geometric terms, then apply the geometric series sum formula to each and combine the results.

Are there any special techniques or formulas needed to sum this series?

Yes, you need to recognize geometric series and use their sum formulas, along with algebraic manipulation to handle the numerator's linear pattern.

Can this series be related to any well-known series or mathematical concepts?

Yes, it can be related to the sum of a linear numerator over exponential denominators, often seen in generating functions and in the study of power series.

How do I verify my sum calculation for the series to N terms?

You can compute partial sums for small values of N and compare them with the formula result to ensure correctness.

Additional Resources

Sum Up The Series 2/3 + 5/9 + 8/27 + 11/81 + ... To N Terms: A Comprehensive Guide

Understanding how to sum up the series 2/3 + 5/9 + 8/27 + 11/81 + ... to N terms is an intriguing mathematical problem that combines elements of arithmetic and geometric progressions. This series presents a unique blend of increasing numerators and denominators that grow exponentially, making it a fascinating example for learners and professionals alike. In this guide, we will explore the structure of the series, derive formulas for its sum, and look at practical methods to evaluate the sum for any number of terms.

Introduction: The Nature of the Series

The series in question is:

$$ S_N = \frac{2}{3} + \frac{5}{9} + \frac{8}{27} + \frac{11}{81} + \dots + \text{up to N terms} $$

At first glance, the pattern of the numerators and denominators may seem complex, but a closer look reveals underlying patterns that can be leveraged to derive a summation formula. Recognizing these patterns allows us to break down the series into manageable components.

Understanding the Series Components

Numerator Pattern

The numerators are:

2, 5, 8, 11, ...

This sequence increases by 3 each time, indicating an arithmetic progression.

- First term: $a_1 = 2$
- Common difference: $d = 3$

The nth numerator, N_n, can be expressed as:

$$N_n = 2 + (n - 1) \times 3 = 3n - 1$$

Denominator Pattern

The denominators are:

3, 9, 27, 81, ...

These are powers of 3:

- 3 = 3^1
- 9 = 3^2
- 27 = 3^3
- 81 = 3^4

The pattern indicates the denominator for the nth term is:

$$D_n = 3^n$$

Formulating the General Term

Putting the numerator and denominator together, the nth term, T_n, of the series is:

$$T_n = \frac{3n - 1}{3^n}$$

Thus, the series becomes:

$$S_N = \sum_{n=1}^N \frac{3n - 1}{3^n}$$

Our goal is to find a closed-form expression for S_N.

Breaking Down the Series

To evaluate S_N, it helps to split the sum into two separate sums:

$$
S_N = \sum_{n=1}^N \frac{3n}{3^n} - \sum_{n=1}^N \frac{1}{3^n}
$$

which simplifies to

$$
S_N = 3 \sum_{n=1}^N \frac{n}{3^n} - \sum_{n=1}^N \frac{1}{3^n}
$$

Now, our focus shifts to evaluating these two sums separately.

Evaluating the Sums

Sum 1: $\sum_{n=1}^N \frac{n}{3^n}$

This is a standard sum involving n over a geometric denominator. The sum:

$$
\sum_{n=1}^N n r^n
$$

has a known closed-form, especially when r is less than 1.

The finite sum:

$$
\sum_{n=1}^N n r^n = \frac{r [1 - (N+1) r^N + N r^{N+1}]}{(1 - r)^2}
$$

Applying this with $r = \frac{1}{3}$:

$$
\sum_{n=1}^N \frac{n}{3^n} = \frac{\frac{1}{3} \left[1 - (N+1) \left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^{N+1}\right]}{\left(1 - \frac{1}{3}\right)^2}
$$

Simplify denominator:

$$
1 - \frac{1}{3} = \frac{2}{3} \quad \Rightarrow \quad (1 - r)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}
$$

Thus, the sum becomes:

$$
\sum_{n=1}^N \frac{n}{3^n} = \frac{\frac{1}{3} \left[1 - (N+1) \left(\frac{1}{3}\right)^N + N \left(\frac{1}{3}\right)^{N+1}\right]}{\frac{4}{9}} = \frac{\frac{1}{3} \left[1 - (N+1) \frac{1}{3^N} + N \frac{1}{3^{N+1}}\right]}{\frac{4}{9}}
$$

Multiply numerator and denominator:

$$
= \frac{\frac{1}{3} \left[1 - (N+1) \frac{1}{3^N} + N \frac{1}{3^{N+1}}\right]}{\frac{4}{9}} = \left(\frac{1}{3} \times \frac{9}{4} \right) \left[1 - (N+1) \frac{1}{3^N} + N \frac{1}{3^{N+1}} \right]
$$

Simplify the coefficient:

$$
\frac{1/3 \times 9}{4} = \frac{3}{4}
$$

Therefore,

$$
\sum_{n=1}^N \frac{n}{3^n} = \frac{3}{4} \left[1 - (N+1) \frac{1}{3^N} + N \frac{1}{3^{N+1}} \right]
$$

Sum 2: $\sum_{n=1}^N \frac{1}{3^n}$

This is a geometric series with ratio $r = \frac{1}{3}$:

$$
\sum_{n=1}^N r^n = r \frac{1 - r^N}{1 - r}
$$

Substituting:

$$
\sum_{n=1}^N \frac{1}{3^n} = \frac{\frac{1}{3} \left(1 - \left(\frac{1}{3}\right)^N \right)}{1 - \frac{1}{3}} = \frac{\frac{1}{3} (1 - 3^{-N})}{\frac{2}{3}} = \frac{\frac{1}{3} (1 - 3^{-N}) \times 3}{2} = \frac{1 - 3^{-N}}{2}
$$

Final Sum Expression

Putting everything together:

$$
S_N = 3 \times \left[\frac{3}{4} \left(1 - (N+1) \frac{1}{3^N} + N \frac{1}{3^{N+1}} \right) \right] - \frac{1 - 3^{-N}}{2}
$$

Simplify the first term:

$$
3 \times \frac{3}{4} = \frac{9}{4}
$$

So,

```
\[
S_N = \frac{9}{4} \left[ 1 - (N+1) \frac{1}{3^N} + N \frac{1}{3^{N+1}}
\right] - \frac{1 - 3^{-N}}{2}
\]

Expanding:

\[
S_N = \frac{9}{4} - \frac{9}{4} (N+1) \frac{1}{3^N} + \frac{9}{4} N
\frac{1}{3^{N+1}} - \frac{1}{2} + \frac{3^{-N}}{2}
\]

Express \( \frac{9}{4} N \frac{1}{3^{N+1}} \) as:

\[
\frac{9 N}{4} \times \frac{1}{3^{N+1}} = \frac{9 N}{4} \times \frac{1}{3^{N}
\times 3} = \frac{9 N}{4} \times \frac{1}{3^{N} \times 3} = \frac{9 N}{4
\times 3} \times \frac{1}{3^{N}} = \frac{3 N}{4} \times \frac{1}{3^{N}}
\]

Now, combine the terms involving \( 1/3^{N} \):

\[
- \frac{9}{4} (N+1) \frac{1}{3^{N}} +
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