The Expressionx^2-9/x2+6x+8 Is Equal To Zero When X = .

The Expression x^2 - 9 / x^2 + 6x + 8 Is Equal To Zero When X = .

Understanding how to solve algebraic expressions set equal to zero is fundamental in algebra. This particular expression, involving quadratic terms both in the numerator and denominator, presents an interesting challenge for students and enthusiasts of mathematics. In this article, we will explore in detail how to analyze, simplify, and solve the equation where the expression equals zero, specifically focusing on finding the values of x that satisfy the condition.

Understanding the Expression

Before diving into solving the equation, it’s essential to understand the structure of the given expression:

$$\frac{x^2 - 9}{x^2 + 6x + 8} = 0$$

This is a rational expression, meaning it is a ratio of two algebraic expressions. The numerator is $x^2 - 9$, which is a difference of squares, and the denominator is $x^2 + 6x + 8$, a quadratic trinomial.

Key Concepts Involved

- Zero of a Rational Expression: The entire expression equals zero when the numerator is zero and the denominator is not zero.

- Domain Restrictions: The values of x that make the denominator zero are not in the domain, as division by zero is undefined.
- Factoring Quadratics: Both numerator and denominator can often be factored to find zeros more easily.

Step-by-Step Solution Process

To find the values of x for which the expression equals zero, follow these systematic steps:

1. Set the Numerator Equal to Zero

Since the fraction is zero when the numerator is zero (and the denominator is not zero), start by solving:

$$x^2 - 9 = 0$$

This simplifies to:

$$x^2 = 9$$

Taking square roots:

$$x = \pm 3$$

Note: These are potential solutions, but we must verify that they do not make the denominator zero.

2. Factor the Denominator and Check for Restrictions

Factor the denominator to identify values of x that are excluded:

$$ x^2 + 6x + 8 $$

Factor as:

$$ (x + 2)(x + 4) $$

The denominator is zero when:

$$ x + 2 = 0 \Rightarrow x = -2 $$

$$ x + 4 = 0 \Rightarrow x = -4 $$

These are the excluded values; the expression is undefined at these points.

3. Verify the Solutions Against Restrictions

Now, check if $x = 3$ or $x = -3$ corresponds to any restrictions:

- $x = 3$: Does not make the denominator zero.
- $x = -3$: Does not make the denominator zero.

Therefore, both solutions are valid.

Final Answer

The expression equals zero when:

$$ x = 3 \quad \text{or} \quad x = -3 $$

with the understanding that $x \neq -2$ and $x \neq -4$, as these values are undefined in the original expression.

Additional Considerations

Why Are Some Solutions Invalid?

Any solution that makes the denominator zero is invalid because division by zero is undefined. In this case, the critical points are:

- $x = -2$
- $x = -4$

Since our solutions $x = 3$ and $x = -3$ do not coincide with these, they are valid solutions.

Graphical Interpretation

Plotting the rational function can provide visual insight:

- The zeros of the numerator ($x = \pm 3$) correspond to x-intercepts.

- The points where the denominator is zero ($x = -2, -4$) are vertical asymptotes.
- The function approaches infinity or negative infinity near these asymptotes.

Practice Problems

To reinforce understanding, consider working through these related problems:

1. Solve $\frac{x^2 - 16}{x^2 - 4} = 0$
2. Find the values of x for which $\frac{2x^2 + 3}{x + 1} = 0$
3. Determine the solutions to $\frac{x^2 + 5x + 6}{x^2 - 9} = 0$

Summary

In conclusion, solving the rational equation $\frac{x^2 - 9}{x^2 + 6x + 8} = 0$ involves:

- Factoring numerator and denominator.
- Finding where the numerator equals zero.
- Ensuring these solutions do not make the denominator zero.
- Recognizing domain restrictions.

The solutions are $x = 3$ and $x = -3$, with the valid domain restrictions being $x \neq -2$ and $x \neq -4$.

Final Thoughts

Mastering the process of solving rational equations enhances problem-solving skills and deepens understanding of algebraic concepts. Always remember to check for restrictions and verify solutions to avoid extraneous roots. With practice, tackling similar problems becomes more intuitive, paving the way for success in advanced mathematics topics such as calculus and algebraic functions.

Keywords: algebra, rational expressions, quadratic equations, solving equations, domain restrictions, factoring quadratics, zero of a rational function

Frequently Asked Questions

How do I solve the equation $(x^2 - 9) / (x^2 + 6x + 8) = 0$ for x?

Since a fraction equals zero when its numerator is zero (and denominator is not zero), set $x^2 - 9 = 0$, which gives $x = \pm 3$. Ensure the denominator $x^2 + 6x + 8 \neq 0$ at these points.

What are the restrictions on x when solving $(x^2 - 9) / (x^2 + 6x + 8) = 0$?

The denominator cannot be zero. Factor $x^2 + 6x + 8$ as $(x + 2)(x + 4)$. So, $x \neq -2$ and $x \neq -4$.

What are the solutions to the equation $(x^2 - 9) / (x^2 + 6x + 8) = 0$?

The solutions are $x = 3$ and $x = -3$, provided these do not make the denominator zero. Since the denominator is zero at $x = -2$ and -4, these are excluded. Therefore, solutions are $x = 3$ and $x = -3$.

Can the equation $(x^2 - 9) / (x^2 + 6x + 8) = 0$ be true if the denominator is zero?

No, the fraction is undefined when the denominator is zero. So, solutions must be checked to ensure they do not make the denominator zero.

How do I verify if $x = 3$ satisfies the original equation?

Plug $x = 3$ into the numerator and denominator: numerator = $3^2 - 9 = 0$, denominator = $(3 + 2)(3 + 4) = 57 = 35$ □ 0. Since numerator is zero and denominator is not zero, $x = 3$ is a valid solution.

Is $x = -3$ a valid solution for the equation?

Yes. Substitute $x = -3$: numerator = $(-3)^2 - 9 = 9 - 9 = 0$; denominator = $(-3 + 2)(-3 + 4) = (-1)(1) = -1$ □ 0. Therefore, $x = -3$ is a valid solution.

Additional Resources

Understanding the Expression $x^2 - 9 / x^2 + 6x + 8 = 0$ When x = ...

When encountering an algebraic expression like $x^2 - 9 / x^2 + 6x + 8 = 0$, it’s essential to understand how to analyze and solve it effectively. This expression involves a rational function, where a quadratic numerator is divided by a quadratic denominator, set equal to zero. Determining the values of x that satisfy this equation requires a careful step-by-step approach, including simplifying the expression, identifying restrictions, and solving the resulting equations. In this guide, we will dissect the expression, explore methods to find solutions, and provide a comprehensive understanding of what it means for x to satisfy the equation.

Breaking Down the Expression

The Original Expression

The given expression is:

$(x^2 - 9) / (x^2 + 6x + 8) = 0$

This is a rational equation. To understand it, we should analyze both the numerator and the denominator, as well as the overall structure.

Recognizing Patterns and Factoring

1. Numerator: $x^2 - 9$

- Recognize as a difference of squares:
- $x^2 - 9 = (x - 3)(x + 3)$

2. Denominator: $x^2 + 6x + 8$

- Factor as a quadratic:
- Find two numbers that multiply to 8 and add up to 6: 2 and 4
- So, $x^2 + 6x + 8 = (x + 2)(x + 4)$

Simplified Form

The expression can be rewritten as:

$[(x - 3)(x + 3)] / [(x + 2)(x + 4)] = 0$

Understanding When the Expression Equals Zero

The Zero of a Rational Expression

A rational expression equals zero when the numerator equals zero and the denominator is not zero at those points. Therefore, to find the solutions:

1. Set the numerator equal to zero:

(x - 3)(x + 3) = 0

2. Solve for x:

- x - 3 = 0 □ x = 3
- x + 3 = 0 □ x = -3

3. Check that these values do not make the denominator zero:

- For x = 3: Denominator is (3 + 2)(3 + 4) = 5 7 □ 0 □
- For x = -3: Denominator is (-3 + 2)(-3 + 4) = (-1) 1 □ 0 □

Therefore, x = 3 and x = -3 are solutions to the original equation.

Addressing Domain Restrictions

When is the Denominator Zero?

Since division by zero is undefined, any x that makes the denominator zero must be excluded from solutions.

- Solve for when the denominator equals zero:

(x + 2)(x + 4) = 0

- x + 2 = 0 □ x = -2
- x + 4 = 0 □ x = -4

Thus, $x \neq -2$ and $x \neq -4$.

Summary of Solutions

- Solutions to the equation: $x = 3$ and $x = -3$
- Excluded values: $x \neq -2, -4$

Since neither solution makes the denominator zero, both are valid solutions.

Additional Insights: What if the Expression Were Set Equal to Other Values?

While our focus was on $x^2 - 9 / x^2 + 6x + 8 = 0$, similar steps can be used for equations where the rational expression equals a different constant, say k:

$(x^2 - 9) / (x^2 + 6x + 8) = k$

In such cases:

1. Multiply both sides by the denominator to clear fractions:

$x^2 - 9 = k(x^2 + 6x + 8)$

2. Expand and rearrange:

$x^2 - 9 = kx^2 + 6kx + 8k$

3. Collect like terms:

$x^2 - kx^2 + (-9 - 8k) = 6kx$

4. Rewrite as a quadratic in x and solve accordingly.

Practical Tips for Solving Rational Equations

- Always factor the numerator and denominator if possible, to identify zeros and restrictions easily.
- Set the numerator equal to zero to find potential solutions when the rational expression equals zero.
- Exclude any x-values that make the denominator zero, as these are not in the domain.
- Check solutions by plugging them back into the original expression to verify they satisfy the equation and do not violate domain restrictions.

Real-World Applications and Contexts

Understanding how to analyze equations like $x^2 - 9 / x^2 + 6x + 8 = 0$ is fundamental in many fields, including:

- Engineering: When solving for stability conditions where the system's behavior is described by rational functions.
- Economics: In models where ratios of quadratic functions represent cost or revenue functions.
- Physics: When dealing with relationships involving quadratic ratios, such as in motion or energy equations.

Final Thoughts

In conclusion, solving $x^2 - 9 / x^2 + 6x + 8 = 0$ involves factoring, understanding the domain restrictions, and solving the resulting simple equations. Recognizing patterns such as difference of squares and quadratic factorization streamlines the process. Always remember to verify that solutions do not violate the restrictions posed by zero denominators. Mastery of these techniques empowers you to analyze and interpret a wide range of rational equations encountered across various disciplines.

Key Takeaways:

- Rational equations are solved by focusing on the numerator and denominator separately.
- Set the numerator equal to zero to find potential solutions when the expression equals zero.
- Always check the domain restrictions imposed by the denominator.
- Factor fully to simplify and identify solutions efficiently.

By following these structured steps, you can confidently analyze and solve complex rational expressions like the one discussed here.

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