What Equations Are Equivalent To 3(5x+4)=12x+18

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Understanding equivalent equations is fundamental in algebra, as they allow us to manipulate and solve equations efficiently while maintaining the same solution set. When presented with an equation like 3(5x+4)=12x+18, the question often arises: What other equations are equivalent to it? Equivalence in algebra means that different equations, possibly expressed in different forms, share exactly the same solution(s). This article explores how to identify, create, and verify equations equivalent to 3(5x+4)=12x+18, along with step-by-step methods, common practices, and tips for solving such equations effectively.

Understanding the Given Equation

Before exploring equivalent equations, it’s essential to understand the structure and solution of the original equation:

Original Equation:

$3(5x + 4) = 12x + 18$

Step 1: Expand the left side

Using distributive property:

$3\ 5x + 3\ 4 = 15x + 12$

So, the equation becomes:

$15x + 12 = 12x + 18$

Step 2: Simplify and solve

Subtract 12x from both sides:

$15x - 12x + 12 = 18$

which simplifies to:

$3x + 12 = 18$

Subtract 12 from both sides:

$3x = 6$

Divide both sides by 3:

$x = 2$

Solution: $x = 2$

This solution will be the key to verifying whether other equations are equivalent.

Defining Equivalent Equations

Equivalent equations are different algebraic expressions that share the same solution(s). If two equations are equivalent, any solution that satisfies one will satisfy the other.

Key points about equivalent equations:

- They can be derived from each other through valid algebraic operations such as addition, subtraction, multiplication, or division by non-zero constants.
- They may look different but have the same solution set.
- They maintain the same solutions even after transformations.

Example:

- Original: $3(5x + 4) = 12x + 18$
- Equivalent: $15x + 12 = 12x + 18$ (expanded form)
- Also equivalent: $3(5x + 4) - (12x + 18) = 0$ (set equal to zero)

How to Find Equations Equivalent to 3(5x+4)=12x+18

Creating equivalent equations involves applying algebraic operations that do not change the solution set. These include:

1. Addition or Subtraction of the Same Expression

Adding or subtracting the same expression from both sides maintains equivalence.

- Example:

\[
3(5x+4) = 12x + 18

\]

Add 5 to both sides:

$$
3(5x+4) + 5 = 12x + 18 + 5
$$

Simplifies to:

$$
15x + 12 + 5 = 12x + 23
$$

Resulting Equation:

$$
15x + 17 = 12x + 23
$$

Since adding the same number to both sides preserves the solution set, this new equation is equivalent.

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2. Multiplication or Division by Non-zero Constants

Multiplying or dividing both sides of the equation by a non-zero number also preserves equivalence.

- Example:

Starting with:

$$
15x + 12 = 12x + 18
$$

Divide both sides by 3:

$$
\frac{15x}{3} + \frac{12}{3} = \frac{12x}{3} + \frac{18}{3}
$$

Simplifies to:

$$
5x + 4 = 4x + 6
$$

This new equation is equivalent to the original because we divided both sides by 3, a non-zero constant.

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3. Re-arranging Terms (Adding or Subtracting Terms)

You can also rearrange the equation to different forms by moving terms around, as long as you perform the same operation on both sides.

- Example:

From:

$$
15x + 12 = 12x + 18
$$

Subtract 12x from both sides:

$$
3x + 12 = 18
$$

Subtract 12 from both sides:

$$
3x = 6
$$

This is an equivalent simplified form.

Step-by-Step Approach to Generate Equivalent Equations

To systematically find equations equivalent to the original, follow these steps:

1. **Start from the original equation:** 3(5x + 4) = 12x + 18
2. **Expand or simplify as needed:** 15x + 12 = 12x + 18
3. **Apply algebraic operations:**

- Add or subtract constants to both sides
- Multiply or divide both sides by a non-zero constant
- Rearrange terms to isolate variables

4. **Verify the solution set:** Solve the transformed equation to confirm $x=2$ is still valid
5. **Repeat with different operations:** Generate multiple equivalent forms for practice and understanding

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Examples of Equations Equivalent To 3(5x+4)=12x+18

Here are several equations that are equivalent to the original:

Example 1: Simplified form

$$
3x = 6
$$

Derived by subtracting 12 from both sides and dividing by 3.

Example 2: Expanded form

$$
15x + 12 = 12x + 18
$$

Starting from the original and expanding.

Example 3: Re-arranged form

$$
15x + 12 - 12x - 18 = 0
$$

which simplifies to:

$$
3x - 6 = 0
$$

Adding all terms to one side.

Example 4: Equation in terms of zero

$$
3(5x + 4) - (12x + 18) = 0
$$

Subtracting the right side from the left.

Verifying Equivalence of Equations

It is crucial to verify that the equations are truly equivalent, particularly after transformations.

Methods for verification include:

- Solving both equations: If both yield the same solution(s), they are equivalent.
- Substituting the solution(s): Plug solutions into each equation to check if they satisfy both.
- Using algebraic equivalence rules: Confirm operations performed are valid and preserve solutions.

Example:

- For the equations:

$$
3(5x + 4) = 12x + 18
$$

and

$$
3x = 6
$$

test x=2:

$$
3(52 + 4) = 122 + 18 \rightarrow 3(10 + 4) = 24 + 18 \rightarrow 314=42 \rightarrow 42=42
$$

and

$$
32=6 \rightarrow 6=6
$$

Both true, confirming equivalence.

Common Mistakes To Avoid When Working With Equivalent Equations

While manipulating equations to find equivalents, avoid these frequent errors:

- Dividing by zero: Never divide both sides by a variable expression that could be zero; always check for extraneous solutions.
- Changing the solution set unintentionally: Operations like multiplying both sides by a negative number require attention to inequality signs (not relevant here but important in inequalities).
- Incorrectly adding or subtracting terms: Make sure to perform the same operation on both sides.
- Misapplying the distributive property: Always expand expressions properly before transforming.

Practical Tips and Best Practices

- Always verify the solutions after transforming equations.
- Keep track of operations performed to ensure they are valid and reversible.
- Practice transforming equations in multiple ways to build flexibility.
- Use substitution to verify that different forms are equivalent.
- Understand the properties of equality: Additive, multiplicative, and distributive properties are key to safe transformations.

Conclusion

Equations equivalent to $3(5x+4)=12x+18$ can be generated through various algebraic manipulations such as expansion, factoring, adding or subtracting constants, and dividing or multiplying by non-zero constants.

Frequently Asked Questions

What is the simplified form of the equation 3(5x + 4) = 12x + 18?

The simplified form is 15x + 12 = 12x + 18.

Are the equations 3(5x + 4) = 12x + 18 and 15x + 12 = 12x + 18 equivalent?

Yes, they are equivalent because the first equation simplifies to the second.

Which equations are equivalent to 3(5x + 4) = 12x + 18?

Any equation that can be derived by simplifying or manipulating 3(5x + 4) = 12x + 18, such as 15x + 12 = 12x + 18 or 3(5x + 4) - 12x - 18 = 0.

How can I verify if two equations are equivalent?

You can verify by simplifying both equations to their simplest form and checking if they match or by solving for x in both equations to see if they produce the same solution set.

What is the solution to the equation 3(5x + 4) = 12x + 18?

Solving gives 3(5x + 4) = 12x + 18 → 15x + 12 = 12x + 18 → 15x - 12x = 18 - 12 → 3x = 6 → x = 2.

Can different forms of an equation represent the same solution set?

Yes, different algebraic forms of an equation can be equivalent and have the same solution set, even if they look different.

Additional Resources

Understanding Equivalence of Equations: What Equations Are Equivalent To 3(5x + 4) = 12x + 18

When exploring algebra, one of the foundational concepts is the notion of equivalent equations. These are equations that, although they may look different, have the same solution set. Understanding which equations are equivalent to a given equation like 3(5x + 4) = 12x + 18 is crucial for mastering algebraic manipulations and problem-solving techniques. In this comprehensive review, we will delve into the concept of equivalent equations, analyze the given equation thoroughly, and explore various ways to determine and generate equations equivalent to it.

What Are Equivalent Equations? An In-Depth Explanation

Before diving into the specific equation 3(5x + 4) = 12x + 18, it’s essential to establish a clear understanding of what makes two equations equivalent.

Definition of Equivalent Equations

Two equations are considered equivalent if they have the same solution set. That is, any value of the variable that satisfies one equation also satisfies the other, and vice versa.

For example:
- \(2x + 4 = 10 \) and \(x = 3 \) are equivalent because both have the solution \(x=3 \).
- However, \(2x + 4 = 10 \) and \(2x = 10 \) are not equivalent because the second equation's solution is \(x=5 \), which does not satisfy the first.

How to Determine if Two Equations Are Equivalent

To check if two equations are equivalent:
1. Solve both equations explicitly for the variable.
2. Compare their solution sets.
3. If the solution sets are identical, the equations are equivalent.

Alternatively, algebraic transformations can help verify equivalence without solving each time, provided the transformations are valid and reversible.

Transformations That Preserve Equivalence

Some common operations that preserve the solution set include:
- Adding or subtracting the same expression from both sides.
- Multiplying or dividing both sides by a non-zero constant.
- Simplifying expressions, such as expanding or combining like terms.

These operations help in converting equations into different forms while maintaining their solution sets.

Analyzing the Given Equation: 3(5x + 4) = 12x + 18

Let’s analyze and simplify the given equation step-by-step:

Step 1: Expand the Left Side

$$
3(5x + 4) = 15x + 12
$$

This is achieved by distributing the 3 across the sum inside the parentheses.

Step 2: Rewrite the Equation

$$
15x + 12 = 12x + 18
$$

Now, the original equation is simplified to a linear form that is easier to analyze.

Step 3: Isolate the Variable to Find the Solution

Subtract $12x$ from both sides:

$$
15x - 12x + 12 = 18
$$

$$
3x + 12 = 18
$$

Subtract 12 from both sides:

$$
3x = 6
$$

Divide both sides by 3:

$$
x = 2
$$

This solution indicates that the original equation has a unique solution, $x=2$.

Implication for Equivalence

Any equation that simplifies to $x=2$ or has the same solution set is equivalent to the original. For instance, the following are all equivalent:
- $3(5x + 4) = 12x + 18$
- $15x + 12 = 12x + 18$
- $3x = 6$
- $x = 2$

Generating Equations Equivalent To the Original

Knowing that the solution set is $x=2$, we can construct multiple equations that are equivalent to the original by applying valid algebraic operations.

1. Equations Derived by Valid Transformations

- Multiply both sides of the original equation by 1 (trivially keeps the equation unchanged).
- Add or subtract identical expressions to both sides:
- For example, add 0: $3(5x + 4) = 12x + 18 + 0$
- Subtract or add a multiple of the solution $x=2$ (though care must be taken not to change the solution set).
- Multiply both sides by a non-zero constant, provided the solution remains consistent:
- For instance, multiply both sides by 2:

$$
2 \times 3(5x + 4) = 2 \times (12x + 18)
$$

$$
6(5x + 4) = 24x + 36
$$

- Simplify:

$$
30x + 24 = 24x + 36
$$

- Solve:

$$
30x - 24x = 36 - 24
$$

$$
6x = 12
$$

$$
x=2
$$

- Since the solution remains $x=2$, this new equation is equivalent.

2. Equations with Same Solution Set in Different Forms

- Equations obtained by rearranging or factoring:
- $5x + 4 = \frac{12x + 18}{3}$
- $5x + 4 = 4x + 6$
- $5x - 4x = 6 - 4$
- $x = 2$

All these equations are equivalent because they lead to the same solution, $x=2$.

3. Constructing Equations with Known Solution

To generate equations equivalent to the original, you can start from the solution and work backwards:
- Since $x=2$, any linear equation that simplifies to $x=2$ is equivalent.
- For example:
- $2x - 4 = 0$ (since substituting $x=2$ yields 0)
- $3x - 6 = 0$
- $10x - 20 = 0$

All of these equations share the same solution set $x=2$, making them equivalent.

Methods to Verify Equation Equivalence

When working with numerous equations, verifying their equivalence efficiently is essential. Here are some methods:

Method 1: Solve Both Equations

- Solve each equation separately for x.
- If the solutions match, the equations are equivalent.

Method 2: Algebraic Transformation

- Convert both equations into simplified canonical forms.
- For linear equations, this often means solving for x.
- If the simplified forms are identical, the equations are equivalent.

Method 3: Substitution Test

- Choose a value of x (preferably the solution $x=2$), and verify whether both equations are satisfied.
- If both are true for the same x, they are likely equivalent, especially for linear equations.

Method 4: Graphical Analysis

- Graph both equations.
- If the graphs coincide (i.e., are the same line), the equations are equivalent.
- For linear equations, this is a straightforward visual check.

Understanding the Solution Set and Its Significance

Since the original equation simplifies to $x=2$, it has a single unique solution. This characteristic influences the nature of equivalent equations:

- Equations with the same single solution are equivalent.
- Equations with different solutions are not equivalent.

This highlights a critical point: algebraic manipulations that preserve the solution set are key to establishing equivalence.

Common Mistakes and Pitfalls in Determining Equivalence

When working through the problem of equivalence, watch out for:

- Dividing both sides by an expression that could be zero: For example, dividing by $x-2$ would eliminate the solution $x=2$ if not handled carefully.
- Altering the solution set unintentionally: For example, adding or subtracting expressions that depend on x without solving or verifying the effect.
- Misapplication of transformations: Such as multiplying both sides by zero or non-zero constants without considering their impact.

Understanding these pitfalls helps in correctly identifying equations that are truly equivalent.

Summary and Key Takeaways

- Equivalent equations share the same solution set; for linear equations, this typically means they simplify to the same solution.
- The given equation $3(5x + 4) = 12x + 18$ simplifies to $x=2$, indicating all equivalent equations must also yield $x=2$.
- To generate equivalent equations, apply valid algebraic transformations such as multiplying both sides by a non-zero constant, adding or subtracting the same expression from both sides, or rearranging terms.
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