What Is The Binomial Expansion Of (x +2y)7

What Is The Binomial Expansion Of (x + 2y)^7

Understanding the binomial expansion of algebraic expressions is a fundamental concept in mathematics, especially in algebra and combinatorics. Specifically, the binomial expansion of $(x + 2y)^7$ involves expanding the expression into a sum of terms that are products of powers of x and $2y$, multiplied by binomial coefficients. This process allows us to express the power of a binomial as a sum, which simplifies calculations and provides insight into the structure of algebraic expressions.

In this article, we will explore the binomial expansion of $(x + 2y)^7$ in detail. We will discuss the binomial theorem, how to apply it to this specific expression, and the step-by-step process of deriving the expanded form. Additionally, we will examine the significance of binomial coefficients, how to compute them, and practical applications of binomial expansion in various fields such as mathematics, physics, and engineering.

Understanding the Binomial Theorem

What Is the Binomial Theorem?

The binomial theorem provides a formula for expanding the powers of a binomial, which is an algebraic expression with two terms. The general form of the binomial theorem is:

$$
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k
$$

where:
- a and b are any algebraic expressions,
- n is a non-negative integer,
- $\binom{n}{k}$ is a binomial coefficient, read as "n choose k," representing the number of ways to choose k elements from a set of n.

This theorem is crucial because it simplifies the process of expanding large powers without manually multiplying the binomial multiple times.

Components of the Binomial Expansion

The binomial expansion involves three key components:
1. Binomial Coefficients ($\binom{n}{k}$): These are the combinatorial coefficients that

determine the number of ways to select terms.
2. Powers of a and b: Each term in the expansion involves decreasing powers of a and increasing powers of b.
3. Summation: The expansion is a sum of all these terms from $k=0$ to $k=n$.

Applying the Binomial Theorem to $(x + 2y)^7$

Step 1: Recognize the Components

In the expression $(x + 2y)^7$, the binomial components are:
- $a = x$
- $b = 2y$
- $n = 7$

Using the binomial theorem, the expansion becomes:

$$
(x + 2y)^7 = \sum_{k=0}^{7} \binom{7}{k} x^{7-k} (2y)^k
$$

Step 2: Write the General Term

The general term T_{k+1} of the expansion is:

$$
T_{k+1} = \binom{7}{k} x^{7-k} (2y)^k
$$

for each k from 0 to 7.

Step 3: Simplify Each Term

Each term involves computing the binomial coefficient and raising $2y$ to the power of k:

$$
T_{k+1} = \binom{7}{k} x^{7-k} 2^k y^k
$$

Thus, the full expansion is:

$$
(x + 2y)^7 = \sum_{k=0}^{7} \binom{7}{k} x^{7-k} 2^k y^k
$$

Calculating Binomial Coefficients $\binom{7}{k}$

Understanding Binomial Coefficients

Binomial coefficients are calculated using the formula:

$$
\binom{n}{k} = \frac{n!}{k!(n-k)!}
$$

where:
- $n!$ denotes factorial of n,
- $k!$ is factorial of k.

For $n=7$, the binomial coefficients are:

k	$\binom{7}{k}$	Calculation
0	1	$\frac{7!}{0!7!} = 1$
1	7	$\frac{7!}{1!6!} = 7$
2	21	$\frac{7!}{2!5!} = 21$
3	35	$\frac{7!}{3!4!} = 35$
4	35	$\frac{7!}{4!3!} = 35$
5	21	$\frac{7!}{5!2!} = 21$
6	7	$\frac{7!}{6!1!} = 7$
7	1	$\frac{7!}{7!0!} = 1$

This symmetric pattern reflects the properties of Pascal's triangle.

Explicit Binomial Coefficients for $(x + 2y)^7$

Using the above, the coefficients are:

- $\binom{7}{0} = 1$
- $\binom{7}{1} = 7$
- $\binom{7}{2} = 21$
- $\binom{7}{3} = 35$
- $\binom{7}{4} = 35$
- $\binom{7}{5} = 21$
- $\binom{7}{6} = 7$
- $\binom{7}{7} = 1$

Constructing the Full Expansion

Step 1: Write Each Term

Using the general term:

$$
T_{k+1} = \binom{7}{k} x^{7-k} 2^k y^k
$$

we expand each term:

- For $k=0$:
$$
\binom{7}{0} x^{7} (2)^0 y^0 = 1 \cdot x^7 \cdot 1 \cdot 1 = x^7
$$

- For $k=1$:
$$
7 \cdot x^{6} \cdot 2^1 \cdot y = 7 \cdot x^6 \cdot 2 \cdot y = 14 x^6 y
$$

- For $k=2$:
$$
21 \cdot x^{5} \cdot 2^2 \cdot y^2 = 21 \cdot x^5 \cdot 4 \cdot y^2 = 84 x^5 y^2
$$

- For $k=3$:
$$
35 \cdot x^{4} \cdot 2^3 \cdot y^3 = 35 \cdot x^4 \cdot 8 \cdot y^3 = 280 x^4 y^3
$$

- For $k=4$:
$$
35 \cdot x^{3} \cdot 2^4 \cdot y^4 = 35 \cdot x^3 \cdot 16 \cdot y^4 = 560 x^3 y^4
$$

- For $k=5$:
$$
21 \cdot x^2 \cdot 2^5 \cdot y^5 = 21 \cdot x^2 \cdot 32 \cdot y^5 = 672 x^2 y^5
$$

- For $k=6$:
$$
7 \cdot x \cdot 2^6 \cdot y^6 = 7 \cdot x \cdot 64 \cdot y^6 = 448 x y^6
$$

- For $k=7$:
$$
1 \cdot x^{0} \cdot 2^7 \cdot y^7 = 1 \cdot 1 \cdot 128 \cdot y^7 = 128 y^7
$$

Step 2: Write the Full Expanded Expression

Combining all terms, the binomial expansion of \($(x + 2y)^7$\) is

Frequently Asked Questions

What is the binomial expansion of $(x + 2y)^7$?

$(x + 2y)^7 = \sum_{k=0}^7 C(7, k)\, x^{7-k} (2y)^k$, which expands to:
$x^7 + 72 y x^6 + 214 y^2 x^5 + 358 y^3 x^4 + 3516 y^4 x^3 + 2132 y^5 x^2 + 764 y^6 x + 128 y^7$.

How do you use the binomial theorem to expand $(x + 2y)^7$?

Apply the binomial theorem formula: $(a + b)^n = \sum_{k=0}^n C(n, k)\, a^{n-k} b^k$. Here, a = x and b = 2y, so the expansion sums terms of the form $C(7, k)\, x^{7-k} (2y)^k$.

What are the binomial coefficients for the expansion of $(x + 2y)^7$?

The binomial coefficients are C(7, 0)=1, C(7, 1)=7, C(7, 2)=21, C(7, 3)=35, C(7, 4)=35, C(7, 5)=21, C(7, 6)=7, C(7, 7)=1.

Can you provide the full expanded form of $(x + 2y)^7$?

Yes. The expansion is:
$x^7 + 14 x^6 y + 168 x^5 y^2 + 1120 x^4 y^3 + 4480 x^3 y^4 + 10752 x^2 y^5 + 15120 x y^6 + 12800 y^7$.

What is the coefficient of $x^4 y^3$ in the expansion of $(x + 2y)^7$?

The coefficient is $C(7, 3)\, (2)^3 = 35\ 8 = 280$.

How does the binomial expansion relate to Pascal's triangle in this problem?

The binomial coefficients for $(x + 2y)^7$ are taken from the 8th row (index 7) of Pascal's triangle: 1, 7, 21, 35, 35, 21, 7, 1.

What is the significance of the term $(2y)^k$ in the binomial expansion?

The term $(2y)^k$ accounts for the coefficient 2 raised to the power k multiplied by y^k, scaling each term appropriately in the expansion.

How can the binomial expansion be used in algebraic problem solving?

It helps expand expressions raised to powers, simplifies polynomial multiplication, and facilitates calculations involving binomials in algebra, calculus, and combinatorics.

Additional Resources

Binomial Expansion of (x + 2y)^7 is a fundamental concept in algebra that exemplifies the power and elegance of mathematical binomial theorems. It is a classic example often used to illustrate how algebraic expressions can be expanded systematically into a sum of terms, each involving coefficients and powers of the variables. This article provides a comprehensive exploration of the binomial expansion of (x + 2y)^7, delving into the theoretical foundation, the step-by-step expansion process, applications, and analytical insights that illuminate its significance in mathematics.

Understanding the Binomial Theorem

What Is the Binomial Theorem?

The binomial theorem is a fundamental principle in algebra that provides a formula for expanding expressions raised to a power, specifically binomials—algebraic expressions with two terms. The theorem states that for any positive integer n:

$$ (a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^{k} $$

where:
- $\binom{n}{k}$ is the binomial coefficient, read as "n choose k."
- a and b are any algebraic expressions or constants.

This formula effectively breaks down the power of a binomial into a sum of terms, each involving a combination of powers of a and b.

Binomial Coefficients and Pascal's Triangle

The binomial coefficients $\binom{n}{k}$ are central to the expansion. They are calculated as:

$$ \binom{n}{k} = \frac{n!}{k! (n - k)!} $$

where $n!$ denotes the factorial of n. These coefficients can be visualized using Pascal's Triangle, a powerful tool that displays binomial coefficients in a triangular format. For $n=7$, the row of Pascal's Triangle is:

$$1, 7, 21, 35, 35, 21, 7, 1$$

which directly corresponds to the coefficients in the expansion of $(a + b)^7$.

Why Is Binomial Expansion Important?

The binomial expansion is not just a theoretical construct; it has numerous applications across different fields:
- Simplifying algebraic expressions.
- Calculating probabilities in combinatorics and statistics.
- Deriving polynomial identities.
- Solving problems in calculus involving series expansions.
- Modeling real-world phenomena in physics, economics, and engineering.

Understanding how to systematically expand binomials like $(x + 2y)^7$ enhances one's problem-solving toolkit, enabling mathematicians and scientists to analyze complex expressions efficiently.

Step-by-Step Expansion of (x + 2y)^7

Applying the Binomial Theorem

To expand $(x + 2y)^7$, we directly apply the binomial theorem formula:

$$(x + 2y)^7 = \sum_{k=0}^{7} \binom{7}{k} x^{7 - k} (2y)^k$$

Each term in the sum corresponds to a specific value of k, from 0 to 7.

Calculating Binomial Coefficients for n=7

Using Pascal's Triangle or the factorial formula:

k	$\binom{7}{k}$	Calculation
0	1	$\frac{7!}{0!7!} = 1$
1	7	$\frac{7!}{1!6!} = 7$
2	21	$\frac{7!}{2!5!} = 21$

3	35	$\frac{7!}{3!4!} = 35$
4	35	$\frac{7!}{4!3!} = 35$
5	21	$\frac{7!}{5!2!} = 21$
6	7	$\frac{7!}{6!1!} = 7$
7	1	$\frac{7!}{7!0!} = 1$

Expanding Each Term

Now, for each k, compute:

$$\binom{7}{k} x^{7 - k} (2y)^k$$

Note that $(2y)^k = 2^k y^k$. Accordingly, the terms are:

1. k=0:
$$\binom{7}{0} x^7 (2y)^0 = 1 \times x^7 \times 1 = x^7$$

2. k=1:
$$7 \times x^6 \times 2 y = 7 \times 2 \times x^6 y = 14 x^6 y$$

3. k=2:
$$21 \times x^5 \times 2^2 y^2 = 21 \times 4 \times x^5 y^2 = 84 x^5 y^2$$

4. k=3:
$$35 \times x^4 \times 2^3 y^3 = 35 \times 8 \times x^4 y^3 = 280 x^4 y^3$$

5. k=4:
$$35 \times x^3 \times 2^4 y^4 = 35 \times 16 \times x^3 y^4 = 560 x^3 y^4$$

6. k=5:
$$21 \times x^2 \times 2^5 y^5 = 21 \times 32 \times x^2 y^5 = 672 x^2 y^5$$

7. k=6:
$$7 \times x \times 2^6 y^6 = 7 \times 64 \times x y^6 = 448 x y^6$$

8. k=7:
$$1 \times 1 \times 2^7 y^7 = 1 \times 128 \times y^7 = 128 y^7$$

Complete Expanded Form

Combining all terms, the binomial expansion of $(x + 2y)^7$ is:

$$
\boxed{
x^7 + 14 x^6 y + 84 x^5 y^2 + 280 x^4 y^3 + 560 x^3 y^4 + 672 x^2 y^5 + 448 x y^6 + 128 y^7
}
$$

\]

This explicit form reveals the distribution of powers and coefficients, illustrating how the binomial theorem efficiently decomposes a high-power expression into manageable terms.

Analytical Insights and Observations

Pattern Recognition and Coefficient Growth

The coefficients in the expansion increase rapidly, peaking at the middle terms. This pattern stems from the binomial coefficients' symmetry and combinatorial properties:

- The coefficients are symmetric: $\binom{7}{k} = \binom{7}{7 - k}$.
- The largest coefficients are at $k=3$ and $k=4$, aligning with the middle of Pascal's Triangle.

This symmetry reflects the combinatorial nature of choosing k elements from 7, which directly influences the expansion's structure.

Impact of Variable Scaling (2y)

The presence of $2y$ instead of just y significantly affects the expansion:

- Each term involving y^k is scaled by 2^k, amplifying the contribution of higher powers of y.
- The coefficients are multiplied accordingly, leading to larger numerical values compared to a standard expansion like $(x + y)^7$.

This underscores the importance of carefully handling coefficients and powers when dealing with scaled variables in binomial expansions.

Applications in Probability and Statistics

The binomial expansion's structure provides foundational skills for understanding binomial probability distributions. For example, the probability of obtaining exactly k successes in n independent Bernoulli trials with success probability p is:

$$P(X=k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

The coefficients $\binom{n}{k}$ and the powers of p and $(1-p)$ mirror the terms in the binomial expansion, establishing a direct link between algebraic expansion and probabilistic modeling.

Advanced Topics and Extensions

Multinomial Theorem

While the binomial theorem deals with two-term expressions, the multinomial theorem generalizes this

What Is The Binomial Expansion Of X 2y 7

What Is The Binomial Expansion Of X 2y 7

Related Articles

- Draw The Structure Of CH3CH2CH2CH2CH2CHCH2
- What Are The Two Types Of Microscopes?+
- How Does Climate Change Make Weather More Extreme?

Back to Home