What Is The Product Of (4 + 3 I) And (12 2 I)

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When dealing with complex numbers, one of the fundamental operations is multiplication. The product of two complex numbers involves combining their real and imaginary parts following specific algebraic rules. Understanding how to multiply complex numbers like (4 + 3i) and (12 + 2i) is essential not only in pure mathematics but also in fields like engineering, physics, and computer science, where complex numbers are used to describe oscillations, waveforms, and signal processing.

In this article, we will explore how to find the product of the complex numbers (4 + 3i) and (12 + 2i), breaking down each step in detail. We will discuss the underlying algebraic principles, demonstrate the calculation process, and explain the significance of the result in various contexts. This comprehensive guide aims to provide clarity for students, educators, and professionals working with complex numbers.

Understanding Complex Numbers

What Are Complex Numbers?

Complex numbers extend the real number system to include imaginary numbers. A complex number has two parts:

- Real part: A real number representing the "horizontal" component.
- Imaginary part: A real number multiplied by the imaginary unit, denoted as i.

A complex number is generally written as:

$$ z = a + bi $$

where:

- a is the real part.
- b is the imaginary coefficient.
- i is the imaginary unit, satisfying $i^2 = -1$.

For example, in $4 + 3i$:

- Real part: 4
- Imaginary part: 3

Similarly, in $12 + 2i$:

- Real part: 12
- Imaginary part: 2

The Importance of Multiplying Complex Numbers

Complex number multiplication is crucial in several applications:

- Signal processing: Representing and manipulating wave signals.
- Control systems: Analyzing system stability.
- Electromagnetism: Describing oscillating fields.
- Mathematics: Studying polynomial roots and complex analysis.

Understanding the product of two complex numbers helps in transforming and simplifying complex expressions, solving equations, and interpreting physical phenomena.

Step-by-Step Calculation of the Product

Given the two complex numbers:

$$ (4 + 3i) \quad \text{and} \quad (12 + 2i) $$

we want to compute:

$$ (4 + 3i) \times (12 + 2i) $$

Using the Distributive Property (FOIL Method)

The multiplication of two binomials follows the distributive property, often remembered by the FOIL acronym:

- F: First terms
- O: Outer terms
- I: Inner terms
- L: Last terms

Applying FOIL to the complex numbers:

$$ (4 + 3i)(12 + 2i) = (4 \times 12) + (4 \times 2i) + (3i \times 12) + (3i \times 2i) $$

Calculating each term:

1. $4 \times 12 = 48$
2. $4 \times 2i = 8i$
3. $3i \times 12 = 36i$
4. $3i \times 2i = 6i^2$

Now, combine these results:

$$48 + 8i + 36i + 6i^2$$

Next, combine like terms:

$$48 + (8i + 36i) + 6i^2$$
$$48 + 44i + 6i^2$$

Recall that $i^2 = -1$, so:

$$6i^2 = 6 \times (-1) = -6$$

Substitute back:

$$48 + 44i - 6$$

Finally, combine the real parts:

$$(48 - 6) + 44i = 42 + 44i$$

Result:

$$(4 + 3i)(12 + 2i) = \boxed{42 + 44i}$$

Interpreting the Result

The product of the two complex numbers is $42 + 44i$, which is itself a complex number with:

- Real part: 42
- Imaginary part: 44

This result can be visualized in the complex plane as a point located at coordinates (42, 44).

Additional Insights and Contexts

Magnitude and Phase of the Product

Understanding the magnitude (also called modulus) and phase (angle) of the product provides deeper insights into the nature of complex multiplication.

- Magnitude (|z|) of a complex number $z = a + bi$:

$$|z| = \sqrt{a^2 + b^2}$$

- Phase (θ):

$$ \theta = \arctan \left(\frac{b}{a} \right) $$

Applying this to the product $ 42 + 44i $:

- Magnitude:

$$ |42 + 44i| = \sqrt{42^2 + 44^2} = \sqrt{1764 + 1936} = \sqrt{3700} \approx 60.83 $$

- Phase:

$$ \theta = \arctan \left(\frac{44}{42} \right) \approx \arctan(1.0476) \approx 46.4^\circ $$

This indicates the product's vector in the complex plane has a length of approximately 60.83 units and is positioned at an angle of about 46.4 degrees from the positive real axis.

Multiplication in Polar Coordinates

Complex numbers can also be expressed in polar form:

$$ z = r (\cos \theta + i \sin \theta) $$

where $ r $ is the magnitude, and $ \theta $ is the argument (phase).

Multiplying two complex numbers in polar form involves:

- Multiplying their magnitudes.
- Adding their angles.

For the original numbers:

- Convert to polar form:
- $ 4 + 3i $:

$$ r_1 = \sqrt{4^2 + 3^2} = 5 $$
$$ \theta_1 = \arctan \left(\frac{3}{4} \right) \approx 36.87^\circ $$

- $ 12 + 2i $:

$$ r_2 = \sqrt{12^2 + 2^2} = \sqrt{144 + 4} = \sqrt{148} \approx 12.17 $$
$$ \theta_2 = \arctan \left(\frac{2}{12} \right) \approx 9.59^\circ $$

- Multiply magnitudes:

$$ r_{product} = r_1 \times r_2 \approx 5 \times 12.17 = 60.83 $$

- Add angles:

$$\theta_{product} = \theta_1 + \theta_2 \approx 36.87^\circ + 9.59^\circ = 46.46^\circ$$

This matches the earlier calculation of the magnitude and phase, confirming the consistency of complex multiplication across different representations.

Applications of Complex Number Multiplication

Understanding how to multiply complex numbers, such as (4 + 3i) and (12 + 2i), is vital across various disciplines:

- Electrical Engineering: Analyzing AC circuits where impedance and phasors are represented as complex numbers.
- Quantum Mechanics: Wave functions involve complex probability amplitudes; their interactions often involve multiplication.
- Control Theory: Stability analysis relies on the roots of characteristic equations, which are complex numbers.
- Signal Processing: Fourier transforms utilize complex multiplication to analyze frequency components.

Practical Example: Signal Modulation

Suppose an engineer is working with two signals represented by complex numbers. Multiplying these signals corresponds to combining their amplitudes and phases, which is essential in modulation techniques like amplitude modulation (AM). The calculation of their product allows for understanding how signals interact and combine in the frequency domain.

Conclusion

The product of the complex numbers (4 + 3i) and (12 + 2i) is $42 + 44i$. This calculation involves applying the distributive property, recognizing that $i^2 = -1$, and simplifying the expression accordingly. The process demonstrates the fundamental algebraic rules governing complex numbers and highlights the importance of both rectangular and polar forms.

Understanding complex multiplication is not only a mathematical skill but also a gateway to numerous practical applications in science and engineering. By mastering the step-by-step process outlined here, students and professionals can confidently work with complex numbers and harness their power in diverse fields.

Summary of the key steps:

1. Expand the product using FOIL.
2. Simplify each term, especially replacing i^2 with -1.
3. Combine like

Frequently Asked Questions

What is the product of (4 + 3i) and (12 + 2i)?

The product is (48 + 8i + 36i + 6i^2), which simplifies to (48 + 44i - 6) because i^2 = -1, resulting in 42 + 44i.

How do I multiply two complex numbers like (4 + 3i) and (12 + 2i)?

Multiply each term using the distributive property: (4)(12) + (4)(2i) + (3i)(12) + (3i)(2i), then simplify, remembering that i^2 = -1.

What is the step-by-step process to find the product of (4 + 3i) and (12 + 2i)?

First, multiply: 4×12 = 48, 4×2i = 8i, 3i×12 = 36i, 3i×2i = 6i^2. Since i^2 = -1, replace 6i^2 with -6. Then, combine like terms: 48 - 6 + (8i + 36i) = 42 + 44i.

What is the final simplified form of (4 + 3i) (12 + 2i)?

The simplified product is 42 + 44i.

Can you provide a quick answer to what (4 + 3i)(12 + 2i) equals?

Yes, it equals 42 + 44i.

Why do we replace i^2 with -1 when multiplying complex numbers?

Because i is the imaginary unit defined by i^2 = -1, which helps simplify expressions involving imaginary numbers.

Additional Resources

What Is The Product Of (4 + 3i) And (12 - 2i)?

When exploring complex numbers, understanding how to multiply them is a fundamental skill that opens the door to a deeper grasp of algebraic operations involving imaginary components. The product of (4 + 3i) and (12 - 2i) is a classic example that illustrates the process of multiplying complex numbers, combining real and imaginary parts, and simplifying the result. This article provides a comprehensive guide to calculating this product, breaking down each step with clarity and detail to ensure a solid understanding of the underlying concepts.

Understanding Complex Numbers

Before diving into the multiplication process, it's essential to grasp what complex numbers are and how they are represented.

What Are Complex Numbers?

Complex numbers are numbers that consist of two parts:
- A real part (a real number)
- An imaginary part (a real number multiplied by the imaginary unit i)

The standard form of a complex number is written as:

$a + bi$

where:
- a = real part
- b = imaginary part
- $i = \sqrt{-1}$, the imaginary unit

Visualizing Complex Numbers

Complex numbers can be visualized on the complex plane, where:
- The horizontal axis (x-axis) represents the real part.
- The vertical axis (y-axis) represents the imaginary part.

This visualization aids in understanding operations like addition, subtraction, and multiplication.

The Multiplication of Complex Numbers: A Step-by-Step Approach

Multiplying complex numbers involves applying the distributive property, often called FOIL (First, Outer, Inner, Last) in algebraic terms.

Given two complex numbers:

- $(a + bi)$
- $(c + di)$

their product is:

$$(a + bi)(c + di) = ac + adi + bci + bdi^2$$

Remember that $i^2 = -1$, which helps simplify the expression.

Calculating the Product of $(4 + 3i)$ and $(12 - 2i)$

Let's now apply this process to our specific numbers:

$(4 + 3i) \times (12 - 2i)$

Step 1: Expand the expression using FOIL

Applying the distributive property:

- First: $4 \times 12 = 48$
- Outer: $4 \times (-2i) = -8i$
- Inner: $3i \times 12 = 36i$
- Last: $3i \times (-2i) = -6i^2$

Step 2: Simplify each term

- The real parts: 48
- The imaginary parts: $-8i + 36i = 28i$
- The last term involves i^2, which simplifies as $-6i^2 = -6 \times (-1) = +6$

Step 3: Combine like terms

Adding the real parts and the imaginary parts:

- Real part: $48 + 6 = 54$
- Imaginary part: $28i$

Final result:

$54 + 28i$

Summary of the Calculation

Step	Expression	Explanation	Result
1	$(4 + 3i)(12 - 2i)$	Expand using FOIL	$(4)(12) + (4)(-2i) + (3i)(12) + (3i)(-2i)$
2	$48 - 8i + 36i - 6i^2$	Write out all terms	$48 - 8i + 36i - 6i^2$
3	Simplify imaginary terms	Combine like terms	$48 + 28i - 6i^2$
4	Replace i^2 with -1	Since $i^2 = -1$	$48 + 28i - 6 \times (-1)$
5	Final simplification	Calculate	$48 + 28i + 6 = 54 + 28i$

Visualizing the Result

The product $54 + 28i$ is a complex number with:
- Real part: 54
- Imaginary part: 28

In the complex plane, this point is located 54 units along the real axis and 28 units along the imaginary axis.

Applications of Multiplying Complex Numbers

Understanding how to multiply complex numbers isn’t just an academic exercise; it has practical applications across multiple fields:

1. Signal Processing
Complex numbers are used to represent signals, and multiplication corresponds to combining signals or analyzing frequency components.

2. Electrical Engineering
Impedance in AC circuits is expressed as complex numbers, and multiplying them relates to calculating power and other properties.

3. Control Systems
Complex multiplication plays a role in system stability analysis, where poles and zeros are represented as complex numbers.

4. Quantum Mechanics
Wave functions and probability amplitudes involve complex arithmetic, including multiplication.

Additional Tips for Multiplying Complex Numbers

- Always remember that $i^2 = -1$: This is crucial for simplifying the product.
- Distribute carefully: Use the FOIL method to ensure all terms are accounted for.
- Combine like terms: Real parts with real parts and imaginary parts with imaginary parts.
- Express the final answer in standard form: a + bi.

Practice Problems

To reinforce the concept, here are some practice problems:

1. Calculate the product of (2 + 5i) and (3 - 4i).
2. Find the product of (7 + 2i) and (1 + 3i).
3. Multiply (-3 + 4i) by (2 - i).

Try solving these to strengthen your understanding of multiplying complex numbers.

Conclusion

The product of (4 + 3i) and (12 - 2i) is 54 + 28i. This calculation demonstrates the importance of systematic expansion, correctly handling the imaginary unit i, and simplifying the expression to its standard form. Mastering such operations not only enhances algebraic skills but also prepares you for more advanced topics involving complex analysis, engineering, and physics. With practice, multiplying complex numbers becomes an intuitive part of your mathematical toolkit, enabling you to analyze and solve problems involving the fascinating world of complex numbers with confidence.

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