What Is The Product Of (-a + 3)(a + 4)?

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Understanding how to multiply algebraic expressions is fundamental in mathematics, especially in algebra. One common operation involves multiplying binomials—expressions with two terms. The product of binomials like (-a + 3)(a + 4) appears frequently in algebraic equations, polynomial expansion, and solving algebraic problems. This article explores the detailed process of multiplying such binomials, provides step-by-step instructions, and discusses their applications, ensuring a comprehensive understanding suitable for students, educators, and math enthusiasts alike.

Introduction to Binomials and Their Products

Before diving into the specific product (-a + 3)(a + 4), it's essential to understand what binomials are and the general rules for multiplying them.

What Are Binomials?

- Binomials are polynomial expressions containing exactly two terms separated by a plus or minus sign.
- Examples include (x + 5), (2a - 7), (-a + 3), and (x - y).

Why Multiply Binomials?

- Binomial multiplication is used to expand expressions, simplify algebraic equations, and solve problems involving polynomial products.
- It lays the foundation for quadratic equations and polynomial factorization.

Understanding the Product (-a + 3)(a + 4)

The expression (-a + 3)(a + 4) involves two binomials: one with terms -a and 3, and the other with a and 4. To understand the product thoroughly, we need to apply the distributive property, often simplified through the FOIL method.

What Is FOIL?

- FOIL stands for First, Outer, Inner, Last — the steps for multiplying two binomials.
- It involves multiplying the terms in each binomial in a specific sequence.

Step-by-Step Expansion of (-a + 3)(a + 4)

Let's systematically expand the product:

1. First Terms: Multiply the first terms of each binomial: (-a) (a) = -a2
2. Outer Terms: Multiply the outer terms: (-a) 4 = -4a
3. Inner Terms: Multiply the inner terms: 3 (a) = 3a
4. Last Terms: Multiply the last terms: 3 4 = 12

Now, combine these results:

- (-a2) + (-4a) + (3a) + 12

Next, simplify by combining like terms:

- -a2 + (-4a + 3a) + 12
- -a2 - a + 12

Thus, the product of (-a + 3)(a + 4) simplifies to:

-a2 - a + 12

In-Depth Explanation of the Expansion Process

Understanding the process in detail helps solidify the concept:

1. Multiplying the First Terms

- The first terms are -a and a.
- Multiplication: (-a) (a) = -a2
- Sign consideration: Negative times positive gives negative, and a a = a2.

2. Multiplying Outer Terms

- Outer terms: -a (from the first binomial) and 4 (from the second).
- Multiplication: (-a) 4 = -4a
- Sign: Negative times positive yields negative.

3. Multiplying Inner Terms

- Inner terms: 3 (from the first binomial) and a (from the second).
- Multiplication: 3 a = 3a
- Sign: Both positive, so the result is positive.

4. Multiplying the Last Terms

- Last terms: 3 and 4.
- Multiplication: 3 4 = 12

5. Combining the Results

- The expanded form: -a2 - 4a + 3a + 12
- Combining like terms: -a2 - a + 12

Visual Representation of the Multiplication

Using a grid or box method can help visualize the multiplication process:

	a	4
-a	(-a) a = -a2	(-a) 4 = -4a
3	3 a = 3a	3 4 = 12

Summing all four products yields the expanded polynomial.

Applications of the Product (-a + 3)(a + 4)

The expansion of binomials like (-a + 3)(a + 4) has several real-world and academic applications:

1. Polynomial Factoring and Expansion

- Understanding binomial multiplication is essential for factoring quadratic equations and expanding polynomial expressions.

2. Solving Algebraic Equations

- Expanding binomials enables solving equations where expressions are multiplied or factored.

3. Graphing Quadratic Functions

- The expanded quadratic form (-a2 - a + 12) can be graphed to analyze parabola properties, roots, and vertex.

4. Algebraic Problem Solving

- Many word problems involve setting up and expanding binomials to find unknowns.

General Formula for Multiplying Binomials

The process demonstrated with (-a + 3)(a + 4) can be generalized:

- For any binomials (x + y)(z + w), the product is xz + xw + yz + yw.
- Simplify by combining like terms if necessary.

Additional Examples for Practice

To deepen understanding, here are some similar binomial multiplication examples:

1. (x - 5)(x + 2)
- Expansion: x2 + 2x - 5x - 10 = x2 - 3x - 10

2. (-b + 6)(b - 3)
- Expansion: -b2 + 3b + 6b - 18 = -b2 + 9b - 18

3. (2a + 7)(a - 4)
- Expansion: 2a2 - 8a + 7a - 28 = 2a2 - a - 28

Practicing these helps reinforce the multiplication rules and the importance of careful sign management.

Conclusion

The product of (-a + 3)(a + 4) exemplifies fundamental algebraic principles involving binomial multiplication. By systematically applying the distributive property, especially the FOIL method, we expand the expression to a quadratic form: -a2 - a + 12. This process not only enhances algebraic manipulation skills but also lays the groundwork for more advanced topics like polynomial factoring, quadratic equations, and graphing functions.

Understanding the step-by-step expansion and the underlying concepts enables learners to approach similar problems confidently. Whether solving equations, simplifying expressions, or analyzing functions, mastering binomial multiplication is an essential skill in mathematics.

Meta Description:
Learn how to multiply binomials with a detailed, step-by-step explanation of the product (-a + 3)(a + 4). Discover algebraic expansion techniques, applications, and practice examples to strengthen your understanding of binomial multiplication.

Frequently Asked Questions

What is the expanded form of the product (-a + 3)(a + 4)?

The expanded form is -a^2 + 5a + 12.

How do I find the product of (-a + 3)(a + 4)?

Use the distributive property (FOIL method): (-a)(a) + (-a)(4) + 3(a) + 3(4), which simplifies to -a^2 - 4a + 3a

+ 12, then combine like terms to get -a^2 - a + 12.

What is the coefficient of the quadratic term in (-a + 3)(a + 4)?

The coefficient of the quadratic term -a^2 is -1.

Does the product of (-a + 3)(a + 4) result in a quadratic expression?

Yes, because multiplying two binomials results in a quadratic expression.

How can I factor the expression (-a + 3)(a + 4)?

First expand it to get -a^2 - a + 12, then factor if possible. Since it's quadratic, it can be factored as (-a + 4)(a + 3).

What is the value of (-a + 3)(a + 4) when a = 2?

Substitute a = 2 into the expanded form: - (2)^2 - 2 + 12 = -4 - 2 + 12 = 6.

Can the product (-a + 3)(a + 4) be written as a single binomial?

No, the product expands to a quadratic expression, which is generally expressed as a trinomial, not a binomial.

What is the significance of the signs in the product (-a + 3)(a + 4)?

The signs determine the terms' signs in the expanded quadratic expression, affecting its shape and roots.

How does the product (-a + 3)(a + 4) relate to solving quadratic equations?

Expanding this product gives a quadratic equation, which can then be solved for a using factoring, completing the square, or quadratic formula.

What are the roots of the quadratic obtained from (-a + 3)(a + 4)?

The roots are when the expression equals zero: (-a + 3)(a + 4) = 0, which gives a = 3 and a = -4.

Additional Resources

What Is The Product Of (-a + 3)(a + 4)?
Understanding the product of algebraic expressions is fundamental to mastering algebraic manipulation, solving equations, and exploring more complex mathematical concepts. In particular, the expression (-a +

3)(a + 4) is a common example that illustrates the application of the distributive property, also known as the FOIL method for binomials. This guide will walk you through the process step-by-step, explaining key concepts, and providing practical tips to help you confidently evaluate similar expressions.

Introduction to the Expression

When we see an expression like (-a + 3)(a + 4), our goal is to find its product, which means multiplying the two binomials to produce a simplified algebraic expression. This process involves applying algebraic rules and understanding how to expand and combine like terms.

Why Is This Important?

- Foundation for Polynomial Operations: Multiplying binomials is a core skill in algebra, forming the basis for more advanced topics like quadratic equations and polynomial factoring.
- Application in Real-World Problems: Many real-life scenarios involve algebraic expressions, such as calculating areas, modeling growth, or analyzing relationships between variables.
- Preparation for Higher Mathematics: Mastering these basic operations prepares students for calculus, linear algebra, and beyond.

Understanding Binomials and Their Products

What Are Binomials?

A binomial is an algebraic expression with two terms, typically joined by addition or subtraction. In our case:

- First binomial: -a + 3
- Second binomial: a + 4

The Product of Binomials

Multiplying two binomials involves applying the distributive property, often remembered through the FOIL method:

- First: Multiply the first terms
- Outer: Multiply the outer terms
- Inner: Multiply the inner terms
- Last: Multiply the last terms

Applying this method systematically ensures no terms are missed.

Step-by-Step Guide to Multiplying $(-a + 3)(a + 4)$

Let's break down the multiplication process:

Step 1: Multiply the First Terms

Multiply -a (from the first binomial) with a (from the second):

- $-a \times a = -a^2$

Step 2: Multiply the Outer Terms

Multiply -a with 4:

- $-a \times 4 = -4a$

Step 3: Multiply the Inner Terms

Multiply 3 with a:

- $3 \times a = 3a$

Step 4: Multiply the Last Terms

Multiply 3 with 4:

- $3 \times 4 = 12$

Step 5: Combine All Results

Now, write all the products together:

- $-a^2$ (from first terms)
- $-4a$ (outer)
- $+ 3a$ (inner)
- $+ 12$ (last)

Simplifying the Result

At this stage, combine like terms:

- $-4a + 3a = -a$

So, the fully expanded form is:

$-a^2 - a + 12$

This is the simplified product of $(-a + 3)(a + 4)$.

Final Answer

The product of $(-a + 3)(a + 4)$ is:

$-a^2 - a + 12$

This quadratic expression captures the result of multiplying the two binomials.

Deeper Insights and Variations

Understanding the Significance of the Result

- The $-a^2$ term indicates a quadratic component, suggesting the product is a parabola opening downward.
- The $-a$ term shows a linear component.
- The constant 12 shifts the graph vertically.

Exploring Special Cases

- If $a = 0$: The expression becomes $-0 + 3$ times $0 + 4 \to 3 \times 4 = 12$.
- If $a = 1$: The product becomes $(-1 + 3)(1 + 4) = 2 \times 5 = 10$, which matches the expanded form: $-1^2 - 1 + 12 = -1 - 1 + 12 = 10$.
- If $a = -1$: The product becomes $(-(-1) + 3)((-1) + 4) = (1 + 3)(3) = 4 \times 3 = 12$. The expanded form: $-(-1)^2 - (-1) + 12 = -1 + 1 + 12 = 12$.

These substitutions verify the correctness of our expansion.

Practical Applications and Related Concepts

Factoring Quadratic Expressions

Knowing how to expand binomials helps in factoring quadratic expressions. For example, recognizing that:

$-a^2 - a + 12$

can be factored further depending on the context.

Solving Equations

Suppose you have an equation like:

$(-a + 3)(a + 4) = 0$

You can set each factor equal to zero to find solutions for a:

- $-a + 3 = 0 \rightarrow a = 3$
- $a + 4 = 0 \rightarrow a = -4$

This approach demonstrates how expansion aids in solving algebraic equations.

Tips for Mastering Binomial Multiplication

- Memorize the FOIL method for quick expansion.
- Pay attention to signs—negative signs can be tricky but are essential for correct calculations.
- Practice with different binomials to become comfortable with various coefficients and signs.
- Verify your work by plugging in values for a to see if both original expressions produce the same result.

Summary

The product of $(-a + 3)(a + 4)$, when expanded, yields $-a^2 - a + 12$. This process involves applying the distributive property systematically, combining like terms, and understanding how binomials multiply to produce quadratic expressions. Mastering this technique is crucial for progressing in algebra, solving equations, and analyzing mathematical relationships involving polynomials.

By practicing these steps and understanding the underlying principles, you'll develop confidence and proficiency in handling similar algebraic expressions, setting a strong foundation for future mathematical endeavors.

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