What Is The Range Of The Function F(x)=3/4|x|-3

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Understanding the range of a function is a fundamental aspect of algebra and calculus, as it tells us all the possible output values that a function can produce based on its input values. When analyzing the function F(x)=3/4|x|-3, determining its range helps us grasp how the function behaves across different x-values, especially considering the absolute value component which influences its shape and output. In this article, we will explore in detail what the range of F(x)=3/4|x|-3 is, how to derive it, and what implications it has for graphing and understanding the function.

Understanding the Function F(x)=3/4|x|-3

Before delving into the range, it's important to understand the structure of the function itself.

Breaking Down the Function

- Absolute Value Component: |x|
The absolute value of x, denoted as |x|, is always non-negative. This means |x| ≥ 0 for all real numbers x. The absolute value function is symmetric about the y-axis, creating a "V" shape when graphed.

- Coefficient 3/4:
Multiplied by |x|, the term 3/4|x| scales the absolute value, influencing the slope of the graph's arms.

- Vertical Shift (-3):
Subtracting 3 shifts the entire graph downward by 3 units.

The overall structure of the function is a V-shaped graph opening upwards, scaled by 3/4, and shifted downward.

Graphical Behavior of F(x)=3/4|x|-3

Understanding the graph helps in visualizing the range.

Shape and Symmetry

- The graph of F(x) resembles an "upward-opening V".
- It is symmetric about the y-axis because |x| is symmetric.

- The vertex of the V occurs at x=0, where |x|=0.

Vertex of the Graph

- At x=0, |x|=0, so F(0)=3/4 0 -3 = -3.
- The vertex point is (0, -3).

Behavior as x approaches infinity

- As |x| increases, F(x) increases proportionally to |x| because of the 3/4 coefficient.
- The function's output becomes larger without bound as x approaches positive or negative infinity.

Determining the Range of F(x)=3/4|x|-3

The range of a function is the set of all possible output values (F(x)) for x in the domain.

Step-by-Step Derivation of the Range

1. Identify the domain:
Since the function involves |x|, which is defined for all real x, the domain is all real numbers, (-∞, ∞).

2. Analyze the minimum value:
The term 3/4|x| is always ≥ 0, so the smallest value occurs when |x|=0.
- F(0) = 3/4 0 - 3 = -3.

3. Determine the maximum value of F(x):
Because |x| can grow without bound, 3/4|x| can become arbitrarily large.
- As |x| → ∞, F(x) → ∞.

4. Conclusion on the range:
The smallest output value is -3, and the function can produce arbitrarily large values.
- Therefore, the range is all real numbers greater than or equal to -3.

Formal Expression of the Range

\[
\boxed{
\text{Range of } F(x) = \left[-3, \infty \right)
}
\]

This notation indicates that the function's outputs include -3 and all real numbers larger than -3.

Key Points about the Range of F(x)=3/4|x|-3

- Minimum value: -3 at x=0.
- Unbounded above: The function increases without limit as |x| increases.
- Interval notation: The range is $[-3, \infty)$.
- Symmetry: The function is symmetric about the y-axis because of the absolute value.

Applications and Implications of the Range

Understanding the range of this function has practical applications in various fields such as physics, engineering, and data analysis, where modeling with absolute value functions is common.

Graphing the Function

- The vertex at (0, -3) serves as the lowest point.
- The arms of the graph extend upward, with slopes of $\pm 3/4$.
- As x moves away from zero, the function's value increases linearly.

Real-World Examples

- Distance Measurement: If x represents a physical distance, then F(x) might represent a cost or energy that increases with distance.
- Error Analysis: The absolute value component models error magnitudes, with the downward shift indicating a base cost or baseline.

Comparison with Similar Functions

Functions involving absolute values often have similar range characteristics.

Examples:

1. F(x)=|x| - 3
- Range: $[-3, \infty)$ (minimum at x=0, value = -3).

2. F(x)=k|x| + c
- Range: $[c, \infty)$ if c is real and the coefficient of |x| is positive.

3. F(x)=-|x| + d

- Range: $(-\infty, d]$, with maximum at x=0.

This comparison illustrates how coefficients and constants shift and scale the range.

Summary

To summarize, the range of the function F(x)=3/4|x|-3 is all real numbers greater than or equal to -3. This is because the absolute value ensures the minimum output occurs at x=0, where the function value is -3, and as |x| increases, the function's output increases without bound.

Key takeaways:

- The minimum of the function is at x=0, with F(0)=-3.
- The function is symmetric about the y-axis.
- The range is $[-3, \infty)$.

Final Thoughts

Understanding the range of functions like F(x)=3/4|x|-3 is essential for graphing, analyzing behavior, and applying these concepts in real-world scenarios. Recognizing how the absolute value, coefficient, and vertical shift influence the range helps students and professionals alike to interpret and utilize similar functions effectively.

Optimized for SEO Keywords:

- Range of F(x)=3/4|x|-3
- Absolute value function range
- How to find the range of a function
- Graph of F(x)=3/4|x|-3
- Understanding the behavior of absolute value functions
- Function analysis and range determination
- Mathematical properties of F(x)=3/4|x|-3

By mastering how to determine and interpret the range of functions like F(x)=3/4|x|-3, learners can deepen their understanding of algebraic concepts and enhance their problem-solving skills in mathematics.

Frequently Asked Questions

What is the general form of the function F(x) = (3/4)|x| - 3?

It is a linear transformation of the absolute value function, scaled by 3/4 and shifted downward by 3 units.

How do you find the range of F(x) = (3/4)|x| - 3?

Since |x| ≥ 0, the minimum value of F(x) occurs at |x|=0, which is -3. As |x| increases, F(x) increases without bound. Therefore, the range is [-3, ∞).

What is the minimum value of F(x) = (3/4)|x| - 3?

The minimum value is -3, which occurs at x=0.

Does the function F(x) = (3/4)|x| - 3 have any maximum value?

No, since |x| can grow without limit, F(x) approaches infinity as |x| increases, so there is no maximum value.

At what point is the minimum of F(x) achieved?

The minimum is achieved at x=0, where F(0) = -3.

Is the range of F(x) continuous?

Yes, the range is continuous, covering all values from -3 to infinity.

How does the absolute value in F(x) affect its range?

The absolute value ensures the function is always non-negative inside the transformation, resulting in a range starting at -3 and extending upward without bound.

If you graph F(x) = (3/4)|x| - 3, how would the graph look in terms of its range?

The graph is a V-shaped graph with its vertex at (0, -3), opening upwards, with the range being all y-values from -3 upwards to infinity.

Additional Resources

What Is The Range Of The Function F(x) = $\frac{3}{4}|x| - 3$

Understanding the behavior of mathematical functions is fundamental not only to algebra but also to numerous applied sciences, including physics, engineering, and computer science. Among the various functions studied, absolute value functions stand out due to their symmetry and distinctive shape. Today, we delve into a specific absolute value-based function: $F(x) = \frac{3}{4}|x| - 3$, aiming to determine its range—the set of all possible output values.

Introduction to the Function

Before exploring the range, it is essential to understand the structure of the function itself. The given function is:

$$
F(x) = \frac{3}{4} |x| - 3
$$

This function combines a scaled absolute value of x with a constant shift. The absolute value $|x|$ is a piecewise function:

$$
|x| =
\begin{cases}
x, & \text{if } x \geq 0 \\
- x, & \text{if } x < 0
\end{cases}
$$

Due to the absolute value, $F(x)$ exhibits symmetry about the y-axis, meaning the function's value at x is the same as at $-x$. The multiplication by $\frac{3}{4}$ determines the steepness or slope of the 'V' shape, while the subtraction of 3 shifts the entire graph downward by 3 units.

Visualizing the Function: The Graph’s Shape

To comprehend the range, visualizing the graph of $F(x)$ is immensely helpful. The graph of $F(x)$ resembles a 'V' shape, opening upwards, with its vertex (the lowest point) at the point where the inner expression $\frac{3}{4}|x|$ is minimized.

- Vertex Point: Occurs at $x=0$, where the absolute value is zero.

- At $x=0$:
$$
F(0) = \frac{3}{4} \times 0 - 3 = -3
$$
So, the vertex of the graph is at $(0, -3)$.

- Behavior as $x \to \pm \infty$:
$$
|x| \to \infty \implies F(x) \to \infty
$$
The function grows without bound as the absolute value of x increases.

This shape and behavior are characteristic of absolute value functions, which are symmetric and have their minimum or maximum at the vertex point.

Determining the Range: Mathematical Analysis

The range of a function is the set of all possible output values $F(x)$ can take as x varies over its domain. Since the domain of $F(x)$ is all real numbers ($\mathbb{R}$), we analyze how the output $F(x)$ behaves across this domain.

Step 1: Identify the minimum value

Given the shape of the graph and the vertex at $x=0$, the minimum value occurs at the vertex:

$$
F(0) = -3
$$

Because the absolute value term cannot be negative, and the entire function is increasing on either side of the vertex (due to the positive coefficient $\frac{3}{4}$), this minimum is the lowest point on the graph.

Step 2: Behavior as x moves away from zero

As x moves away from zero in either positive or negative direction:

$$
|x| \to \infty
$$
$$
F(x) = \frac{3}{4}|x| - 3 \to \infty
$$

This indicates that the function's output increases without bound as x increases or decreases.

Step 3: Formal range conclusion

Considering the above, the outputs:

- Achieve the minimum at $F(0) = -3$.
- Increase without limit to infinity as x tends toward $\pm \infty$.

Thus, the range of the function is:

$$
\boxed{[-3, \infty)}
$$

which includes all real numbers greater than or equal to -3.

Practical Implications of the Range

Knowing the range of a function like $F(x) = \frac{3}{4}|x| - 3$ has practical applications:

- Optimization problems: If this function models cost or height, understanding the minimum ensures

you know the lowest value achievable.
- Graphing and data analysis: Recognizing the range allows for accurate plotting and interpretation of data.
- Constraint setting: In real-world scenarios, knowing the possible outputs helps in setting bounds for feasible solutions.

The Role of Transformations in Shaping the Range

Understanding how transformations affect the range is vital:

- Vertical shifts: Adding or subtracting constants shifts the range vertically.
- In our case, subtracting 3 shifts the graph downward, changing the minimum from 0 (for the basic $|x|$ function) to -3.

- Scaling factors: Multiplying by $\frac{3}{4}$ affects the steepness but not the overall shape or the minimum point, only the rate at which the function increases away from the vertex.

- Reflections: Although not present here, reflections across axes also alter the range's bounds.

Comparing with Similar Functions

To deepen understanding, consider other functions with similar structures:

Function	Description	Range	Notes
$f(x) = \lvert x\rvert$	Basic absolute value	$[0, \infty)$	Vertex at (0,0)
$g(x) = 2\lvert x\rvert + 1$	Vertical stretch and shift	$[1, \infty)$	Vertex at (0,1)
$h(x) = -\frac{1}{2}\lvert x\rvert + 4$	Reflection and shift	$(-\infty, 4]$	Open downward

By comparing these, the effect of coefficients and constants on the range becomes clearer, reinforcing the understanding of how transformations influence the possible output values.

Summary and Final Thoughts

In conclusion, the function $F(x) = \frac{3}{4}|x| - 3$ is a classic example of how absolute value functions can be manipulated through scaling and shifting. Its graph is a "V" shape symmetric about the y-axis, with the vertex at $(0, -3)$. The function's outputs are bounded below by -3, and as x moves towards positive or negative infinity, the outputs grow arbitrarily large.

Therefore, the range of $F(x)$ is:

$$
\boxed{[-3, \infty)}
$$

This understanding not only helps in graphing and analyzing the function but also offers insights into how modifications to basic functions can tailor their behavior to fit specific mathematical or real-world contexts.

Final Remarks

Mathematical functions like $F(x) = \frac{3}{4}|x| - 3$ exemplify how simple transformations—scaling and shifting—can dramatically alter a function’s output set. Mastering these concepts allows students, engineers, and scientists to predict, manipulate, and optimize functions effectively across multiple disciplines. Whether designing a physical structure, analyzing economic models, or programming algorithms, understanding the range remains a fundamental skill in the mathematical toolkit.

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