What Is The Range Of The Function F(x) = 3x2 6x 8?

What Is The Range Of The Function $F(x) = 3x^2 + 6x + 8$?

Understanding the range of a function is essential in mathematics because it tells us all the possible output values the function can produce for real input values. When dealing with quadratic functions like $F(x) = 3x^2 + 6x + 8$, determining the range involves analyzing the parabola's shape and position on the coordinate plane. This article provides a comprehensive explanation of how to find the range of the given quadratic function, along with relevant concepts, step-by-step procedures, and practical examples.

Understanding Quadratic Functions

Definition of a Quadratic Function

A quadratic function is a polynomial function of degree 2, typically expressed in the form:
- $F(x) = ax^2 + bx + c$

where:
- a, b, and c are real numbers,
- $a \neq 0$.

In our case, $F(x) = 3x^2 + 6x + 8$, with $a = 3$, $b = 6$, and $c = 8$.

Graph of a Quadratic Function

The graph of a quadratic function is a parabola. The parabola opens upward if the coefficient 'a' is positive and downward if 'a' is negative. Since $a = 3 > 0$ in our function, the parabola opens upward.

Analyzing the Function $F(x) = 3x^2 + 6x + 8$

Identifying the Vertex

The vertex of a parabola provides critical information about its maximum or minimum point, which directly affects the range.

The x-coordinate of the vertex is given by:

- $x = -b / (2a)$

For our function:
- $x = -6 / (2 3) = -6 / 6 = -1$

To find the y-coordinate of the vertex, substitute $x = -1$ into the original function:

$F(-1) = 3(-1)^2 + 6(-1) + 8$
$= 31 - 6 + 8$
$= 3 - 6 + 8$
$= 5$

Therefore, the vertex is at point (-1, 5).

Shape and Opening of the Parabola

Since the coefficient $a = 3$ is positive, the parabola opens upward, indicating that the vertex represents the minimum point of the parabola.

Determining the Range of $F(x) = 3x^2 + 6x + 8$

What Is the Range?

The range of a function consists of all possible output values (y-values) that the function can produce.

Given that our parabola opens upward and the vertex is at (-1, 5), the minimum value of the function is 5.

Because the parabola opens upward and extends infinitely upwards, the function can take on any value greater than or equal to 5.

Thus, the range is:
- $y \geq 5$

Expressing the Range in Interval Notation

Using interval notation, the range is:
- $[5, \infty)$

This indicates that the function outputs all real numbers starting from 5 and extending to infinity.

Step-by-Step Method to Find the Range of a Quadratic Function

Step 1: Write the Function in Standard Form

Our function is already in standard quadratic form:
- $F(x) = 3x^2 + 6x + 8$

Step 2: Find the x-coordinate of the Vertex

Calculate using:
- $x = -b / (2a)$

For our function:
- $x = -6 / (23) = -1$

Step 3: Find the Corresponding y-value at the Vertex

Substitute $x = -1$ into the function:
- $F(-1) = 3(-1)^2 + 6(-1) + 8 = 5$

Step 4: Determine the Nature of the Parabola

Since $a = 3 > 0$, the parabola opens upward, and the vertex is a minimum point.

Step 5: Write the Range

The range is all y-values greater than or equal to the y-coordinate of the vertex:
- $y \geq 5$

Expressed as:
- $[5, \infty)$

Examples and Practice Problems

Example 1:

Find the range of the function $F(x) = 2x^2 - 4x + 1$.

Solution:
1. Identify a, b, c:
- a=2, b=-4, c=1
2. Find x-coordinate of vertex:
- $x = -(-4) / (22) = 4/4 = 1$
3. Find y-value at x=1:
- $F(1) = 2(1)^2 - 4(1) + 1 = 2 - 4 + 1 = -1$

4. Parabola opens upward ($a=2 > 0$), so the vertex is a minimum.
5. Range:
- $y \geq -1$
- Range: $[-1, \infty)$

Practice Problem:

Determine the range of $F(x) = -5x^2 + 10x - 3$.

Solution:
(Students are encouraged to follow the steps outlined above)

Additional Considerations in Range Determination

Dealing with Different Forms of Quadratic Functions

Quadratic functions can sometimes be expressed in different forms, such as factored form or vertex form. Regardless of the form, the process of finding the range involves identifying the vertex and the parabola's opening direction.

Transforming to Vertex Form

Converting a quadratic into vertex form:
- $F(x) = a(x - h)^2 + k$

Where (h, k) is the vertex. To convert our function, complete the square:

$F(x) = 3x^2 + 6x + 8$

Factor out 3:
- $F(x) = 3(x^2 + 2x) + 8$

Complete the square inside:
- $x^2 + 2x = x^2 + 2x + 1 - 1 = (x + 1)^2 - 1$

Rewrite:
- $F(x) = 3[(x + 1)^2 - 1] + 8$
- $F(x) = 3(x + 1)^2 - 3 + 8$
- $F(x) = 3(x + 1)^2 + 5$

The vertex form confirms the vertex at $(-1, 5)$, and since the coefficient of the squared term is positive, the parabola opens upward.

Summary and Key Takeaways

- The range of a quadratic function depends on the parabola's vertex and its opening direction.
- For upward-opening parabolas, the range starts at the y-coordinate of the vertex and extends to infinity.
- To find the range, identify the vertex via the x-coordinate formula, compute the y-value, and determine whether the parabola opens upward or downward.
- Converting to vertex form can simplify range determination.

Conclusion

The function $F(x) = 3x^2 + 6x + 8$ has a minimum value at its vertex (−1, 5). Because the parabola opens upward, the range includes all real numbers greater than or equal to 5. In interval notation, the range is $[5, \infty)$. Understanding how to analyze the vertex and the parabola’s shape is crucial in accurately determining the range of any quadratic function.

Mastering these concepts enables students and mathematicians to analyze quadratic functions efficiently, which is essential in various applications such as physics, engineering, economics, and beyond.

Frequently Asked Questions

What is the range of the function F(x) = 3x^2 + 6x + 8?

The range of the function is all real numbers greater than or equal to 2, i.e., $[2, \infty)$.

How do we find the minimum value of the quadratic function F(x) = 3x^2 + 6x + 8?

By completing the square or using the vertex formula, the minimum occurs at $x = -1$, giving $F(-1) = 2$.

What is the vertex of the parabola represented by F(x) = 3x^2 + 6x + 8?

The vertex is at the point (−1, 2), which is the minimum point of the parabola.

Does the function F(x) = 3x^2 + 6x + 8 have any maximum value?

No, since the parabola opens upward (coefficient of x^2 is positive), it has no maximum value, only a minimum.

How does the leading coefficient of the quadratic

affect the range of $F(x) = 3x^2 + 6x + 8$?

A positive leading coefficient (3) means the parabola opens upward, so the range starts at the vertex minimum value and extends to infinity.

Can the range of $F(x)$ be expressed in interval notation?

Yes, the range is $[2, \infty)$, including 2 and extending infinitely upward.

What methods can be used to determine the range of a quadratic function like $F(x) = 3x^2 + 6x + 8$?

Methods include completing the square, using the vertex formula, or analyzing the parabola’s graph to identify the minimum point.

Additional Resources

What Is The Range Of The Function $F(x) = 3x^2 + 6x + 8$?

Understanding the range of a function is fundamental in algebra and calculus, as it reveals all the possible output values a function can produce given its domain. In this detailed exploration, we analyze the quadratic function $F(x) = 3x^2 + 6x + 8$, delving into the methods of determining its range, the properties of quadratic functions, and the implications of its characteristics.

Introduction to Quadratic Functions and Their Ranges

Quadratic functions are polynomial functions of degree two, commonly expressed in the form:

$$
f(x) = ax^2 + bx + c
$$

where $a \neq 0$, and b, c are real numbers. Their graphs are parabolas, which open upward if $a > 0$ and downward if $a < 0$.

Why is the range important?
The range indicates all the possible output values y that the function can produce as x varies over its domain, typically all real numbers unless specified otherwise.

Analyzing the Function $F(x) = 3x^2 + 6x + 8$

Given the quadratic function:

$$
F(x) = 3x^2 + 6x + 8
$$

we recognize:

- The leading coefficient $a = 3$ is positive, implying the parabola opens upward.
- The domain of F is all real numbers: $(-\infty, \infty)$.

Our goal is to find the set of all y-values (the range) for which there exists some x such that $y = F(x)$.

Step 1: Find the Vertex of the Parabola

The vertex of a parabola $y = ax^2 + bx + c$ gives the minimum (if $a > 0$) or maximum (if $a < 0$) point, which is crucial for determining the range.

The x-coordinate of the vertex:

$$
x_{v} = -\frac{b}{2a}
$$

For our function:

$$
x_{v} = -\frac{6}{2 \times 3} = -\frac{6}{6} = -1
$$

Now, find the y-coordinate of the vertex:

$$
F(-1) = 3(-1)^2 + 6(-1) + 8 = 3(1) - 6 + 8 = 3 - 6 + 8 = 5
$$

Thus, the vertex is at $(-1, 5)$.

Since $a = 3 > 0$, the parabola opens upward, meaning the vertex at $(-1, 5)$ is the minimum point on the graph.

Step 2: Determine the Range

Because the parabola opens upward and the vertex is the lowest point, the function's output values are all real numbers greater than or equal to the y-coordinate of the vertex:

$$
\boxed{
\text{Range of } F(x) = [5, \infty)
}
$$

This indicates that:

- The minimum value of $F(x)$ is 5, attained at $x = -1$.
- The function approaches infinity as $x \to \pm \infty$.

Alternative Method: Using the Discriminant to Find the Range

Another way to verify the range, especially for quadratics, involves analyzing the quadratic equation $F(x) = y$ for various y-values.

Rearranged:

$$
3x^2 + 6x + (8 - y) = 0
$$

For y to be in the range, this quadratic in x must have real solutions, i.e., its discriminant $D \geq 0$.

The discriminant:

$$
D = b^2 - 4ac
$$

Here:

- $a = 3$
- $b = 6$
- $c = 8 - y$

Calculating:

$$
D = (6)^2 - 4 \times 3 \times (8 - y) = 36 - 12(8 - y) = 36 - 96 + 12y = -60 + 12y
$$

Set $D \geq 0$:

$$
-60 + 12y \geq 0 \implies 12y \geq 60 \implies y \geq 5
$$

Thus, the set of \(y \)-values for which the quadratic in \(x \) has real solutions is \(y \geq 5 \). Hence, the range:

\[
\boxed{
[5, \infty)
}
\]

which confirms our earlier conclusion based on the vertex.

Implications and Visualizations

Visualizing the parabola:

- The vertex at \((-1, 5) \) is the lowest point.
- As \(x \to \pm \infty \), \(F(x) \to \infty \).
- The parabola opens upward, covering all \(y \)-values from 5 upwards.

This visualization helps in understanding the behavior of the function and its outputs.

Summary of Key Findings

- The quadratic function \(F(x) = 3x^2 + 6x + 8 \) opens upward.
- The vertex is at \((-1, 5) \), which is the minimum point.
- The range of \(F(x) \) is all real numbers \(y \geq 5 \).
- The method using the vertex formula and the discriminant approach both lead to the same conclusion.

Final answer:

\[
\boxed{
\text{Range of } F(x) = [5, \infty)
}
\]

Additional Considerations: Transformations and Applications

Understanding the range is crucial beyond pure mathematics. For example, in physics or engineering, quadratic functions model projectile trajectories, energy states, or optimization problems. Recognizing the minimum or maximum value allows practitioners to make informed decisions based on the output constraints.

Transformations of the quadratic, such as shifting or scaling, alter the vertex's position and the parabola's opening, which directly influences the range. For instance, changing the coefficient \(a \) affects the parabola's steepness and the rate at which \(F(x) \to \infty \).

Conclusion

Determining the range of a quadratic function like \(F(x) = 3x^2 + 6x + 8 \) involves analyzing its vertex and discriminant. Both methods confirm that the function attains a minimum value of 5 at \(x = -1 \), with the output extending to infinity thereafter. The systematic approach outlined ensures clarity and precision in understanding the behavior of quadratic functions, which is fundamental in advanced mathematics and numerous applied fields.

In essence, the range of the function \(F(x) = 3x^2 + 6x + 8 \) is \([5, \infty)\).

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