What Is The Zero Of The Function Below?f(x) = 3x - 15

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Understanding the concept of zeros of functions is fundamental in algebra and calculus, as it helps in analyzing the behavior of functions and solving equations efficiently. When working with linear functions like f(x) = 3x - 15, identifying the zero of the function is a crucial step in understanding where the function intersects the x-axis. This point, known as the zero or root of the function, provides insights into the solution of the equation f(x) = 0 and plays a vital role in graphing and problem-solving contexts.

In this comprehensive article, we will explore what the zero of the function f(x) = 3x - 15 is, how to determine it, its significance, and related concepts. Whether you're a student beginning to learn algebra or someone seeking a deeper understanding of linear functions, this guide will walk you through the process step-by-step, ensuring clarity and mastery of the topic.

Understanding the Concept of Zero of a Function

What Is a Zero of a Function?

A zero of a function f(x) is a value of x for which the function's output is zero. In other words, it is the x-value at which the graph of the function crosses or touches the x-axis. Mathematically, the zero of a function f(x) is the solution to the equation:

f(x) = 0

For example, if f(x) = 3x - 15, then the zero of this function is the value of x that makes the expression equal to zero.

Why Are Zeros Important?

Zeros are significant because they:

- Indicate the points where the function intersects the x-axis.
- Help in graphing the function accurately.
- Are solutions to equations involving the function.
- Provide insights into the behavior of the function, such as where it

changes from positive to negative or vice versa.
- Play a role in solving real-world problems involving optimization, equilibrium points, and more.

Analyzing the Function f(x) = 3x - 15

Understanding the Function's Structure

The given function f(x) = 3x - 15 is a linear function, characterized by a straight-line graph. Its general form is:

f(x) = mx + b

where:

- m is the slope of the line.
- b is the y-intercept.

For our function:

- Slope (m) = 3
- Y-intercept (b) = -15

This indicates that the line crosses the y-axis at -15 and rises 3 units vertically for each 1 unit increase in x.

Graphing the Function

Graphing helps visualize the function and locate its zero:

1. Plot the y-intercept at (0, -15).
2. Use the slope to find another point, for example:
- From (0, -15), move 1 unit right to x=1.
- Move 3 units up (since slope = 3) to y= -12.
- Plot (1, -12).
3. Draw a straight line through these points.

The x-intercept, where the line crosses the x-axis, is the zero of the function.

How to Find the Zero of the Function f(x) = 3x - 15

Method 1: Algebraic Solution

The most straightforward way to find the zero of a linear function is to set the function equal to zero and solve for x:

f(x) = 0

Applying this to our function:

3x - 15 = 0

Now, solve for x:

1. Add 15 to both sides:

3x = 15

2. Divide both sides by 3:

x = 15 / 3

x = 5

Therefore, the zero of the function is at x = 5.

Method 2: Graphical Approach

- Plot the function as described earlier.
- Observe the point where the line crosses the x-axis.
- The x-coordinate of this point is the zero, which in this case is at x=5.

Method 3: Using the Zero Formula for Linear Functions

For a linear function in form f(x) = mx + b, the zero can be found using:

x = -b / m

Applying this formula:

x = -(-15) / 3 = 15 / 3 = 5

This confirms the algebraic solution.

Significance of the Zero in the Context of the Function

Graphical Interpretation

The zero at x=5 means the graph of f(x) crosses the x-axis at the point (5, 0). This point is where the function transitions from positive to negative or vice versa, depending on the slope.

Real-World Applications

Linear functions model various real-world scenarios such as:

- Calculating break-even points in economics.
- Determining time when a certain level of profit or loss occurs.
- Finding the point where a physical quantity reaches zero, like the point at which a car's speed hits zero.

In the context of f(x) = 3x - 15, if this function represented profit based on units sold, then:

- When units sold (x) equals 5, profit (f(x)) is zero.
- Selling fewer than 5 units results in a loss.
- Selling more than 5 units results in profit.

Importance in Solving Equations

Knowing the zero helps in solving related equations, optimization problems, and understanding the behavior of the function across different x-values.

Additional Related Concepts

Zero of a Function vs. Root of an Equation

- The zero of a function is the x-value where $f(x) = 0$.
- The root of an equation is the solution to the equation, which in this context is the same as the zero of the function.

Zero of a Function in Different Types of Functions

- For quadratic functions, zeros are points where the parabola intersects the x-axis.
- For polynomial functions of higher degree, there can be multiple zeros.
- For non-polynomial functions, zeros might involve more complex solutions.

Factoring to Find Zero

While in this case, direct algebraic methods suffice, sometimes factoring is useful:

$f(x) = 3x - 15$

Factor out 3:

$f(x) = 3(x - 5)$

Set equal to zero:

$3(x - 5) = 0$

$x - 5 = 0$

$x = 5$

This confirms the zero.

Summary and Key Takeaways

- The zero of the function $f(x) = 3x - 15$ is the x-value where the function equals zero.
- Solving the equation $3x - 15 = 0$ gives $x = 5$.
- The graph of the function crosses the x-axis at $(5, 0)$.
- The zero provides insights into the function's behavior, solutions to equations, and real-world applications.

- The process of finding zeros involves algebraic manipulation, graphing, or using formulas.

Conclusion

Understanding how to find the zero of a function is a fundamental skill in mathematics, essential for graphing, solving equations, and analyzing functions. For the linear function $f(x) = 3x - 15$, the zero is straightforward to determine and occurs at $x=5$. Recognizing this point enhances comprehension of the function's behavior and its applications in various fields.

Whether you are solving simple equations or analyzing complex functions, mastering the concept of zeros equips you with a powerful tool to interpret and manipulate mathematical models effectively. Remember, the key steps involve setting the function equal to zero and solving for x, a method applicable to a wide range of functions beyond just linear ones.

Keywords: zero of a function, f(x) = 3x - 15, solve for zero, linear function, x-intercept, algebraic solution, graph of a function, mathematical analysis, roots of equations, solving linear equations, zero of a function formula

Frequently Asked Questions

What is the zero of the function f(x) = 3x - 15?

The zero of the function is $x = 5$ because substituting $x = 5$ into the function gives $f(5) = 3(5) - 15 = 15 - 15 = 0$.

How do you find the zero of the linear function f(x) = 3x - 15?

To find the zero, set $f(x) = 0$ and solve for x: $0 = 3x - 15$, which leads to $x = 5$.

What is the significance of the zero of a function like f(x) = 3x - 15?

The zero represents the x-value where the function crosses the x-axis, which in this case is at $x = 5$.

Is the zero of f(x) = 3x - 15 unique?

Yes, since this is a linear function, it has only one zero, which is at x = 5.

Can you interpret the zero of f(x) = 3x - 15 graphically?

Graphically, the zero is the point where the line crosses the x-axis, which occurs at x = 5.

What is the value of the zero of the function f(x) = 3x - 15?

The value of the zero is x = 5.

If f(x) = 3x - 15, for what value of x does the function equal zero?

f(x) equals zero when x = 5.

Additional Resources

What Is The Zero Of The Function Below? f(x) = 3x - 15

Understanding the concept of a zero of a function is fundamental in algebra and calculus. It forms the basis for analyzing the behavior of functions, solving equations, and modeling real-world phenomena. In this investigative review, we will delve into the specifics of the function f(x) = 3x - 15, exploring what its zero is, how to find it, and what significance it holds in broader mathematical contexts.

Introduction to Function Zeros: Conceptual Foundations

Before analyzing the specific function f(x) = 3x - 15, it is essential to understand what a zero of a function entails.

Definition of a Zero of a Function

A zero of a function f(x) is a value c in the domain of f such that:

$f(c) = 0$

In other words, it's the x-value where the function intersects the x-axis on a graph, representing the solution to the equation $f(x) = 0$.

Why Are Zeros Important?

Zeros are critical because they:

- Help identify roots or solutions of equations.
- Indicate points where a function changes sign.
- Are used in calculus to find critical points, maxima, minima, and points of inflection.
- Play a vital role in applications across physics, economics, engineering, and more.

Analyzing the Function $f(x) = 3x - 15$

The function in question is a linear function, characterized by its constant rate of change.

General Form and Characteristics

The function $f(x) = 3x - 15$ is in slope-intercept form:

$f(x) = mx + b$

where:
- $m = 3$ (the slope)
- $b = -15$ (the y-intercept)

This form makes it straightforward to analyze its zeros.

Graphical Representation

Graphically, the function is a straight line crossing the y-axis at (0, -15), rising with a slope of 3. Its zero is the point where the line crosses the x-axis, i.e., where $y = 0$.

Methodology to Find the Zero of f(x) = 3x - 15

Finding the zero involves solving the equation:

$f(x) = 0$

which translates to:

$3x - 15 = 0$

This is a simple linear equation that can be solved via algebraic manipulation.

Step-by-Step Solution Process

1. Start with the equation:

$3x - 15 = 0$

2. Add 15 to both sides to isolate the term with x:

$3x = 15$

3. Divide both sides by 3 to solve for x:

$x = 15 / 3$

4. Simplify:

$x = 5$

Thus, the zero of the function is at $x = 5$.

Verification

To confirm, substitute $x = 5$ back into the function:

$f(5) = 3(5) - 15 = 15 - 15 = 0$

The calculation confirms the zero at $x = 5$.

Mathematical Significance of the Zero

The zero at $x = 5$ indicates the point where the function intersects the x-axis. This point, (5, 0), is crucial in understanding the behavior of the function and its graph.

Graphical Interpretation

- The line crosses the x-axis at (5, 0).
- The y-intercept is at (0, -15).
- The slope of 3 indicates the line rises 3 units vertically for every 1 unit horizontally.

Implications in Real-World Contexts

Suppose this function models a specific scenario, such as profit over units sold, with the equation indicating profit per unit minus fixed costs. The zero point would represent the break-even point, where profit transitions from negative to positive.

Extensions and Broader Applications

While the analysis here is straightforward, understanding zeros in more complex functions involves additional techniques.

Zeros in Polynomial and Nonlinear Functions

- For higher-degree polynomials, zeros are found using factoring, synthetic division, or numerical methods.
- For transcendental functions, zeros may require iterative methods or graphing tools.

Relevance to Calculus and Analysis

Zeros are used in:
- Finding critical points where derivatives are zero.
- Determining intervals of increase and decrease.
- Solving optimization problems.

Example Applications

- Engineering: determining load points where systems fail.
- Economics: identifying break-even points.
- Physics: solving for equilibrium states.

Summary and Conclusions

The function f(x) = 3x - 15 has a single zero at x = 5. This point signifies where the function's graph intersects the x-axis, and it carries significant implications in both pure and applied mathematics.

Key takeaways:
- The zero of a function is found by setting f(x) = 0 and solving.
- For f(x) = 3x - 15, the zero is x = 5.
- The zero point is (5, 0), marking the function's x-intercept.
- Understanding zeros aids in graphing, solving equations, and interpreting real-world scenarios.

In conclusion, the investigation confirms that the zero of the linear function f(x) = 3x - 15 is at x = 5, exemplifying the straightforward nature of linear equations and their zeros. This foundational concept underpins more complex analyses and applications across scientific disciplines, reinforcing the importance of mastering such fundamental algebraic techniques.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Larson, R., & Edwards, B. H. (2012). Precalculus with Limits. Cengage Learning.
- Khan Academy. (n.d.). "Finding the zero of a function." Retrieved from https://www.khanacademy.org/math/algebra

Author's Note: For further exploration, students and researchers are encouraged to examine how zeros behave in nonlinear functions and the methods used to approximate zeros in complex scenarios.

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