Which Graph Represents Z = (3 + 4i) (2 + I)

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Understanding complex numbers and their graphical representations is an intriguing aspect of mathematics that combines algebra, geometry, and analysis. When asked to determine which graph corresponds to a given complex number expression such as Z = (3 + 4i) (2 + i), it involves both performing the multiplication and interpreting the result geometrically. This article provides a comprehensive guide to understanding how to approach this problem, the significance of the complex number operations involved, and how to accurately identify its representation on the complex plane.

Introduction to Complex Numbers and Their Geometric Representation

What Are Complex Numbers?

Complex numbers are numbers that have both a real part and an imaginary part, expressed in the form:

$$z = a + bi$$

where:
- a is the real part,
- b is the imaginary part,
- i is the imaginary unit, satisfying $i^2 = -1$.

For example, in the expression $3 + 4i$, the real part is 3, and the imaginary part is 4.

Graphical Representation: The Complex Plane

Complex numbers can be visualized geometrically on the complex plane, also called the Argand plane. Here:
- The horizontal axis represents the real component.
- The vertical axis represents the imaginary component.

Any complex number $a + bi$ corresponds to a point (a, b) on this plane. The magnitude (or modulus) of the complex number is given by:

$$|z| = \sqrt{a^2 + b^2}$$

and the argument (or angle) \(\theta \) is:

\[\theta = \arctan \left(\frac{b}{a} \right) \]

This geometric interpretation allows us to perform operations like multiplication and division through transformations involving magnitudes and angles.

Step-by-Step Solution to Find Z = (3 + 4i)(2 + i)

Step 1: Expand the Expression

To find the value of \(Z \), we need to multiply the two complex numbers:

\[
Z = (3 + 4i)(2 + i)
\]

Applying distributive property (FOIL method):

\[
Z = 3 \times 2 + 3 \times i + 4i \times 2 + 4i \times i
\]

Calculating each term:

- \(3 \times 2 = 6 \)
- \(3 \times i = 3i \)
- \(4i \times 2 = 8i \)
- \(4i \times i = 4i^2 \)

Recall that \(i^2 = -1 \), so:

\[
4i^2 = 4 \times (-1) = -4
\]

Putting everything together:

\[
Z = 6 + 3i + 8i - 4
\]

Combine like terms:

\[

$$
Z = (6 - 4) + (3i + 8i) = 2 + 11i
$$

Thus, the complex number Z simplifies to:

$$
Z = 2 + 11i
$$

Step 2: Interpret the Result Geometrically

This result indicates that the point representing Z on the complex plane is at:

- Real part: 2
- Imaginary part: 11

This point can be plotted at the coordinates $(2, 11)$.

Understanding the Geometric Representation of Z = 2 + 11i

Plotting the Point

On the complex plane:
- The x-coordinate (horizontal axis) corresponds to the real part, which is 2.
- The y-coordinate (vertical axis) corresponds to the imaginary part, which is 11.

The point $(2, 11)$ is located 2 units to the right of the origin and 11 units above it.

Calculating Magnitude and Argument

To better understand the position of Z, we can compute its magnitude and argument:

- Magnitude:

$$
|Z| = \sqrt{2^2 + 11^2} = \sqrt{4 + 121} = \sqrt{125} \approx 11.18
$$

- Argument:

$$
\theta = \arctan \left(\frac{11}{2} \right) \approx \arctan(5.5) \approx 79.63^\circ
$$

This indicates that Z is located approximately 11.18 units from the origin at an angle close to 80° from the positive real axis.

Understanding the Graphs and Their Features

Types of Graphs for Complex Numbers

When visualizing complex numbers, various types of graphs can represent their properties:

- Point Plot: Shows the exact location of the complex number as a point on the plane.
- Vector (Arrow): From the origin to the point, illustrating magnitude and direction.
- Transformations: Graphs demonstrating multiplication, such as scaling and rotation.

Features to Recognize in the Graphs

When selecting the correct graph based on the complex number Z, look for:

- The point located at $(2, 11)$.
- An arrow from the origin pointing to $(2, 11)$.
- The length of the arrow approximately 11.18 units.
- The angle with respect to the positive real axis roughly 80°.

Graphs that depict these features accurately are the correct representations.

How to Identify the Correct Graph

Step-by-Step Approach

To determine which graph represents Z:

1. Locate the point $(2, 11)$ on the complex plane.
2. Check the position relative to the axes:
- Is it 2 units along the real axis?
- Is it 11 units along the imaginary axis?

3. Verify the magnitude:
- Is the length of the arrow from the origin approximately 11.18 units?
4. Verify the angle:
- Is the angle from the positive real axis close to 80°?
5. Compare with the provided options:
- Eliminate graphs that do not match these criteria.
- Focus on the one accurately showing the point or vector at $(2, 11)$.

Common Mistakes to Avoid

- Misreading the axes or scale.
- Confusing the point's location (e.g., plotting at $(11, 2)$ instead of $(2, 11)$).
- Choosing a graph that shows the wrong magnitude or angle.
- Ignoring the directionality if vectors are involved.

Additional Insights: Multiplication of Complex Numbers as Rotation and Scaling

Complex multiplication combines two transformations:
- Scaling: The magnitude of the product is the product of the magnitudes.
- Rotation: The argument (angle) of the product is the sum of the arguments.

In our case:

$$
|3 + 4i| = \sqrt{3^2 + 4^2} = 5
$$
$$
|2 + i| = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5} \approx 2.236
$$
$$
|Z| = |3 + 4i| \times |2 + i| = 5 \times 2.236 \approx 11.18
$$

And the argument:

$$
\arg(3 + 4i) = \arctan \left(\frac{4}{3} \right) \approx 53.13^\circ
$$
$$
\arg(2 + i) = \arctan \left(\frac{1}{2} \right) \approx 26.57^\circ
$$
$$
\arg(Z) = 53.13^\circ + 26.57^\circ \approx 79.70^\circ
$$

This matches the earlier calculation, confirming the geometric interpretation.

Conclusion: Which Graph Represents Z = (3 + 4i)(2 + i)?

The correct graph will depict the point at \((2, 11) \) on the complex plane, with an arrow from the origin to this point, approximately 11.18 units long, and at an angle close to 80° from the positive real axis. Recognizing these features ensures accurate identification of the graph corresponding to \(Z \).

By understanding the multiplication of complex numbers as a combination of scaling and rotation, you gain a deeper appreciation of their geometric representations. Whether for academic purposes or practical applications like signal processing and physics, visualizing complex numbers enhances comprehension and problem-solving skills.

Summary of Key Points:
- The simplified value of \(Z = (3 + 4i)(2 + i) \) is \(2 + 11i \).
- The point \((2, 11) \) on the complex plane

Frequently Asked Questions

What is the main purpose of representing complex numbers on a graph?

Representing complex numbers on a graph helps visualize their real and imaginary components, making it easier to understand their magnitude, phase, and operations like addition or multiplication visually.

How do you interpret the graph of Z = (3 + 4i) (2 + i)?

The graph of Z represents the point corresponding to the product of the two complex numbers, showing how multiplication affects magnitude and angle in the complex plane.

What is the process to find the graphical representation of Z = (3 + 4i)(2 + i)?

First, multiply the complex numbers algebraically to find the resulting complex number, then plot the resulting point on the complex plane with its real and imaginary parts as coordinates.

How do you multiply two complex numbers like (3 + 4i) and (2 + i)?

Use the distributive property: (3 + 4i)(2 + i) = 32 + 3i + 4i2 + 4ii = 6 + 3i + 8i + 4i^2. Since i^2 = -1, the result simplifies to 6 + 11i - 4 = 2 + 11i.

What is the magnitude and argument of the product Z = (3 + 4i)(2 + i)?

First, compute the product to get Z = 2 + 11i. The magnitude is √(2^2 + 11^2) = √(4 + 121) = √125 ≈ 11.18, and the argument is arctangent(11/2) ≈ 79.7 degrees.

Which graph best illustrates the product of (3 + 4i) and (2 + i)?

The graph should show a point at (2, 11) on the complex plane, representing the real part 2 and imaginary part 11, reflecting the multiplication result.

How can understanding the graph of complex multiplication aid in visualizing complex functions?

It helps in understanding how multiplication affects magnitude and phase, visualizing transformations like rotations and scaling in the complex plane, which is crucial in complex analysis and signal processing.

Additional Resources

Which Graph Represents Z = (3 + 4i) (2 + i)

Understanding complex numbers and their geometric representations is a fundamental aspect of advanced mathematics, particularly in fields like engineering, physics, and computer science. When asked to find the graph representing a product of two complex numbers, such as Z = (3 + 4i) (2 + i), it is essential to analyze the algebraic operation and translate it into a visual form. This article explores the process of determining the correct graphical representation of this complex multiplication, providing a comprehensive breakdown of the concepts, methods, and features involved.

Introduction to Complex Numbers and Their Graphical Representation

Complex numbers are numbers of the form a + bi, where a and b are real numbers, and i is the imaginary unit satisfying $i^2 = -1$. These numbers can be represented graphically on the complex plane, where the horizontal axis (the real axis) corresponds to the real part, and the vertical axis (the imaginary axis) corresponds to the imaginary part.

Key features of complex numbers in the complex plane:
- Real part (a): The x-coordinate.
- Imaginary part (b): The y-coordinate.
- Magnitude (modulus): The distance from the origin, calculated as $|z| = \sqrt{(a^2 + b^2)}$.
- Argument (angle): The angle θ that the line from the origin to the point makes with the positive real axis, measured counter-clockwise.

Complex multiplication involves combining these properties, often expressed in polar form, which simplifies understanding rotations and scalings.

Understanding the Given Complex Numbers

Before analyzing the product, it's crucial to understand each component:

- First complex number: 3 + 4i
- Real part: 3
- Imaginary part: 4
- Magnitude: $|3 + 4i| = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$
- Argument: $\theta_1 = \text{arctangent}(4/3) \approx 53.13°$

- Second complex number: 2 + i
- Real part: 2
- Imaginary part: 1
- Magnitude: $|2 + i| = \sqrt{(2^2 + 1^2)} = \sqrt{(4 + 1)} = \sqrt{5} \approx 2.236$
- Argument: $\theta_2 = \text{arctangent}(1/2) \approx 26.57°$

These properties help facilitate their multiplication in polar form, where the magnitude of the product is the product of the magnitudes, and the argument of the product is the sum of the arguments.

Calculating the Product Z = (3 + 4i) (2 + i)

The algebraic method involves multiplying out the two complex numbers:

$Z = (3 + 4i)(2 + i)$

Applying distributive property:

$Z = 3\times2 + 3\times i + 4i\times2 + 4i\times i$
$Z = 6 + 3i + 8i + 4i^2$

Recall that $i^2 = -1$:

$Z = 6 + (3i + 8i) + 4\times(-1)$
$Z = 6 + 11i - 4$
$Z = (6 - 4) + 11i$
$Z = 2 + 11i$

Result: The product $Z = 2 + 11i$

This algebraic calculation shows that Z is a complex number with:

- Real part: 2
- Imaginary part: 11

Graphical Representation of Z = 2 + 11i

Visualizing Z involves plotting the point (2, 11) on the complex plane:

- The point is located 2 units along the real axis (to the right of the origin).
- It is 11 units along the imaginary axis (above the real axis).

This position indicates a point far above the real axis, corresponding to a large imaginary component.

Features of this point:
- Magnitude: $|Z| = \sqrt{2^2 + 11^2} = \sqrt{4 + 121} = \sqrt{125} \approx 11.18$
- Argument: $\theta = \text{arctangent}(11/2) \approx 79.48°$, which indicates a steep angle close to perpendicular to the real axis.

Understanding the Geometric Meaning of the Multiplication

Complex multiplication can be interpreted geometrically as a combination of scaling and rotation:

- Scaling: The magnitude of the product is the product of the magnitudes of the factors.
- Rotation: The argument of the product is the sum of the arguments of the factors.

For the given numbers:

- Magnitude of the first: 5
- Magnitude of the second: $\sqrt{5} \approx 2.236$
- Magnitude of the product: $5 \times 2.236 \approx 11.18$, matching the calculated

magnitude of Z.

- Argument of the first: ≈ 53.13°
- Argument of the second: ≈ 26.57°
- Sum: 53.13° + 26.57° ≈ 79.70°, very close to the calculated argument of Z (~79.48°)

This confirms the geometric interpretation: multiplying two complex numbers results in a new point that is scaled by the product of their magnitudes and rotated by the sum of their angles.

Graphical Features of the Correct Representation

When selecting the correct graph from multiple options, consider these features:

Position of the Point
- Located at (2, 11): 2 units along the real axis, 11 units along the imaginary axis.
- Should be in the upper-right quadrant (Quadrant I), since both real and imaginary parts are positive.

Magnitude and Angle
- The point should have a magnitude close to 11.18.
- The angle should be approximately 79.48°, indicating a steep upward tilt from the positive real axis.

Relative to Other Points
- The point should be consistent with the multiplication properties, not just a random point.
- The size and angle should match the calculations based on the original numbers.

Visual Clarity
- The graph should clearly show the point at the correct coordinates.
- Axes should be labeled or scaled appropriately to reflect the magnitude.

Comparing Options
- Graphs that depict points in the correct quadrant with the accurate coordinates are likely correct.
- Graphs with points far from (2, 11) or in other quadrants are incorrect.
- Graphs displaying the magnitude and angle consistent with the calculations are preferred.

Conclusion: Which Graph Represents Z = (3 + 4i)(2 + i)?

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The algebraic and geometric analyses converge on the conclusion that the
graph representing Z should display a point at approximately (2, 11). This
point corresponds to the complex number resulting from the multiplication of
(3 + 4i) and (2 + i). Its magnitude (~11.18) and argument (~79.48°) confirm
the geometric interpretation that the product involves scaling and rotating
the original numbers.

In summary:
- The correct graph should show a point in the upper-right quadrant at (2,
11).
- The point’s distance from the origin should be close to 11.18.
- The angle with respect to the positive real axis should be near 79.48°.
- The graph should accurately reflect the algebraic and geometric properties
discussed.

By understanding these features, one can confidently identify the graph that
correctly represents Z = (3 + 4i)(2 + i). This process exemplifies the power
of combining algebraic calculations with geometric intuition to interpret
complex numbers visually.
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Which Graph Represents Z 3 4i 2 I

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