1/1*3+1/3*5+.......+1/47*49 NEED HELP ASAP!!!!!!!!!!!!!!!

1/13+1/35+.......+1/4749 NEED HELP ASAP!!!!!!!!!!!!!!!

If you're working on a mathematics problem involving the sum of the series 1/13 + 1/35 + ... + 1/4749, you're not alone. Many students and math enthusiasts encounter series summation problems that seem complex at first glance but can be simplified through understanding patterns, algebraic manipulation, and known techniques. In this article, we'll explore how to evaluate this particular series step by step, making it easier to understand and solve. Whether you're preparing for exams or just want to improve your series summation skills, this comprehensive guide will help you grasp the concepts thoroughly.

Understanding the Series: 1/13 + 1/35 + ... + 1/4749

Breaking Down the Series

The series in question is:

$$ S = \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{47 \times 49} $$

This sequence involves reciprocals of the product of two odd numbers, starting from 1 and 3, then 3 and 5, and so on, up to 47 and 49.

Identifying the Pattern

Notice that:
- The denominators are products of two consecutive odd numbers.
- The first term is $\frac{1}{1 \times 3}$.
- The last term is $\frac{1}{47 \times 49}$.
- The numerator in each term is 1, and the denominators are formed as consecutive odd numbers multiplied together.

The key is to find a way to express each term in a form that simplifies the sum.

Techniques for Summing the Series

Partial Fraction Decomposition

Partial fraction decomposition is a common technique in summing series involving rational functions. It involves expressing a fraction as a sum of simpler fractions that can cancel out when summed over a sequence.

For the general term:

$$ T_k = \frac{1}{(2k-1)(2k+1)} $$

where k runs from 1 to 23 (since the last term corresponds to $2 \times 23 - 1 = 47$).

Step-by-Step Solution

Expressing the General Term

Let's write the general term explicitly:

$$ T_k = \frac{1}{(2k - 1)(2k + 1)} $$

Our goal is to decompose T_k into simpler fractions.

Partial Fraction Decomposition

Set:

$$ \frac{1}{(2k - 1)(2k + 1)} = \frac{A}{2k - 1} + \frac{B}{2k + 1} $$

Multiply both sides by $(2k - 1)(2k + 1)$:

$$ 1 = A(2k + 1) + B(2k - 1) $$

Now, solve for A and B:

$$ 1 = 2A k + A + 2B k - B $$

Group like terms:

$$ 1 = (2A + 2B)k + (A - B) $$

Since the left side is constant (1), the coefficient of k on the right must be zero, and the constant term must be 1:

\[
\begin{cases}

2A + 2B = 0 \\
A - B = 1
\end{cases}
\]

Solve the system:

1. From the first equation:

\[2A + 2B = 0 \Rightarrow A + B = 0 \Rightarrow B = -A \]

2. Substitute into the second equation:

\[A - (-A) = 1 \Rightarrow A + A = 1 \Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2} \]

Then:

\[B = -A = -\frac{1}{2} \]

So, the decomposition is:

\[T_k = \frac{1/2}{2k - 1} - \frac{1/2}{2k + 1} \]

or

\[T_k = \frac{1}{2} \left(\frac{1}{2k - 1} - \frac{1}{2k + 1} \right) \]

Summing the Series

Expressing the Series as a Telescoping Sum

The original sum:

\[S = \sum_{k=1}^{23} T_k = \frac{1}{2} \sum_{k=1}^{23} \left(\frac{1}{2k - 1} - \frac{1}{2k + 1} \right) \]

Break into two sums:

\[S = \frac{1}{2} \left(\sum_{k=1}^{23} \frac{1}{2k - 1} - \sum_{k=1}^{23} \frac{1}{2k + 1} \right) \]

Let's write out the terms explicitly:

- First sum:

\[\sum_{k=1}^{23} \frac{1}{2k - 1} = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \cdots +

\frac{1}{45} + \frac{1}{47} \]

- Second sum:

\[\sum_{k=1}^{23} \frac{1}{2k + 1} = \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{47} + \frac{1}{49} \]

Notice that many terms cancel when we subtract the sums:

\[S = \frac{1}{2} \left(\left(1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{45} + \frac{1}{47}\right) - \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{47} + \frac{1}{49}\right) \right) \]

After cancellation:

\[S = \frac{1}{2} \left(1 - \frac{1}{49} \right) \]

because all intermediate terms \(\frac{1}{3}, \frac{1}{5}, \dots, \frac{1}{47} \) cancel out.

Final Calculation and Result

Calculate:

\[1 - \frac{1}{49} = \frac{49}{49} - \frac{1}{49} = \frac{48}{49} \]

Therefore,

\[S = \frac{1}{2} \times \frac{48}{49} = \frac{48}{98} \]

Simplify numerator and denominator:

\[\frac{48}{98} = \frac{24}{49} \]

Final answer:

\[
\boxed{
\frac{24}{49}
}
\]

Summary and Key Takeaways

- Expressing the general term using partial fractions simplifies the summation process.
- Recognizing telescoping sums can drastically reduce the complexity of series calculations.
- Canceling common terms in telescoping series leaves only the first and last terms, making the sum straightforward to evaluate.
- The sum of the series 1/13 + 1/35 + ... + 1/4749 is $\frac{24}{49}$.

Additional Tips for Series Summation

Practice Recognizing Patterns

Many series involve patterns that can be exploited through algebraic manipulation. Always look for opportunities to rewrite terms in a form conducive to telescoping or partial fractions.

Verify with Smaller Examples

Before tackling large series, test your approach on smaller series to ensure your method works.

Use Known Series and Identities

Familiarity with harmonic series, telescoping series, and partial fraction identities can accelerate problem-solving.

If you're struggling with series summations or need further help, consider consulting math textbooks, online tutorials, or seeking assistance from teachers or tutors. Mastery of these techniques will significantly improve your problem-solving skills and confidence in tackling complex series.

Remember: Practice makes perfect, and breaking down complex series into manageable parts is the key to solving them efficiently.

Frequently Asked Questions

What is the sum of the series 1/13 + 1/35 + ... + 1/4749?

The sum of the series is 1/2.

How can I evaluate the sum 1/13 + 1/35 + ... + 1/4749?

You can use partial fraction decomposition to rewrite each term and then find a telescoping pattern to simplify the sum.

Is there a general formula for the sum of the series 1/(2k-1)(2k+1) for k=1 to n?

Yes, the sum from k=1 to n of 1/[(2k-1)(2k+1)] equals 1/2 minus 1/(2n+1).

How do I derive the sum of 1/[(2k-1)(2k+1)]?

Use partial fractions: 1/[(2k-1)(2k+1)] = 1/2 [1/(2k-1) - 1/(2k+1)]. Then, sum over k and observe telescoping cancellation.

What is the value of the sum 1/13 + 1/35 + ... + 1/4749?

The value is 1/2 because summing the telescoping series yields that result for n=24.

Can I verify the sum 1/2 numerically?

Yes, by calculating each term and adding them up, you'll find the sum approaches 0.5, confirming the result.

Does the pattern of the series help in simplifying the sum?

Yes, recognizing the telescoping pattern through partial fractions simplifies the sum significantly.

What is the importance of recognizing telescoping series in problems like this?

Recognizing telescoping series allows for quick simplification and avoids tedious calculations by cancelling out intermediate terms.

Are there online tools or calculators to evaluate such series?

Yes, symbolic mathematics tools like WolframAlpha or mathematical software can evaluate these series directly when provided the general term.

Additional Resources

Mathematical Series Evaluation: An In-Depth Analysis of the Sum \(\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \ldots + \frac{1}{47 \times 49} \)

Understanding and simplifying complex series is a cornerstone of mathematical problem-solving. The series:

$$
\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \ldots + \frac{1}{47 \times 49}
$$

may appear daunting at first glance, but with a systematic approach, it reveals itself as a manageable and elegant sum. This article provides an extensive exploration of this series, breaking down each step as if analyzing a high-performance product, ensuring clarity and depth for learners and enthusiasts alike.

Understanding the Series: Structure and Pattern

Series Breakdown

At first glance, the series involves a sequence of fractions where the denominators are products of consecutive odd integers:

$$
\frac{1}{1 \times 3}, \quad \frac{1}{3 \times 5}, \quad \frac{1}{5 \times 7}, \quad \ldots, \quad \frac{1}{47 \times 49}
$$

This pattern indicates that the numerator is always 1, while the denominator consists of the product of two odd numbers that increase by 2 each time:

- First term: 1×3
- Second term: 3×5
- Third term: 5×7
- ...
- Last term: 47×49

The sequence of the first factors is: 1, 3, 5, ..., 47

The sequence of the second factors is: 3, 5, 7, ..., 49

Indexing the Series

To analyze this sum rigorously, it helps to define a general term:

$$
T_k = \frac{1}{(2k - 1)(2k + 1)}
$$

where k ranges over the integers starting from 1:

- When $k=1$, numerator: $2(1)-1=1$, denominator: $2(1)+1=3$, matching the first term.
- When $k=23$, numerator: $2(23)-1=45$, denominator: $2(23)+1=47$, matching the last term.

Since the last term involves 47 and 49, we note that:

$$
2k - 1 = 47 \Rightarrow k = 24
$$

Thus, the series runs from $k=1$ to $k=24$:

$$
\sum_{k=1}^{24} \frac{1}{(2k - 1)(2k + 1)}
$$

Methodical Approach: Partial Fraction Decomposition

Why Partial Fractions?

Partial fraction decomposition is a powerful technique used to simplify complex rational expressions into sums of simpler fractions, making summation more straightforward. For this series, it enables us to rewrite each term into a form that reveals telescoping properties.

Decomposing a Typical Term

Let's analyze a generic term:

$$
T_k = \frac{1}{(2k - 1)(2k + 1)}
$$

Applying partial fractions:

$$
\frac{1}{(2k - 1)(2k + 1)} = \frac{A}{2k - 1} + \frac{B}{2k + 1}
$$

Multiplying through by the denominator:

$$
1 = A(2k + 1) + B(2k - 1)
$$

Expanding:

$$
1 = 2A k + A + 2B k - B
$$

Grouping like terms:

$$
1 = (2A + 2B)k + (A - B)
$$

Since the left side is constant, the coefficients of k must be zero:

$$
2A + 2B = 0 \Rightarrow A + B = 0
$$

And the constant term:

$$
A - B = 1
$$

From $A + B = 0$, we get $B = -A$. Substituting into the second:

$$
A - (-A) = 1 \Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}
$$

And $B = -A = -\frac{1}{2}$.

Therefore:

$$
\frac{1}{(2k - 1)(2k + 1)} = \frac{1/2}{2k - 1} - \frac{1/2}{2k + 1}
$$

Or simplified:

$$
T_k = \frac{1}{2} \left(\frac{1}{2k - 1} - \frac{1}{2k + 1} \right)
$$

This decomposition is the key to telescoping the series.

Summation and Telescoping Series

Expressing the Entire Series

Now, the sum S becomes:

$$
S = \sum_{k=1}^{24} T_k = \frac{1}{2} \sum_{k=1}^{24} \left(\frac{1}{2k - 1} - \frac{1}{2k + 1} \right)
$$

Expanding the sum:

$$
S = \frac{1}{2} \left[\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \ldots + \left(\frac{1}{47} - \frac{1}{49}\right) \right]
$$

Note how the terms cancel in a telescoping fashion:

$$
S = \frac{1}{2} \left(\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{47} - \frac{1}{49} \right)
$$

All intermediate terms cancel out, leaving only the first positive term and the last negative term:

$$
S = \frac{1}{2} \left(1 - \frac{1}{49} \right)
$$

Final Calculation and Result

Calculating the Sum

Simplify:

$$
S = \frac{1}{2} \times \left(1 - \frac{1}{49} \right) = \frac{1}{2} \times \frac{49 - 1}{49} =
$$

\frac{1}{2} \times \frac{48}{49}
\]

Multiplying:

\[
S = \frac{48}{98} = \frac{24}{49}
\]

Therefore, the sum of the series:

\[
\boxed{
\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \ldots + \frac{1}{47 \times 49} = \frac{24}{49}
\]

Summary and Takeaways

- Pattern Recognition: Identifying the structure of the series as involving products of consecutive odd numbers was crucial.

- Partial Fraction Decomposition: Transforming each term into a telescoping form simplifies the sum dramatically.

- Telescoping Series: Recognizing telescoping patterns allows for cancellation of intermediate terms, reducing complex sums to simple differences.

- Final Result: The sum evaluates neatly to \(\frac{24}{49} \), approximately 0.4898, which is a neat, rational value—highlighting the elegance of mathematical series.

Additional Tips for Series Evaluation

- Always look for patterns in denominators or numerators.
- Consider substitution and index shifts to clarify the series structure.
- Use partial fractions to decompose complex rational expressions.
- Recognize telescoping sums early to simplify calculations.
- Verify the limits of summation carefully, especially when dealing with sequences involving quadratic or higher-order terms.

This detailed exploration of the series \(\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \ldots +

\frac{1}{47 \times 49} \) demonstrates how systematic approaches—like algebraic decomposition and recognizing telescoping patterns—can unlock elegant solutions. Whether you're studying series for academic purposes or seeking their applications in advanced mathematics, mastering these techniques enhances both understanding and problem-solving efficiency.

1 1 3 1 3 5 1 47 49 Need Help Asap

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