A Line Passes Through The Point (-3, 1) And Has A Slope Of 6

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Understanding the fundamental principles of straight lines in coordinate geometry is essential for students, teachers, and professionals working in fields like mathematics, engineering, and physics. When analyzing a line that passes through a specific point with a given slope, such as the point (-3, 1) with a slope of 6, it becomes crucial to understand how to derive the equation of the line, interpret its properties, and apply it to real-world problems. This comprehensive guide aims to provide an in-depth explanation of this particular line, covering its equation derivation, graphical representation, and various applications.

Understanding the Basics of Line Equations

Before diving into the specifics of the line passing through (-3, 1) with a slope of 6, it's important to review the fundamental concepts related to straight lines in the coordinate plane.

The Slope-Intercept Form

The slope-intercept form of a line's equation is one of the most commonly used in algebra:

$$ y = mx + b $$

Where:
- y is the dependent variable
- x is the independent variable
- m is the slope of the line
- b is the y-intercept, the point where the line crosses the y-axis

Knowing the slope m and a point (x_1, y_1) on the line allows us to determine the line's equation directly.

The Point-Slope Form

Alternatively, the point-slope form is particularly useful when a point on the line and the slope are known:

$$ y - y_1 = m(x - x_1) $$

This form emphasizes the relationship between a specific point and the slope, making it straightforward to derive the line's equation.

Deriving the Equation of the Line

Given the problem statement: the line passes through the point $(-3, 1)$ and has a slope of 6, we can use the point-slope form to find its equation.

Step-by-Step Derivation

1. Identify the known values:
- Point $(x_1, y_1) = (-3, 1)$
- Slope $m = 6$

2. Apply the point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 6(x - (-3))$$

3. Simplify the expression:

$$y - 1 = 6(x + 3)$$

4. Distribute the slope:

$$y - 1 = 6x + 18$$

5. Solve for y to convert to slope-intercept form:

$$y = 6x + 19$$

Result: The equation of the line passing through $(-3, 1)$ with a slope of 6 is:

$$\boxed{ y = 6x + 19 }$$

This linear equation fully describes the line's behavior in the coordinate plane.

Graphing the Line

Visual representation provides an intuitive understanding of the line's properties and its position relative to the coordinate axes.

Plotting Key Points

To graph the line $y = 6x + 19$:

- Point (-3, 1): Given in the problem statement, verify it lies on the line:

$$y = 6(-3) + 19 = -18 + 19 = 1$$

which confirms the point is on the line.

- Y-intercept (0, 19): When $x = 0$,

$$y = 6(0) + 19 = 19$$

- Additional Point: Choose $x = 1$:

$$y = 6(1) + 19 = 6 + 19 = 25$$

Plotting these points provides a clear depiction of the line's orientation.

Drawing the Graph

Use graph paper or graphing software to plot the points:

- (-3, 1)
- (0, 19)
- (1, 25)

Connect these points with a straight line, extending it across the coordinate plane to visualize the entire line.

Analyzing the Line's Properties

Understanding the properties of this line aids in grasping its behavior and applications.

Slope Interpretation

- The slope $m = 6$ indicates the line rises 6 units vertically for every 1 unit it moves horizontally.
- The large positive slope means the line is steep and inclines upward from left to right.

Y-Intercept

- The y-intercept $b = 19$ is the point where the line crosses the y-axis.
- This high y-intercept indicates the line starts at a high point on the y-axis when $x = 0$.

Line's Equation Summary

Property	Description
Equation	$y = 6x + 19$
Slope m	6
Y-intercept b	19
Passing through	(-3, 1), (0, 19), (1, 25)

Applications of the Line in Real-World Contexts

Linear equations are fundamental in modeling relationships in various fields. Understanding lines like $y = 6x + 19$ enables practical applications.

1. Physics: Velocity and Displacement

- In kinematics, the equation of motion often follows a linear pattern when acceleration is constant.
- For example, if y represents displacement and x time, a slope of 6 could signify a constant velocity of 6 units per time interval.

2. Economics: Cost and Revenue Analysis

- Linear functions model cost functions where the slope indicates the rate of increase in costs with production.
- If y is total cost and x is units produced, a slope of 6 could represent a cost increase of 6 dollars per unit.

3. Engineering: Material Stress Analysis

- In material science, stress-strain relationships often exhibit linear behavior within elastic limits.
- The slope might represent the modulus of elasticity, indicating how much

stress increases with strain.

4. Business and Marketing

- Revenue models can be linear, where the slope indicates profit per unit sold.
- The intercept represents fixed costs or baseline revenue.

Solving Problems Involving the Line

Understanding how to manipulate and solve equations related to this line is vital in various problem-solving scenarios.

Finding Coordinates of Specific Points

- To find y for a given x, substitute x into the equation $y = 6x + 19$.
- Conversely, to find x for a given y, rearrange:

$$y = 6x + 19 \Rightarrow x = \frac{y - 19}{6}$$

Determining the Intersection with Other Lines

- To find where this line intersects with another line, set their equations equal and solve for x and y.

Parallel and Perpendicular Lines

- Lines parallel to this line share the same slope $m = 6$.
- Lines perpendicular have slopes that are negative reciprocals, i.e., $-\frac{1}{6}$.

Extensions and Advanced Topics

For those interested in deeper exploration, several advanced topics stem from the basic understanding of this line.

1. Line Equations in Different Forms

- Standard form: $Ax + By + C = 0$

Converting $y = 6x + 19$:

$$6x - y + 19 = 0$$

- Two-point form: Using points (x_1, y_1) and (x_2, y_2):

$$y - y_1 = m(x - x_1)$$

- General form: Rearranged standard form to include all terms.

2. Distance from a Point to the Line

- To find the shortest distance from a point to this line, apply the point-to-line distance formula.

3. Line Segment and Ray Concepts

- Extending the line to finite segments or rays involves defining boundaries and directionality.

Conclusion

In summary, the line passing through the point (-3, 1) with a slope of 6 is accurately represented by the equation $y = 6x + 19$. This equation encapsulates the geometric and algebraic properties of the line, allowing for visualization, analysis, and application across various disciplines. Whether used in physics to model motion, in economics to analyze costs, or in engineering for material analysis, understanding how to derive and interpret the equation of such a line is a fundamental skill in mathematics. Mastery of these concepts paves

Frequently Asked Questions

How do you find the equation of a line passing through the point (-3, 1) with a slope of 6?

Use the point-slope form: $y - y_1 = m(x - x_1)$. Substituting, $y - 1 = 6(x + 3)$.

Simplify to get y = 6x + 19.

What is the slope-intercept form of the line passing through (-3, 1) with slope 6?

The equation is y = 6x + 19.

How can you verify that a point lies on the line with slope 6 passing through (-3, 1)?

Substitute the point's coordinates into the line's equation. For (-3, 1), check if y = 6x + 19 gives 1 = 6(-3) + 19, which simplifies to 1 = -18 + 19, confirming the point lies on the line.

What is the significance of the point (-3, 1) in the line's equation?

It's the point through which the line passes, serving as a reference point for deriving the line's equation.

Can the line passing through (-3, 1) with slope 6 be written in standard form?

Yes. Starting from y = 6x + 19, rearranged as 6x - y + 19 = 0.

How does the slope of 6 influence the steepness of the line?

A slope of 6 indicates a steep line, rising 6 units vertically for every 1 unit horizontally.

If you wanted to find another point on this line, how would you do it?

Choose any value for x and compute y using the equation y = 6x + 19. For example, if x=0, y=19.

What are some real-world scenarios where a line with slope 6 passing through a specific point might be applicable?

It could model rapid growth rates in economics, high-speed projectiles in physics, or steep incline gradients in engineering, all passing through a specific measurement point.

Additional Resources

Line Equation Through (-3, 1) With Slope 6: An In-Depth Exploration

Understanding the geometric and algebraic properties of lines in the coordinate plane is fundamental to numerous fields, from mathematics and engineering to computer graphics and data analysis. In this detailed review, we will explore the specific case of a line passing through the point $(-3, 1)$ with a slope of 6. This scenario exemplifies core principles of linear equations, offering insights into their derivation, properties, and applications. Let's dissect this topic with methodical precision.

Understanding the Basics: The Equation of a Line

Before delving into the specific line, it’s essential to review the foundational concepts underpinning linear equations in two dimensions.

The Slope-Intercept Form

The most common representation of a line is the slope-intercept form:

$$
y = mx + b
$$

where:
- m is the slope of the line,
- b is the y-intercept (the point where the line crosses the y-axis).

This form is particularly useful because it directly encodes the slope and the point where the line intersects the y-axis. However, when we are given a point and the slope, the more versatile form is the point-slope form.

The Point-Slope Form

The point-slope form is expressed as:

$$
y - y_1 = m(x - x_1)
$$

where:
- (x_1, y_1) is a point on the line,
- m is the slope.

This form is ideal when a point on the line and its slope are known, providing a straightforward way to derive the line's equation.

Deriving the Equation: From Point and Slope to Line Equation

Given our specific data:
- Point $(-3, 1)$,
- Slope $m = 6$,

we will derive the explicit equation of the line.

Step-by-Step Derivation

Step 1: Write the point-slope form:

$$
y - 1 = 6(x - (-3))
$$

which simplifies to:

$$
y - 1 = 6(x + 3)
$$

Step 2: Expand the right side:

$$
y - 1 = 6x + 18
$$

Step 3: Isolate y:

$$
y = 6x + 18 + 1
$$

$$
\boxed{y = 6x + 19}

\]

This is the slope-intercept form of the line passing through $(-3, 1)$ with a slope of 6.

Final Equation:

$$
\boxed{y = 6x + 19}
$$

Properties of the Line

Having established the equation, it’s vital to explore its properties and implications.

Slope Analysis

- The slope $m = 6$ indicates a steep, positively inclined line.
- For every unit increase in x, y increases by 6 units.
- The steepness signifies rapid growth; in real-world scenarios, this could represent phenomena like high growth rates, acceleration, or rapid change.

Y-Intercept

- The y-intercept occurs at the point $(0, 19)$.
- This is where the line crosses the y-axis, serving as a crucial reference point for graphing and understanding the line's position.

Point-Through Validation

- Confirm the line passes through $(-3, 1)$:

Substitute $x = -3$:

$$
y = 6(-3) + 19 = -18 + 19 = 1
$$

which matches the given point, confirming correctness.

Graphical Representation and Visualization

Visualizing lines is pivotal for comprehension. The line $y = 6x + 19$ can be plotted with key points and slope.

Key points for graphing:

- Y-intercept: (0, 19)
- Point at $x = -3$: (-3, 1), as given
- Additional points: For a more comprehensive graph, pick other x-values:

x	$y = 6x + 19$	Coordinates
-5	$6(-5)+19 = -30+19 = -11$	(-5, -11)
0	19	(0, 19)
2	$6(2)+19=12+19=31$	(2, 31)

Plotting these points confirms a straight, steep line crossing the y-axis at 19 and passing through $(-3, 1)$.

Graphing Tips:

- Use a graphing calculator or software for precision.
- Mark the y-intercept clearly.
- Draw the line through the known points.
- Extend in both directions for full visualization.

Applications and Significance

Understanding this specific line isn't merely an academic exercise; it has practical relevance across various disciplines.

Real-world Applications

- Physics: Represents constant acceleration or velocity scenarios.
- Economics: Models linear relationships such as cost functions or supply/demand curves.
- Engineering: Used in signal processing and system modeling where linear relationships exist.

- Data Science: Regression lines with given data points and slopes.

Mathematical Significance

- Serves as a fundamental example illustrating how point and slope determine a line.
- Demonstrates the transition from geometric intuition to algebraic form.
- Acts as a building block for understanding more complex concepts like linear transformations, systems of equations, and calculus derivatives.

Extension: Exploring Variations and Related Concepts

This exploration opens avenues for deeper inquiry.

1. Line Perpendicular to Our Line

- The slope of a perpendicular line is the negative reciprocal:

$$
m_{\perp} = -\frac{1}{6}
$$

- Equation passing through $(-3, 1)$:

$$
y - 1 = -\frac{1}{6}(x + 3)
$$

- Simplify for standard form:

$$
y = -\frac{1}{6}x - \frac{1}{6}\times 3 + 1 = -\frac{1}{6}x - \frac{1}{2} + 1 = -\frac{1}{6}x + \frac{1}{2}
$$

2. Parallel Lines

- Any line parallel to our line shares the same slope $m=6$.
- Passing through different points, these lines maintain the same steepness

but shift position.

3. Intersection with Other Lines

- The intersection point of our line with another can be found by solving the system of equations, a fundamental operation in analytic geometry.

Conclusion: The Significance of Precise Line Characterization

This detailed review underscores the importance of methodical derivation, property analysis, and visualization. The line passing through $(-3, 1)$ with a slope of 6 exemplifies core principles of linear equations, illustrating how point and slope uniquely determine a line, and how its properties influence broader applications.

Understanding such lines fosters proficiency in mathematical modeling, problem-solving, and graphical analysis, essential skills in academic and professional contexts. Whether used in theoretical mathematics or practical design, mastering the derivation and interpretation of lines like $y = 6x + 19$ is fundamental to advancing one's quantitative reasoning and analytical capabilities.

A Line Passes Through The Point 3 1 And Has A Slope Of 6

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