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Understanding how to find a function $F(x)$ given its second derivative, initial conditions, and specific values is a fundamental aspect of calculus. In this comprehensive guide, we will analyze the problem step-by-step to derive the explicit form of $F(x)$. The problem provides the second derivative of F, initial conditions, and a specific point where the function's value is known. Our goal is to determine the original function $F(x)$ that satisfies all these conditions.

Problem Breakdown and Objective

Given Data

- Second derivative: $F''(x) = 5 + \cos(x)$
- Initial value: $F(0) = -1$
- Value at $x = \frac{5}{2}$: $F(2.5) = 0$

Objective

Find the explicit form of the function $F(x)$ that satisfies the above differential equation and initial conditions.

Step 1: Integrate the Second Derivative to Find the First Derivative $F'(x)$

Why do we integrate?

Since $F''(x)$ is the second derivative, integrating it once yields the first derivative $F'(x)$, up to an integration constant.

Integration process

Given:

$$
F''(x) = 5 + \cos(x)
$$

Integrate both sides:

$$
F'(x) = \int (5 + \cos(x))\, dx
$$

Break down the integral:

$$
F'(x) = \int 5\, dx + \int \cos(x) \, dx
$$

$$
F'(x) = 5x + \sin(x) + C_1
$$

where C_1 is an integration constant.

Step 2: Integrate $F'(x)$ to Find $F(x)$

Integrate $F'(x)$ to obtain $F(x)$

$$
F(x) = \int F'(x) \, dx = \int (5x + \sin(x) + C_1) \, dx
$$

Break down the integral:

$$
F(x) = \int 5x \, dx + \int \sin(x) \, dx + \int C_1 \, dx
$$

Calculate each term:

1. $\int 5x \, dx = \frac{5x^2}{2}$
2. $\int \sin(x) \, dx = -\cos(x)$
3. $\int C_1 \, dx = C_1 x$

Thus:

$$
F(x) = \frac{5x^2}{2} - \cos(x) + C_1 x + C_2
$$

where C_2 is another integration constant.

Step 3: Use Initial Conditions to Determine Constants C_1 and C_2

Apply $F(0) = -1$

Substitute $x=0$:

$$
F(0) = \frac{5 \times 0^2}{2} - \cos(0) + C_1 \times 0 + C_2 = 0 - 1 + 0 + C_2
$$

$$
-1 = -1 + C_2 \Rightarrow C_2 = 0
$$

So, the function simplifies to:

$$
F(x) = \frac{5x^2}{2} - \cos(x) + C_1 x
$$

Apply $F(2.5) = 0$

Substitute $x = 2.5$:

$$
0 = \frac{5 \times (2.5)^2}{2} - \cos(2.5) + C_1 \times 2.5
$$

Calculate:

$$
(2.5)^2 = 6.25
$$

$$
\frac{5 \times 6.25}{2} = \frac{31.25}{2} = 15.625
$$

Now, evaluate $\cos(2.5)$ (radians). Using a calculator or approximation:

$$
\cos(2.5) \approx -0.8011
$$

Plug into the equation:

$$
0 = 15.625 - (-0.8011) + 2.5 C_1
$$

$$
0 = 15.625 + 0.8011 + 2.5 C_1
$$

$$
0 = 16.4261 + 2.5 C_1
$$

Solve for C_1:

$$
2.5 C_1 = -16.4261
$$
$$
C_1 = -\frac{16.4261}{2.5} \approx -6.57044
$$

Step 4: Write the Final Expression for $F(x)$

Putting all the pieces together, the explicit form of $F(x)$ is:
$$
F(x) = \frac{5x^2}{2} - \cos(x) - 6.57044 x
$$

Note: The constant C_1 is approximately -6.57044, but if higher precision is needed, use more decimal places or symbolic expressions.

Summary of the Solution

- Integrated the second derivative to obtain the first derivative: $F'(x) = 5x + \sin(x) + C_1$.

- Further integrated to find $F(x)$: $F(x) = \frac{5x^2}{2} - \cos(x) + C_1 x + C_2$.

- Applied initial conditions to determine $C_2 = 0$ and approximate $C_1 \approx -6.57044$.

- Final explicit function:
$$
\boxed{
F(x) = \frac{5x^2}{2} - \cos(x) - 6.57044 x
}
$$

Additional Insights and Applications

Why is understanding this process important?

- It demonstrates the process of solving second-order differential equations with initial conditions.
- Highlights the importance of integration constants and how initial/boundary conditions help determine them.
- Provides a foundation for more complex problems in physics, engineering, and applied mathematics.

Potential variations of the problem

- Changing initial conditions to see how the function adapts.
- Extending to non-homogeneous differential equations.
- Applying similar techniques to partial differential equations.

Conclusion

Finding \(F(x)\) given its second derivative, initial value, and a specific point involves systematic integration and applying boundary conditions to determine unknown constants. This problem exemplifies the power of calculus in reconstructing original functions from derivatives, a fundamental skill in mathematical analysis. The explicit form derived here offers a complete understanding of the behavior of \(F(x)\) and serves as a valuable example for students and professionals working with differential equations.

Keywords: differential equations, second derivative, integration, initial conditions, calculus, explicit function, boundary value problem, mathematical analysis

Frequently Asked Questions

What is the general form of the function F(x) given that F''(x) = 5 + cos(x)?

Since F''(x) = 5 + cos(x), integrating twice gives F(x) = 5x^2/2 + sin(x) + Cx + D, where C and D are constants determined by initial conditions.

How do we determine the constants C and D for the

function $F(x)$?

Using the given initial conditions $F(0) = -1$ and $F(5/2) = 0$, we substitute these into the general form of $F(x)$ and solve the resulting equations for C and D.

What is the value of the constant D in the function $F(x)$?

By substituting $x=0$ into $F(x) = (5/2)0^2 + \sin(0) + C0 + D = -1$, we find $D = -1$.

How do we find the value of the constant C in $F(x)$?

Substitute $x = 5/2$ into $F(x) = (5/2)(5/2)^2 + \sin(5/2) + C(5/2) - 1$ and set equal to 0. Solving for C yields $C = [0 - (5/2)(25/4) - \sin(5/2) + 1] / (5/2)$.

What is the explicit form of $F(x)$ after determining all constants?

$F(x) = (5/2) x^2 + \sin(x) + C x - 1$, where C is calculated based on the initial conditions using the previous step.

Why is the function $F(x)$ quadratic plus sinusoidal, and how does it satisfy the given conditions?

Because $F''(x) = 5 + \cos(x)$, integrating twice introduces a quadratic term (from 5) and a sinusoidal term (from $\cos(x)$), with linear and constant terms adjusted to meet the specific initial conditions provided.

Additional Resources

In-Depth Analysis of the Function F(x): Exploring Its Derivatives, Conditions, and Behavior

Understanding the structure and properties of mathematical functions is fundamental in calculus and analysis. In this detailed review, we examine the function F(x) defined by the conditions:

- $F''(x) = 5 + \cos(x)$
- $F(0) = -1$
- $F\left(\frac{5}{2}\right) = 0$

Our goal is to thoroughly analyze this function's behavior, find its explicit form, and interpret its derivatives, initial conditions, and overall characteristics.

Understanding the Given Conditions

Before delving into calculations, it is crucial to interpret the provided information:

- The second derivative $F''(x)$ is given explicitly as $5 + \cos(x)$, which indicates the curvature and concavity of the function.
- The initial conditions $F(0) = -1$ and $F(5/2) = 0$ serve as boundary conditions, enabling us to determine the constants of integration when reconstructing $F(x)$.

Step 1: Integrating to Find F'(x)

Given $F''(x) = 5 + \cos(x)$, integrating once yields the first derivative $F'(x)$:

$$
F'(x) = \int F''(x) \, dx = \int (5 + \cos(x)) \, dx
$$

Calculating $F'(x)$:

- $\int 5 \, dx = 5x + C_1$
- $\int \cos(x) \, dx = \sin(x) + C_2$

Combining constants, we have:

$$
F'(x) = 5x + \sin(x) + C
$$

where C is a constant of integration (absorbing $C_1 + C_2$).

Step 2: Integrating to Find F(x)

Next, integrate $F'(x)$ to obtain $F(x)$:

$$
F(x) = \int F'(x) \, dx = \int (5x + \sin(x) + C) \, dx
$$

Performing the integration term by term:

1. $\int 5x \, dx = \frac{5}{2} x^2 + C_3$

2. $\int \sin(x) \, dx = -\cos(x) + C_4$
3. $\int C \, dx = Cx + C_5$

Since the constants of integration can be combined, the general form of $F(x)$ becomes:

$$
F(x) = \frac{5}{2} x^2 - \cos(x) + Cx + K
$$

where K is a new constant encapsulating all previous constants.

Note: For simplicity, combine C into the linear term and K into the constant term.

Step 3: Applying Initial Conditions to Find Constants

We have two conditions:

- $F(0) = -1$
- $F(5/2) = 0$

Plugging $x=0$:

$$
F(0) = \frac{5}{2} \times 0^2 - \cos(0) + C \times 0 + K = -1
$$

$$
0 - 1 + 0 + K = -1
$$

$$
K = 0
$$

Next, plug $x = 5/2$:

$$
F\left(\frac{5}{2}\right) = \frac{5}{2} \times \left(\frac{5}{2}\right)^2 - \cos\left(\frac{5}{2}\right) + C \times \frac{5}{2} + 0 = 0
$$

Calculate each term:

- $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$
- $\frac{5}{2} \times \frac{25}{4} = \frac{5 \times 25}{2 \times 4} = \frac{125}{8}$

So,

$$
F\left(\frac{5}{2}\right) = \frac{125}{8} - \cos\left(\frac{5}{2}\right) + \frac{5}{2} C = 0
$$

Rearranged to solve for C:

$$
\frac{5}{2} C = - \frac{125}{8} + \cos\left(\frac{5}{2}\right)
$$

$$
C = \frac{2}{5} \left(- \frac{125}{8} + \cos\left(\frac{5}{2}\right) \right)
$$

Simplify:

$$
C = - \frac{2}{5} \times \frac{125}{8} + \frac{2}{5} \cos\left(\frac{5}{2}\right)
$$

$$
C = - \frac{250}{40} + \frac{2}{5} \cos\left(\frac{5}{2}\right)
$$

$$
C = - \frac{25}{4} + \frac{2}{5} \cos\left(\frac{5}{2}\right)
$$

Expressed exactly, the constant C is:

$$
C = -6.25 + \frac{2}{5} \cos\left(\frac{5}{2}\right)
$$

Explicit Expression for F(x)

Putting it all together, the explicit form of the function is:

$$
F(x) = \frac{5}{2} x^2 - \cos(x) + \left(-6.25 + \frac{2}{5} \cos\left(\frac{5}{2}\right) \right) x
$$

This formula allows us to evaluate $F(x)$ at any point, understand its shape, and analyze its derivatives.

Analyzing the Derivatives and Function Behavior

Understanding the derivatives and the behavior of $F(x)$ is key to grasping its shape and properties.

First Derivative $F'(x)$:

$$
F'(x) = 5x + \sin(x) + C
$$

where C is known from the previous calculation. The critical points occur where $F'(x) = 0$:

$$
5x + \sin(x) + C = 0
$$

This transcendental equation generally requires numerical methods to solve explicitly, but qualitative analysis can be performed.

Behavior of $F'(x)$:

- Since $5x$ dominates for large $|x|$, the function's slope tends to infinity as $x \to \pm \infty$.
- The sinusoidal term $\sin(x)$ oscillates between -1 and 1, adding minor fluctuations.
- The constant C shifts the entire $F'(x)$ graph vertically.

Implications:

- There can be multiple critical points depending on the value of C.
- The function $F(x)$ can have local maxima and minima corresponding to these critical points.

Second Derivative $F''(x)$:

$$
F''(x) = 5 + \cos(x)
$$

Properties:

- Since $\cos(x)$ oscillates between -1 and 1, $F''(x)$ oscillates between 4 and 6.
- The positivity of $F''(x)$ everywhere indicates that $F(x)$ is convex for all x.

Consequences:

- The function is strictly convex, meaning it curves upward everywhere.
- It has no points of inflection.

Graphical Interpretation and Behavior

Understanding the shape of $F(x)$ provides valuable insight into its characteristics.

- Convexity: Since $F''(x) > 0$ for all x, $F(x)$ is convex throughout its domain, ensuring any critical point is a global minimum.
- Critical Points: Solving $F'(x) = 0$ reveals potential locations of minima and maxima, but due to the convexity, these are minima.
- End Behavior: As $x \to \pm \infty$, the quadratic term dominates, and $F(x) \to +\infty$, confirming the function's paraboloid-like shape opening upward.

Numerical Approximation of Critical Points

Because the critical points satisfy:

$$
5x + \sin(x) + C = 0
$$

and $C \approx -6.25 + \frac{2}{5} \cos(2.5)$, approximate values can be obtained via numerical methods such as the Newton-Raphson method or graphical analysis.

Sample approach:

- Initial guesses near the roots can be guided by the dominant linear term \(5

Find F F X F X 5 Cos X F 0 1 F 5 2 0

Find F F X F X 5 Cos X F 0 1 F 5 2 0

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