Find The 10th Term Of The Geometric Sequence 8, -24, 72, ...

Find The 10th Term Of The Geometric Sequence 8, -24, 72, ... Understanding how to find specific terms in a geometric sequence is a fundamental skill in mathematics, especially in algebra and sequences. In this article, we will explore the step-by-step process to determine the 10th term of the sequence 8, -24, 72, ... We will delve into the concept of geometric sequences, how to identify the common ratio, and how to apply the formula for the nth term. Whether you're a student preparing for exams or someone interested in mathematical patterns, this comprehensive guide will provide clarity and confidence in solving such problems.

What is a Geometric Sequence?

A geometric sequence is a list of numbers where each term after the first is found by multiplying the previous term by a constant called the common ratio. This type of sequence is prevalent in various fields such as finance, physics, and computer science due to its exponential growth or decay patterns.

Key Characteristics of a Geometric Sequence

- Common Ratio (r): The constant factor between consecutive terms.
- First Term (a_1): The initial term of the sequence.
- General Term Formula: The nth term can be expressed as $a_n = a_1$ r^(n-1).

Identifying the Common Ratio

Given the sequence 8, -24, 72, ..., the first step is to determine the common ratio r.

Calculating the Common Ratio

To find r, divide any term by its preceding term:

- $r = (-24) / 8 = -3$
- $r = 72 / (-24) = -3$

Since both calculations yield the same result, we confirm that the common ratio $r = -3$.

Deriving the General Term Formula

With the first term $a_1 = 8$ and the common ratio $r = -3$, the formula for the nth term becomes:
$a_n = 8 (-3)^{(n-1)}$

This formula allows us to find any term in the sequence, including the 10th term.

Calculating the 10th Term

Applying the general formula for $n = 10$:
$a_{10} = 8 (-3)^{(10-1)} = 8 (-3)^9$

Evaluating (-3)^9

Since $(-3)^9$ involves an odd power, the result will be negative:

- $(-3)^9 = - (3^9)$

Calculate 3^9:

- $3^1 = 3$
- $3^2 = 9$
- $3^3 = 27$
- $3^4 = 81$

- 3^5 = 243

- 3^6 = 729

- 3^7 = 2187

- 3^8 = 6561

- 3^9 = 19683

Therefore:

(-3)^9 = -19683

Now, multiply by 8:

a_{10} = 8 (-19683) = -157464

Result: The 10th term of the sequence is -157464.

Summary of Steps to Find the nth Term in a Geometric Sequence

To systematically find the nth term, follow these key steps:

1. Identify the first term (a_1): Look at the sequence's initial value.
2. Determine the common ratio (r): Divide any term by the previous term.
3. Apply the formula: a_n = a_1 r^(n-1).
4. Calculate the specific term: Plug in the values for n, a_1, and r, then evaluate.

Additional Tips for Working with Geometric Sequences

- Always verify the common ratio by checking multiple pairs of consecutive terms.
- Be cautious with negative ratios, especially when raising to powers, to determine the sign of the result.
- Use a calculator for large exponents to avoid arithmetic errors.
- For sequences with fractional or decimal ratios, ensure calculations are precise.

Real-World Applications of Geometric Sequences

Understanding geometric sequences extends beyond academic exercises; they are essential in various real-world contexts:

- Population growth or decay: Modeling exponential increases or decreases.
- Financial calculations: Compound interest problems often involve geometric sequences.
- Physics: Radioactive decay or wave amplitudes diminish exponentially.
- Computer science: Algorithms that involve recursive multiplication or exponential data growth.

Conclusion: Mastering the 10th Term of a Geometric Sequence

Finding the 10th term of the sequence 8, -24, 72, ... demonstrates the power of understanding the properties of geometric sequences. By identifying the common ratio and applying the nth term formula, you can efficiently determine specific terms within the sequence. Remember to verify your calculations, especially when dealing with negative or large exponents. Mastery of these concepts not only aids in solving sequence problems but also enhances your overall mathematical reasoning.

Quick Recap

- Sequence: 8, -24, 72, ...
- Common ratio (r): -3
- First term (a_1): 8
- General formula: a_n = 8 (-3)^(n-1)
- 10th term: -157464

By following these steps and tips, you will confidently work through similar problems involving geometric sequences, whether in academic settings or real-world applications.

Frequently Asked Questions

What is the common ratio of the geometric sequence 8, -24, 72, ...?

The common ratio is -3, since -24 ÷ 8 = -3 and 72 ÷ -24 = -3.

How do you find the 10th term of the geometric sequence 8, -24, 72, ...?

Use the formula for the nth term: a_n = a_1 r^(n-1). Here, a_1 = 8 and r = -3. So, a_10 = 8 (-3)^(10-1) = 8 (-3)^9.

What is the value of the 10th term in the sequence 8, -24, 72, ...?

The 10th term is 8 (-3)^9 = 8 (-19683) = -157464.

How can I verify the 10th term of the sequence 8, -24, 72, ...?

Calculate the 10th term using the formula: a_10 = 8 (-3)^9, which equals -157464, confirming your answer.

What is the general formula for the nth term of this geometric sequence?

The nth term is given by a_n = 8 (-3)^{n-1}.

Is the sequence 8, -24, 72, ... increasing or decreasing?

The sequence alternates signs due to the negative common ratio, with the magnitude increasing and decreasing alternately, but overall the absolute value grows exponentially.

How does the negative common ratio affect the sequence?

It causes the sequence to alternate signs each term, switching between positive and negative, while the magnitude increases exponentially.

Can the formula for the 10th term be simplified further?

No, the formula is already simplified: a_10 = 8 (-3)^9, which equals -157464.

Additional Resources

Find The 10th Term Of The Geometric Sequence 8, -24, 72, ...

Understanding the intricacies of sequences and series is a fundamental aspect of mathematics, especially in algebra and calculus. Among these, geometric sequences stand out due to their elegant pattern of multiplication. In this article, we will delve into the process of finding the 10th term of a specific geometric sequence: 8, -24, 72, and beyond. Whether you're a student brushing up on your math skills or a curious learner aiming to understand the mechanics behind geometric progressions, this comprehensive guide will walk you through each step with clarity and precision.

What Is a Geometric Sequence?

Before we jump into calculating the 10th term, it’s essential to understand what a geometric sequence is.

Definition:
A geometric sequence is a list of numbers where each term after the first is obtained by multiplying the previous term by a constant called the common ratio.

General form:

$$a, ar, ar^2, ar^3, \ldots$$

Here,

- a is the first term of the sequence,
- r is the common ratio,
- n is the term number.

Key characteristics:

- The ratio r remains constant throughout the sequence.
- The sequence can either grow rapidly if $|r| > 1$ or decay if $|r| < 1$.
- Negative ratios cause the sequence to alternate signs, creating a pattern of positive and negative terms.

In our specific sequence: 8, -24, 72, ..., the pattern follows this structure, which we will explore in detail.

Identifying the First Term and Common Ratio

To find the 10th term, the first step is to identify the initial parameters.

Step 1: Determine the first term, a.

Given: 8, -24, 72, ...

The first term, a, is simply the first number:

$$a = 8$$

Step 2: Find the common ratio, r.

The common ratio is found by dividing a term by its preceding term.

Let's verify this with the first two known terms:

$$ r = \frac{\text{second term}}{\text{first term}} = \frac{-24}{8} = -3 $$

Check with the second and third terms:

$$ r = \frac{72}{-24} = -3 $$

The ratio remains consistent, confirming that:

$$ r = -3 $$

This is a crucial step because it confirms the sequence is geometric with a ratio of -3.

Formulating the General Term of the Sequence

The general term a_n of a geometric sequence is given by:

$$ a_n = a \times r^{n-1} $$

where:

- a is the first term,
- r is the common ratio,
- n is the term number.

Given the values identified:

$$ a = 8 $$
$$ r = -3 $$

The explicit formula for the sequence becomes:

$$ a_n = 8 \times (-3)^{n-1} $$

This formula allows us to find any term in the sequence directly, simply by substituting the value of n.

Calculating the 10th Term

With the general formula in hand, calculating the 10th term involves substituting $n = 10$:

$$ a_{10} = 8 \times (-3)^{10 - 1} = 8 \times (-3)^9 $$

Step 1: Calculate $(-3)^9$.

Since 9 is odd, $(-3)^9$ will be negative.

Calculate $|-3|^9 = 3^9$.

Let's compute 3^9:

- $3^1 = 3$
- $3^2 = 9$
- $3^3 = 27$
- $3^4 = 81$
- $3^5 = 243$
- $3^6 = 729$
- $3^7 = 2187$
- $3^8 = 6561$
- $3^9 = 19683$

Therefore:

$$ (-3)^9 = -19683 $$

Step 2: Multiply by 8.

$$ a_{10} = 8 \times (-19683) = -157464 $$

Thus, the 10th term of the sequence is:

$$ \boxed{-157464} $$

Understanding the Significance of the Result

The negative value indicates the sequence's alternating nature due to the negative common ratio. Each subsequent term in the sequence flips sign and scales by a factor of 3, leading to rapid growth in magnitude.

Sequence overview:

Term Number	Term Value	Sign	Explanation
1	8	Positive	Starting point
2	-24	Negative	8×-3
3	72	Positive	-24×-3
4	-216	Negative	72×-3
...	...	...	Continuing pattern
10	-157464	Negative	As calculated

This pattern highlights how the sequence alternates signs and increases in magnitude exponentially.

Applications and Real-World Contexts of Geometric Sequences

Understanding geometric sequences isn't just an academic exercise; they have practical applications in various fields:

- Finance: Calculating compound interest involves geometric sequences, where each period's amount depends on multiplying by a growth factor.
- Population Dynamics: Models often assume populations grow or shrink exponentially, following geometric progression principles.
- Physics: Decay processes, such as radioactive decay, follow geometric decay patterns.
- Computer Science: Algorithms that divide problems into smaller parts or exponential growth models rely on geometric progressions.

In our case, recognizing the pattern and calculating specific terms can help in predicting future values, understanding growth rates, and modeling real-world scenarios.

Summary and Final Thoughts

- The sequence 8, -24, 72, ... is geometric with a first term $a = 8$ and common ratio $r = -3$.
- The general term is given by $a_n = 8 \times (-3)^{n-1}$.
- The 10th term is calculated as -157464.

Grasping the mechanics of geometric sequences empowers learners to analyze exponential growth and decay, understand patterns, and apply these concepts across disciplines. Whether in academic settings or practical applications, the ability to identify, formulate, and compute terms in geometric sequences remains a vital mathematical skill.

In conclusion, the 10th term of the sequence 8, -24, 72, ... is -157,464. This result not only exemplifies the exponential nature of geometric progressions but also illustrates the importance of understanding ratios, formulas, and sequence behavior in mathematical problem-solving.

If you’re interested in further exploring sequences and series, consider practicing with different common ratios and initial terms, or studying how these sequences underpin complex phenomena in science and finance.

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