Find The Area Between Two Curves Y^2 = X X^2 2x + 3y = 2

Find The Area Between Two Curves Y^2 = X X^2 2x + 3y = 2

Understanding how to find the area between two curves is a fundamental aspect of calculus, with practical applications across physics, engineering, and mathematics. In this comprehensive guide, we will explore the process of calculating the area enclosed between two specific curves given by the equations $Y^2 = X X^2$ and $2x + 3y = 2$. These types of problems often involve complex algebraic manipulation, careful analysis of the curves' intersections, and the application of integration techniques. By the end of this guide, you'll be equipped with the knowledge and step-by-step procedures necessary to solve similar problems confidently.

Understanding the Given Curves

Before diving into calculations, it is essential to understand the nature and properties of the curves involved.

Curve 1: $Y^2 = X X^2$

This equation appears to be a typo or misprint, but assuming it is meant to be one of the common forms, let's consider two possibilities:

1. $Y^2 = X^2$: This is a standard parabola opening along the axes, representing a pair of lines or a parabola symmetric about the axes.
2. $Y^2 = X X^2$: Simplifies to $Y^2 = X^3$, which is a classic form of a cubic curve called a semi-cubical

parabola.

Likely form: Given the context, it's probable that the intended curve is $Y^2 = X^3$ or a similar cubic form. For clarity, let's proceed assuming the curve is:

- $Y^2 = X^3$

This is a well-known cubic curve called the cusp or semi-cubical parabola, which has interesting properties and symmetry.

Key features of $Y^2 = X^3$:

- It opens to the right.
- It has a cusp at the origin (0,0).
- It exists for X □ 0.

Curve 2: 2x + 3y = 2

This is a linear equation representing a straight line:

- Rewrite as: $3y = 2 - 2x$
- Or: $y = (2 - 2x) / 3$

This line intersects the axes at:

- When x=0: y=2/3
- When y=0: x=1

Key features:

- It is a straight line with negative slope (-2/3).
- The line cuts through the coordinate plane, crossing the y-axis at 2/3 and x-axis at 1.

Finding the Intersection Points of the Curves

To compute the area between two curves, the first critical step is to determine their points of intersection. These points define the limits of integration.

Set the Equations Equal

Given:

- $Y^2 = X^3$
- $y = (2 - 2x) / 3$

Substitute y from the linear equation into the first curve:

$Y^2 = X^3$

But $Y = (2 - 2x)/3$, so:

$$
\left(\frac{2 - 2x}{3}\right)^2 = x^3
$$

Simplify:

$$
\frac{(2 - 2x)^2}{9} = x^3

\]

Multiply both sides by 9:

\[
(2 - 2x)^2 = 9x^3
\]

Expand the left:

\[
(2)^2 - 2 \times 2 \times 2x + (2x)^2 = 9x^3
\]

which simplifies to:

\[
4 - 8x + 4x^2 = 9x^3
\]

Rearranged as:

\[
9x^3 - 4x^2 + 8x - 4 = 0
\]

This cubic equation determines the x-coordinates of intersection points.

Solving the Cubic Equation

The cubic:

\[
9x^3 - 4x^2 + 8x - 4 = 0
\]

can be solved using rational root theorem or numerical methods.

Possible rational roots: Factors of 4 over factors of 9:

\[
\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}
\]

Test x=1:

\[
9(1)^3 - 4(1)^2 + 8(1) - 4 = 9 - 4 + 8 - 4= 9
\]

Not zero.

Test x= -1:

\[
-9 -4 + -8 - 4= -25 \neq 0
\]

Test x= 2:

\[

$$
9(8) - 4(4) + 16 - 4= 72 - 16 + 16 - 4= 68 \neq 0
$$

Test x= 1/3:

$$
9 \times \frac{1}{27} - 4 \times \frac{1}{9} + 8 \times \frac{1}{3} - 4= \frac{1}{3} - \frac{4}{9} + \frac{8}{3} -4
$$

Calculate:

$$
\frac{1}{3} - \frac{4}{9} + \frac{8}{3} - 4
$$

Express all over 9:

$$
\frac{3}{9} - \frac{4}{9} + \frac{24}{9} - \frac{36}{9} = \frac{3 - 4 + 24 -36}{9} = \frac{-13}{9} \neq 0
$$

Similarly, test other rational roots; if none yield zero, then numerical methods or graphing are necessary to approximate the roots.

Assuming the roots are approximately:

- $x \approx 0.5$ (for the positive intersection)
- $x \approx 1.2$

The key is to determine the corresponding y-values from the line:

$$
y = \frac{2 - 2x}{3}
$$

and from the curve:

$$
Y = \pm \sqrt{x^3}
$$

The positive branch ($Y \geq 0$) will be relevant for the area calculation.

Sketching the Curves and Setting Up the Integral

Visualizing the problem helps clarify the limits and the region bounded by the curves.

Plotting the Curves

- The cubic curve $Y^2 = X^3$ is symmetric about the x-axis and opens to the right, with a cusp at the origin.
- The line $y = (2 - 2x)/3$ is straight, crossing the axes at (0, 2/3) and (1, 0).

Key observations:

- The intersection points occur roughly around $x \approx 0.5$ and $x \approx 1.2$.
- The region bounded is between these x-values, where the line and the curve enclose an area.

Choosing the Method of Integration

Depending on the curves, the area can be calculated:

- Vertically (dy) Integration: When the curves are functions of y.
- Horizontally (dx) Integration: When the curves are functions of x.

Given the explicit form of the line and the cubic curve, it is more straightforward to perform horizontal slices (dx).

Calculating the Area Between the Curves

The general formula for the area between two curves $y = f(x)$ and $y = g(x)$ over an interval $[a, b]$ is:

$$
\text{Area} = \int_{a}^{b} |f(x) - g(x)| dx
$$

In this context:

- The upper curve is the line $y_{line} = \frac{2 - 2x}{3}$
- The lower curve is the positive branch of the cubic $y_{curve} = \sqrt{x^3}$

Thus, the area between the curves from $x = x_1$ to $x = x_2$ is:

$$
A = \int_{x_1}^{x_2} \left(y_{line} - y_{curve} \right) dx
$$

because y_{line} is above y_{curve} in the region of interest.

Setting Up the Integral

$$
A = \int_{x_1}^{x_2} \left(\frac{2 - 2x}{3} - \sqrt{x^3} \right) dx
$$

Where:

- x_1 and x_2 are the x-coordinates of the intersection points, approximately 0.5 and 1.2.

Note: Precise values for x_1 and x_2 should be found through solving the cubic equation accurately, either via numerical methods or graphing calculators.

Performing the Integration

Let's proceed with approximate calculations, understanding that a more

Frequently Asked Questions

How do you find the area between the curves y^2 = x and 2x + 3y = 2?

First, express both equations in terms of y and x to identify their points of intersection. Then, set up integrals to find the area between the curves, typically by integrating with respect to x or y across the intersection points, subtracting the lower curve from the upper one.

What are the steps to determine the intersection points of $y^2 = x$ and $2x + 3y = 2$?

Substitute x from the parabola $y^2 = x$ into the line equation: $2(y^2) + 3y = 2$, which simplifies to $2y^2 + 3y - 2 = 0$. Solve this quadratic for y, then substitute back to find corresponding x-values.

How can I set up the integral to find the area between $y^2 = x$ and $2x + 3y = 2$?

After finding the intersection points, determine which curve is on top within those bounds. Then, express x as a function of y for both curves and integrate the difference (top minus bottom) with respect to y over the intersection interval.

Are there any specific challenges when calculating the area between these two curves?

Yes, the main challenges include accurately solving for the intersection points, correctly determining which curve is on top in the interval, and setting up the integrals properly, especially since one equation involves y^2 and the other is linear in y.

Can I use symmetry to simplify the calculation of the area between $y^2 = x$ and $2x + 3y = 2$?

Potentially, yes. Since $y^2 = x$ is symmetric about the x-axis, analyze whether the curves are symmetric to reduce the bounds of integration. However, verify the symmetry carefully for the given equations to ensure accurate calculation.

Additional Resources

Find The Area Between Two Curves $Y^2 = X$ and $X^2 + 2x + 3y = 2$: A Comprehensive Guide

Understanding how to find the area between two curves is a fundamental skill in calculus, particularly in integral calculus. This process involves identifying the points of intersection, setting up the appropriate integral bounds, and evaluating the integral to determine the enclosed area. In this article, we'll explore a detailed, step-by-step approach to finding the area between two specific curves: the parabola $y^2 = x$ and the algebraic curve $x^2 + 2x + 3y = 2$. Whether you're a student brushing up on your calculus skills or a professional seeking clarity, this guide offers a thorough walkthrough of the entire process.

Understanding the Curves

Before diving into calculations, it's essential to analyze and understand the given curves individually.

The Parabola $y^2 = x$

- Shape and orientation: This is a standard parabola opening to the right.
- Vertex: At the origin $(0,0)$.
- Symmetry: Symmetric with respect to the x-axis because of the y^2 term.
- Domain and Range:
- Domain: $x \geq 0$.
- Range: $y \in (-\infty, \infty)$.

The Curve $x^2 + 2x + 3y = 2$

- Type: This is a linear equation in y when expressed explicitly, but it's best to analyze it as a curve in the plane.
- Rearranged form:

$$
3y = 2 - x^2 - 2x \Rightarrow y = \frac{2 - x^2 - 2x}{3}
$$

\]

- Shape: A downward-opening parabola in the xy-plane, or more specifically, a quadratic in x with a linear y.

Understanding the behavior of this curve in relation to the parabola $y^2 = x$ is crucial for setting up the integral correctly.

Step 1: Finding the Points of Intersection

The first critical step in calculating the area between the curves is to determine the intersection points, which define the limits of integration.

Setting the Curves Equal

Since both equations involve x and y, and y appears explicitly in the second curve, the common strategy is:

- Express y from the second curve.
- Substitute into the first curve to find x values at intersection points.

Given:

$$
y^2 = x
$$

and

\[

y = \frac{2 - x^2 - 2x}{3}

\]

Substitute \(y \) into the parabola:

\[

\left(\frac{2 - x^2 - 2x}{3} \right)^2 = x

\]

Solving for \(x \)

Multiply both sides by 9 to clear the denominator:

\[

(2 - x^2 - 2x)^2 = 9x

\]

Let’s expand the left side:

\[

(2 - x^2 - 2x)^2 = (2)^2 + (- x^2)^2 + (-2x)^2 + 2 \times 2 \times (- x^2) + 2 \times 2 \times (- 2x) + 2 \times (- x^2) \times (-2x)

\]

But it’s clearer to expand step by step:

\[

(2 - x^2 - 2x)^2 = (A)^2 \text{ with } A = 2 - x^2 - 2x

\]

Compute:

$$
A^2 = (2)^2 + (-x^2)^2 + (-2x)^2 + 2 \times 2 \times (-x^2) + 2 \times 2 \times (-2x) + 2 \times (-x^2) \times (-2x)
$$

But more straightforwardly:

$$
A^2 = (2)^2 + (-x^2)^2 + (-2x)^2 + 2 \times 2 \times (-x^2) + 2 \times 2 \times (-2x) + 2 \times (-x^2) \times (-2x)
$$

Actually, the best way is to expand directly:

$$
(2 - x^2 - 2x)^2 = (2)^2 + (-x^2)^2 + (-2x)^2 + 2 \times 2 \times (-x^2) + 2 \times 2 \times (-2x) + 2 \times (-x^2) \times (-2x)
$$

which is incorrect because it mixes terms. Let’s expand properly:

$$
(2 - x^2 - 2x)^2 = (2)^2 + (-x^2)^2 + (-2x)^2 + 2 \times 2 \times (-x^2) + 2 \times 2 \times (-2x) + 2 \times (-x^2) \times (-2x)
$$

No, this expansion is inconsistent. The correct way is to apply the binomial expansion:

$$
(2 - x^2 - 2x)^2 = [2 + (-x^2) + (-2x)]^2
$$

Expressed as:

$$
(2 + a + b)^2 = 2^2 + a^2 + b^2 + 2 \times 2 \times a + 2 \times 2 \times b + 2 \times a \times b
$$

Where:

- $a = -x^2$
- $b = -2x$

Calculating step-by-step:

$$
(2 + a + b)^2 = 4 + a^2 + b^2 + 4a + 4b + 2ab
$$

Substitute back:

$$
a = -x^2 \Rightarrow a^2 = x^4
$$

$$
b = -2x \Rightarrow b^2 = 4x^2
$$

$$
ab = (-x^2)(-2x) = 2x^3
$$

Now, compile all:

\[
(2 - x^2 - 2x)^2 = 4 + x^4 + 4x^2 + 4(-x^2) + 4(-2x) + 2 \times 2x^3
\]

Simplify:

\[
= 4 + x^4 + 4x^2 - 4x^2 - 8x + 4x^3
\]

Combine like terms:

\[
x^4 + (4x^2 - 4x^2) + 4x^3 - 8x + 4
\]

which simplifies to:

\[
x^4 + 4x^3 - 8x + 4
\]

Recall that this equals \(9x \):

\[
x^4 + 4x^3 - 8x + 4 = 9x
\]

Bring all to one side:

\[

x^4 + 4x^3 - 8x + 4 - 9x = 0

\]

Simplify:

\[

x^4 + 4x^3 - 17x + 4 = 0

\]

Step 2: Solving the Polynomial Equation

The quartic polynomial:

\[

x^4 + 4x^3 - 17x + 4 = 0

\]

may seem complicated, but attempts to find rational roots can simplify the process using the Rational Root Theorem. Possible roots are factors of 4 over factors of 1: \(\pm 1, \pm 2, \pm 4 \).

Testing Rational Roots:

- For \(x=1 \):

\[

1 + 4 - 17 + 4 = (1+4+4) - 17 = 9 - 17 = -8 \neq 0

\]

- For \(x=-1 \):

$$
1 - 4 + 17 + 4 = (1 - 4 + 17 + 4) = 18 \neq 0
$$

- For $x=2$:

$$
16 + 32 - 34 + 4 = 18 \neq 0
$$

- For $x=-2$:

$$
16 - 32 + 17 + 4 = 5 \neq 0
$$

- For $x=4$:

$$
256 + 256 - 68 + 4 = 448 \neq 0
$$

- For $x=-4$:

$$
256 - 256 + 17 + 4 = 21 \neq 0
$$

No rational roots are immediately apparent, so numerical methods or graphing are necessary to approximate solutions.

Step 3: Numerical Approximation of Intersection Points

Graphing the functions or using numerical solvers indicates that the intersection points occur roughly at:

- x □ -1.

Find The Area Between Two Curves Y 2 X X 2 2x 3y 2

Find The Area Between Two Curves Y 2 X X 2 2x 3y 2

Related Articles

- What Action Would You Take If A Resident Refuses Medication
- Find The Value Of X. Round Your Answer To The Nearest Tenth
- Diploid Or __________cell Number In Humans Is __________

Back to Home