Find The Average Value Of F(x)=4x^2 Over The Interval [5,2].

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When studying functions in calculus, one common task is to determine the average value of a function over a specific interval. In this article, we will explore how to find the average value of the function $f(x) = 4x^2$ over the interval [5, 2]. Although the interval appears to be from 5 to 2, which is decreasing, the process remains the same; we just need to carefully interpret the bounds. Understanding this calculation is essential in various applications such as physics, engineering, and economics, where average values provide meaningful insights into the behavior of functions over intervals.

Understanding the Concept of Average Value of a Function

Before diving into the calculation, it’s important to understand what the average value of a function signifies.

Definition of Average Value

The average value of a continuous function f(x) over an interval [a, b] is given by the formula:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
$$

This formula computes the mean of the function's values over the interval, representing a constant function that has the same total area as the original function over that interval.

Why Is It Important?

The average value provides a simplified representation of the function’s overall behavior. For example, in physics, it can represent the average speed over a trip. In economics, it can signify the average cost or revenue over a period.

Analyzing the Function and Interval

In our specific case, the function is $f(x) = 4x^2$, and the interval is [5, 2].

Note on Interval Orientation

Typically, the interval is written with the lower number first, [a, b], where $a < b$. Here, the interval is [5, 2], which suggests the function is considered from $x = 5$ down to $x = 2$. To stay consistent with the standard formula, we can rewrite this as [2, 5], noting that:

$$
\int_{5}^{2} f(x) \, dx = - \int_{2}^{5} f(x) \, dx
$$

This negative sign accounts for the reversed bounds and ensures the integral evaluates correctly.

Calculating the Average Value

Let's proceed step-by-step to find the average value of $f(x) = 4x^2$ over [5, 2].

Step 1: Rewrite the Interval in Standard Form

Since the original interval is [5, 2], we rewrite it as [2, 5]:

$$
f_{avg} = \frac{1}{5 - 2} \int_{2}^{5} 4x^2 \, dx
$$

Note that:

$$
f_{avg} = \frac{1}{3} \int_{2}^{5} 4x^2 \, dx
$$

Step 2: Compute the Integral

The integral to evaluate is:

$$
\int_{2}^{5} 4x^2 \, dx
$$

Pulling out the constant:

$$
4 \int_{2}^{5} x^2 \, dx
$$

The integral of x^2 is:

\[

\int x^2 \, dx = \frac{x^3}{3}
\]

Applying the limits:

\[
4 \left[\frac{x^3}{3} \right]_{2}^{5} = \frac{4}{3} \left(5^3 - 2^3 \right)
\]

Calculate the powers:

\[
5^3 = 125,\quad 2^3 = 8
\]

So,

\[
\frac{4}{3} (125 - 8) = \frac{4}{3} \times 117 = \frac{4 \times 117}{3} = \frac{468}{3} = 156
\]

Step 3: Find the Average Value

Now, divide by the length of the interval, which is 3:

\[
f_{avg} = \frac{1}{3} \times 156 = 52
\]

Therefore, the average value of the function f(x) = 4x^2 over the interval [5, 2] is 52.

Summary of the Calculation

- Reinterpreted the interval from [5, 2] to [2, 5].
- Used the average value formula:

\[
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
\]

- Calculated the integral:

\[
\int_{2}^{5} 4x^2 \, dx = 156
\]

- Divided by the interval length (3):

$$
f_{avg} = \frac{156}{3} = 52
$$

This straightforward process highlights how integral calculus provides a powerful tool for finding the mean or average value of functions over specified intervals.

Additional Tips for Calculating the Average Value of Functions

To ensure accuracy and efficiency when calculating the average value of a function, consider the following tips:

- **Always verify the interval bounds:** Ensure the bounds are correctly ordered; if not, adjust accordingly and account for the negative sign if integrating directly.
- **Use fundamental integration formulas:** Recall basic integrals such as $\int x^n dx = \frac{x^{n+1}}{n+1}$.
- **Simplify constants before integrating:** Factor out constants to streamline calculations.
- **Check your work:** Always re-evaluate the integral and division step to prevent common mistakes.

Practical Applications of Finding the Average Value

Understanding how to find the average value of a function has numerous practical applications:

Physics

Calculating average speed or velocity over a time interval involves integrating the instantaneous speed function and dividing by the duration, similar to the process outlined above.

Economics

Determining the average cost or revenue over a period involves integrating the cost or revenue function across the interval and dividing by the length of that period.

Engineering

Analyzing average stress, strain, or other physical quantities over a material or period leverages this calculus technique.

Conclusion

Finding the average value of a function like f(x) = 4x^2 over an interval involves understanding the integral calculus principles and carefully managing the bounds of integration. For the interval [5, 2], reordering the bounds to [2, 5] and applying the formula yields an average value of 52. Mastering this skill enhances your ability to analyze functions across disciplines, providing meaningful insights into their overall behavior.

Remember, the key steps include rewriting the interval correctly, calculating the definite integral, and dividing by the interval length. With practice, this process becomes intuitive and invaluable for solving real-world problems involving averages and mean values of functions.

Frequently Asked Questions

What is the formula for finding the average value of a function over an interval?

The average value of a function f(x) over the interval [a, b] is given by (1/(b - a)) times the integral of f(x) from a to b, i.e., (1/(b - a)) ∫[a to b] f(x) dx.

How do you evaluate the integral of F(x) = 4x^2 from 5 to 2?

Since the interval is from 5 to 2, you can integrate 4x^2 normally and then evaluate from 2 to 5, remembering to switch the limits if needed, and account for the negative sign if integrating backwards.

What is the value of the integral ∫4x^2 dx?

The integral of 4x^2 with respect to x is (4 x^3) / 3 + C.

How do you compute the average value of F(x) = 4x^2 over [5, 2]?

Calculate the definite integral of 4x^2 from 2 to 5, then divide by (5 - 2) = 3 to find the average value.

What is the integral of 4x^2 evaluated from 2 to 5?

Evaluate (4/3) x^3 from 2 to 5: (4/3)(5^3) - (4/3)(2^3) = (4/3)(125) - (4/3)(8) = (500/3) - (32/3) = (468/3) = 156.

What is the average value of F(x) = 4x^2 over [5, 2]?

Using the integral result, the average value is (1/3) 156 = 52.

Why is the interval [5, 2] considered in reverse order, and how does it affect calculations?

Since the lower limit is 5 and the upper limit is 2, integrating from 5 to 2 is equivalent to negative the integral from 2 to 5, but when computing the average, you take the absolute difference in limits to ensure a positive result.

What is the final answer for the average value of F(x) = 4x^2 over [5, 2]?

The average value is 52.

Additional Resources

Find The Average Value Of F(x)=4x^2 Over The Interval [5,2]

In the realm of calculus, understanding the average value of a function over a specific interval is fundamental. It provides insight into the overall behavior of the function, akin to finding the mean height in a series of measurements or the average temperature over a day. Today, we delve into a particular case: determining the average value of the function $f(x) = 4x^2$ over the interval $[5, 2]$. While the interval might seem unconventional—since the upper limit is less than the lower limit—the process remains straightforward, emphasizing the importance of understanding the foundational concepts of integral calculus.

Understanding the Concept of Average Value of a Function

Before diving into calculations, it's essential to grasp what it means to find the average value of a function over an interval.

What is the Average Value?

The average value of a continuous function $f(x)$ over an interval $[a, b]$ is a value f_{avg} such that the area under the curve from a to b is equivalent to a rectangle of height f_{avg} spanning the same interval. Mathematically, it is expressed as:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
$$

This formula essentially sums up all the infinitesimal values of $f(x)$ across the interval, then divides by the length of the interval to find the mean.

Why Is This Important?

- Practical Applications: Engineers and scientists use this concept to find average quantities like average speed, temperature, or pressure over a specific period or range.
- Mathematical Significance: It bridges the gap between the integral (area under the curve) and the average value, linking geometry with algebra.

Dealing with the Interval: [5, 2]

Typically, intervals are written with the lower limit first and the upper limit second, i.e., $[a, b]$ with $a < b$. Here, the given interval is $[5, 2]$, which suggests that the integration limits are reversed.

How to Handle Reversed Limits?

- Reversal of Limits: The integral from a to b can be rewritten as the negative of the integral from b to a:

$$
\int_{a}^{b} f(x) \, dx = - \int_{b}^{a} f(x) \, dx
$$

- Implication for the Average: When calculating the average value over $[5, 2]$, we can proceed by taking the integral from 2 to 5 and then applying the negative sign, or directly substitute and accept the negative result, bearing in mind that the overall average should be a positive quantity since it represents a mean value.

Note: For clarity and consistency, we will evaluate the integral from 2 to 5 and then adjust accordingly.

Step-by-Step Calculation

Let's now perform the calculation to find the average value of $f(x) = 4x^2$ over $[5, 2]$.

1. Reframe the Interval

Since the interval is $[5, 2]$, we switch the limits:

$$
\text{Average value} = \frac{1}{2 - 5} \int_{5}^{2} 4x^2 \, dx
$$

which simplifies to:

$$
\frac{1}{-3} \int_{5}^{2} 4x^2 \, dx
$$

Alternatively, rewriting:

$$
\frac{1}{-3} \times \left(- \int_{2}^{5} 4x^2 \, dx \right) = \frac{1}{-3} \times - \int_{2}^{5} 4x^2 \, dx = \frac{1}{3} \int_{2}^{5} 4x^2 \, dx
$$

This approach ensures the average value remains positive.

2. Calculate the Integral

The core is to compute:

$$
\int_{2}^{5} 4x^2 \, dx
$$

Let's proceed step-by-step:

- Factor out constants:

$$
4 \int_{2}^{5} x^2 \, dx
$$

- Compute the indefinite integral:

$$
\int x^2 \, dx = \frac{x^3}{3}
$$

- Evaluate definite integral:

$$
4 \left[\frac{x^3}{3} \right]_{2}^{5} = 4 \times \left(\frac{5^3}{3} - \frac{2^3}{3} \right) = 4 \times \left(\frac{125}{3} - \frac{8}{3} \right) = 4 \times \frac{117}{3} = 4 \times 39 = 156
$$

3. Calculate the Average Value

Recall:

$$
f_{avg} = \frac{1}{b - a} \times \text{Integral}
$$

Applying for the original interval:

$$
f_{avg} = \frac{1}{5 - 2} \times 156 = \frac{1}{3} \times 156 = 52
$$

Since the integral was taken from 2 to 5 (reversing the original limits), and the division accounts for the interval length, the average value of $f(x)$ over $[5, 2]$ is:

$$
\boxed{52}
$$

Insights and Implications

The calculation reveals that despite the unusual interval notation, the process is straightforward once the integral limits are correctly interpreted. The average value of $f(x) = 4x^2$ over the interval $[5, 2]$ is 52.

This outcome underscores several critical points:

- Interval Direction Matters: Reversing the limits introduces a negative sign, but since the average value involves division by the length of the interval, which is negative in this case, the final result remains positive.
- Function Behavior: Since $f(x) = 4x^2$ is a parabola opening upwards, its average value over the interval $[2, 5]$ naturally aligns with the mean of its values at these points, weighted by the interval length.

Broader Applications and Considerations

Understanding how to find the average value of a function over an interval has numerous practical applications:

- Physics: Calculating average velocity over a time period.
- Economics: Determining average cost or revenue over a period.
- Engineering: Finding average stress or strain over a material.

In more complex scenarios where the function isn't as straightforward as a polynomial, numerical methods like Simpson's rule or trapezoidal rule may be employed to approximate the average value.

Final Remarks

Calculating the average value of a function, especially over an interval with reversed limits, reinforces the importance of understanding fundamental calculus principles. The process involves integrating the function over the desired interval, paying attention to the direction of limits, and then applying the average value formula.

In the case of $f(x) = 4x^2$ over $[5, 2]$, we found the average to be 52, illustrating that even with unconventional intervals, the core mathematical tools remain robust and reliable. This exercise not only solidifies the concept of average value but also exemplifies the elegance and consistency of calculus in solving real-world problems.

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