Find The Coefficient Of X^4 Y^7 In The Expansion Of (x Y)11?

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Understanding how to determine the coefficient of specific terms in algebraic expansions is a fundamental skill in mathematics, especially in binomial and multinomial theorem applications. In this article, we will explore the process of finding the coefficient of the term $x^4 y^7$ in the expansion of $(x y)^{11}$. We will break down the concepts step-by-step, provide relevant formulas, and illustrate the process with examples to ensure clarity.

Introduction to Binomial and Multinomial Theorems

Before diving into the specific problem, it is essential to understand the foundational principles involved – primarily, the binomial theorem and the multinomial theorem.

Binomial Theorem

The binomial theorem provides a formula for expanding expressions of the form $(a + b)^n$:

$$
(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k
$$

where $\binom{n}{k}$ is the binomial coefficient, calculated as:

$$
\binom{n}{k} = \frac{n!}{k!(n - k)!}
$$

This theorem is particularly useful for expanding binomials and identifying specific terms with given powers.

Multinomial Theorem

When dealing with more than two terms, the multinomial theorem comes into play:

$$
(a_1 + a_2 + \cdots + a_m)^n = \sum \frac{n!}{k_1!k_2!\cdots k_m!} a_1^{k_1} a_2^{k_2} \cdots a_m^{k_m}
$$

where the sum is taken over all non-negative integers $k_1, k_2, \ldots, k_m$ such that:

$$
k_1 + k_2 + \cdots + k_m = n
$$

The multinomial coefficient is:

$$
\frac{n!}{k_1!k_2!\cdots k_m!}
$$

In our case, the expansion involves a binomial raised to a power, which simplifies the process.

Understanding the Problem: Expansion of \((x y)^{11}\)

The problem asks for the coefficient of the term \(x^4 y^7\) in the expansion of \((x y)^{11}\).

Observe that:

\[

(x y)^{11} = (x y)^{11}

\]

This can be rewritten as:

\[

x^{11} y^{11}

\]

but this is just the initial expression. The question is about expanding \((x y)^{11}\) and identifying the coefficient of the specific term \(x^4 y^7\).

However, since the expression is a single power of a product, the expansion is straightforward.

Step-by-Step Solution

Let's analyze the problem carefully.

Step 1: Rewrite the Expression

The expression:

$$
(x y)^{11}
$$

can be rewritten as:

$$
x^{11} y^{11}
$$

which is just a single monomial, not an expanded sum. But the question seems to imply an expansion process, so perhaps the problem is asking about the expansion of $(x y)^n$ in a different context or considering the binomial expansion.

Alternatively, if the problem was to find the coefficient of $x^4 y^7$ in the expansion of $(x + y)^{11}$, the process would involve binomial coefficients. But since the expression is $(x y)^{11}$, it's simply a single term.

Therefore, the key is to clarify whether the problem is to find the coefficient of $x^4 y^7$ in $(x + y)^{11}$ or in $(x y)^{11}$.

Given the wording, it's likely a typographical or conceptual question about the expansion of $(x + y)^{11}$. Let’s assume the problem is to find the coefficient of $x^4 y^7$ in the expansion of $(x + y)^{11}$.

Clarification: Corrected Assumption

Assumed problem: Find the coefficient of $x^4 y^7$ in the expansion of $(x + y)^{11}$.

Solution for the Corrected Problem: Coefficient in \((x + y)^{11}\)

Given the assumption, the problem reduces to:

Find the coefficient of \(x^4 y^7\) in \((x + y)^{11}\).

Step 2: Applying the Binomial Theorem

The binomial expansion of \((x + y)^{11}\) is:

\[
(x + y)^{11} = \sum_{k=0}^{11} \binom{11}{k} x^{11 - k} y^{k}
\]

In this sum, each term looks like:

\[
\binom{11}{k} x^{11 - k} y^{k}
\]

To find the coefficient of \(x^4 y^7\), identify the term where:

\[

11 - k = 4

\]

\[

k = 7

\]

and

\[

y^k = y^7

\]

which matches our desired term.

Step 3: Calculating the Coefficient

Since \(k=7\), the coefficient is:

\[

\binom{11}{7}

\]

Calculate \(\binom{11}{7}\):

\[

\binom{11}{7} = \frac{11!}{7! \times (11 - 7)!} = \frac{11!}{7! \times 4!}

\]

Calculating factorials:

- $11! = 39916800$
- $7! = 5040$
- $4! = 24$

Compute numerator and denominator:

$$
\binom{11}{7} = \frac{39916800}{5040 \times 24}
$$

Calculate denominator:

$$
5040 \times 24 = 120960
$$

Now divide:

$$
\frac{39916800}{120960} = 330
$$

Therefore:

$$
\boxed{\text{Coefficient of } x^4 y^7 \text{ in } (x + y)^{11} = 330}
$$

Summary and Final Remarks

- The question's initial phrasing suggests an exploration into the expansion of $(x y)^{11}$, but as that simplifies to a single monomial $x^{11} y^{11}$, it contains only one term.

- Typically, when asked about specific powers of x and y in an expansion, the expression is often a binomial $(x + y)^n$.

- For $(x + y)^{11}$, the coefficient of $x^4 y^7$ is obtained directly via the binomial coefficient $\binom{11}{7}$.

- The binomial coefficient $\binom{11}{7}$ evaluates to 330, which is the answer.

Additional Notes and Tips for Similar Problems

- Identify the Form of the Expression: Always first determine whether you are dealing with a binomial or multinomial expansion.

- Match the Powers: When finding a specific term, set the exponents to match the desired powers of x and y.

- Use Binomial Coefficients: Recall the binomial coefficient formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

- Check the Exponent Sum: Ensure that the sum of the exponents matches the original power n.

- Verify the Term: Confirm that the selected k corresponds to the powers of x and y.

- Practice with Variations: Try similar problems with different exponents and binomial expansions to strengthen understanding.

Conclusion

Determining the coefficient of a specific term in binomial expansions is a fundamental skill in algebra. By understanding the binomial theorem and carefully matching the exponents, you can efficiently compute the needed coefficients. In the case of expanding $(x + y)^{11}$, the coefficient of $x^4 y^7$ is 330. Mastery of these concepts not only aids in solving algebraic problems but also enhances your overall mathematical reasoning skills.

Meta Description: Learn how to find the coefficient of $x^4 y^7$ in the expansion of $(x + y)^{11}$. Step-by-step explanation of binomial theorem applications, calculations, and tips for similar algebraic problems.

Frequently Asked Questions

What is the general term in the expansion of $(x y)^{11}$?

The general term is given by $C(11, k) (x y)^k$, which simplifies to $C(11, k) x^k y^k$.

How do I determine the coefficient of $x^4 y^7$ in the expansion of (x

y)^11?

Since (x y)^11 expands to terms of x^k y^k, the powers of x and y are equal in each term. To find the coefficient of x^4 y^7, note that the powers do not match, so the term does not exist in this expansion.

Is it possible to find the coefficient of x^4 y^7 in (x y)^11?

No, because in the expansion of (x y)^11, each term has x and y raised to the same power, so the coefficients of x^4 y^7 cannot occur.

What is the coefficient of x^k y^k in the expansion of (x y)^11?

The coefficient of x^k y^k is C(11, k).

Can we find the coefficient of x^4 y^7 in any expansion of the form (x y)^n?

Only if the powers of x and y in the term match, meaning k = 4 for x^4 and k = 7 for y^7, which is impossible. So, no, in (x y)^n, the powers of x and y are always equal, so x^4 y^7 cannot occur.

What is the value of the binomial coefficient C(11, 4)?

C(11, 4) = 330.

Why can't the term with x^4 y^7 appear in (x y)^11?

Because in the expansion of (x y)^11, all terms have x and y raised to the same power k, which ranges from 0 to 11. Since 4 □ 7, such a term does not occur.

What does the term (x y)^11 expand to in terms of powers of x and y?

It expands to sum_{k=0}^{11} C(11, k) x^k y^k, with each term having equal powers of x and y.

Is there any way to find the coefficient of x^4 y^7 in (x y)^11?

No, because the powers of x and y in each term are always equal; thus, x^4 y^7 never appears in the expansion.

What is the overall conclusion about finding the coefficient of x^4 y^7 in (x y)^11?

Since the powers of x and y in the expansion are always equal, the coefficient of x^4 y^7 in (x y)^11 is zero.

Additional Resources

Find The Coefficient Of X^4 Y^7 In The Expansion Of (x Y)11?

In the intricate world of algebra, binomial expansions serve as foundational tools that unlock the secrets of polynomial expressions. Among the many challenges posed by these expansions, one common task is to identify specific coefficients associated with particular terms—an exercise that sharpens both conceptual understanding and problem-solving skills. Today, we focus on a classic example: finding the coefficient of \(X^4 Y^7\) in the expansion of \((xY)^{11}\). At first glance, this problem seems straightforward, but a closer look reveals nuances rooted in binomial theorem principles that merit detailed exploration.

Understanding the Problem Statement

Before diving into calculations, it’s essential to clarify what the question asks. The expression in question is:

\[

$$(xY)^{11}$$

which, when expanded, involves powers of x and Y. The goal is to find the coefficient of the specific term involving $X^4 Y^7$.

Key points:

- The expression is a single binomial raised to the 11th power.
- The term of interest involves X^4 and Y^7.
- The problem is primarily about identifying the coefficient associated with this term in the expansion.

However, notice that the original expression involves lowercase variables, x and Y, but the term we seek involves uppercase X and Y. Typically, this discrepancy indicates a notation issue, or possibly a substitution. For clarity, assume the problem involves the expansion of $(xY)^{11}$, and the task is to identify the coefficient of the term where x is raised to the 4th power, and Y is raised to the 7th power.

Clarifying the Variables and Notation

Are X and x the same?
In algebraic expressions, uppercase and lowercase variables often represent different quantities unless specified otherwise. Given the problem statement, it’s reasonable to interpret:

- X^4 as the same as x^4,
- Y^7 as the same as Y^7.

If so, the task simplifies to finding the coefficient of the term where $x^4 Y^7$ appears in the expansion of $(xY)^{11}$.

What is $(xY)^{11}$?

This is a binomial expression with only one term, but it can be viewed as:

$$
(xY)^{11} = x^{11} Y^{11}
$$

since the exponent applies to the entire product. Therefore, the expansion contains only a single term:

$$
x^{11} Y^{11}
$$

which has no additional terms or combinations.

But wait—this raises an important point.
If the expression is simply $(xY)^{11}$, then the expansion is trivial: a single term with $x^{11} Y^{11}$. The coefficient of any other term, such as $x^4 Y^7$, does not exist, because the expansion contains only that one term.

Reconciling the Notation: Is It a Binomial Expansion?

Given the straightforward nature of $(xY)^{11}$, perhaps the problem intended to ask about the expansion of a different binomial expression involving two terms, such as:

$$
(x + Y)^{11}
$$

or

$$
(a + b)^{11}
$$

which is a standard binomial expansion where coefficients for specific terms are calculated.

Assumption:

Given the context, and common problem structures, it’s likely that the question was meant to be:

"Find the coefficient of $x^4 Y^7$ in the expansion of $(x + Y)^{11}$."

This is a classic binomial coefficient problem.

The Binomial Theorem: A Primer

The binomial theorem states that for any positive integer n,

$$
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{k} b^{n-k}
$$

where:

- $\binom{n}{k}$ is the binomial coefficient, read as "n choose k."
- The general term in the expansion is:

$$

\binom{n}{k} a^{k} b^{n-k}

\]

Applying to our problem:

Suppose we're expanding \((x + Y)^{11}\). The general term is:

\[

\binom{11}{k} x^{k} Y^{11 - k}

\]

To find the coefficient of \(x^4 Y^7\):

- The power of \(x\) is \(4\), so \(k = 4\).
- The power of \(Y\) is \(7\).

Check whether these satisfy the relation:

\[

11 - k = 7 \Rightarrow k = 4

\]

which aligns perfectly.

Calculating the Coefficient

Using the binomial coefficient:

\[

\binom{11}{4} = \frac{11!}{4! \times (11 - 4)!} = \frac{11!}{4! \times 7!}

\]

Calculating step-by-step:

1. Factorials:

\[

11! = 11 \times 10 \times 9 \times 8 \times 7!

\]

2. Substitute into the binomial coefficient:

\[

\binom{11}{4} = \frac{11 \times 10 \times 9 \times 8 \times 7!}{4! \times 7!}

\]

3. Cancel \(7!\):

\[

\binom{11}{4} = \frac{11 \times 10 \times 9 \times 8}{4!}

\]

4. Calculate numerator:

\[

11 \times 10 = 110 \\

110 \times 9 = 990 \\

990 \times 8 = 7920

\]

5. Calculate denominator:

$$
4! = 4 \times 3 \times 2 \times 1 = 24
$$

6. Final calculation:

$$
\frac{7920}{24} = 330
$$

Therefore, the coefficient of $x^4 Y^7$ in $(x + Y)^{11}$ is 330.

Summary of Key Findings

- If the initial problem was $\boxed{\text{Find the coefficient of } x^4 Y^7 \text{ in } (x + Y)^{11}}$, the answer is 330.
- The binomial theorem provides an efficient way to calculate specific coefficients.
- The key step is matching the powers: $k = 4$ for x^4, which implies $Y^{11-4} = Y^7$.

Additional Clarifications and Variations

What if the problem involved $(xY)^{11}$?
As discussed earlier, that expression expands to a single term:

\[

$$x^{11} Y^{11}$$

Hence, the coefficient of $x^4 Y^7$ does not exist in that expansion, making the answer zero.

What if the expression was different?
For example, $(ax + by)^{n}$, the coefficients depend on the multinomial theorem, which generalizes binomial coefficients.

Practical Applications and Significance

Understanding how to find specific coefficients in polynomial expansions is crucial in various fields:

- Mathematics & Algebra: Fundamental for simplifying expressions, solving equations, and understanding combinatorial relationships.
- Statistics & Probability: Binomial coefficients appear in binomial distributions, which model the likelihood of a certain number of successes in independent trials.
- Engineering & Computer Science: Algorithms often involve combinatorial calculations, especially in coding theory, cryptography, and algorithm analysis.

Final Thoughts

While the initial question may seem straightforward, it highlights the importance of carefully interpreting algebraic expressions and their expansions. Recognizing whether an expression is a binomial, multinomial, or a single term is vital for choosing the right approach. The binomial theorem remains one of the most elegant and powerful tools in the mathematician's toolkit, providing quick access to coefficients that otherwise require extensive enumeration.

In our exploration, we've demonstrated how to approach such problems systematically, ensuring clarity at each step. Whether dealing with theoretical exercises or real-world applications, mastery of these concepts empowers students and professionals alike to navigate the intricate tapestry of algebra with confidence.

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