Find The Critical Numbers Of The Function.g(y)=(y-1)/(y2-y+1)

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Understanding how to find the critical numbers of a function is a fundamental skill in calculus. Critical numbers are essential because they help identify points on the graph where the function's behavior changes—these include potential local maxima, minima, or points of inflection. In this comprehensive guide, we will explore the process of finding the critical numbers of the function $g(y) = \frac{y - 1}{y^2 - y + 1}$. We'll discuss the necessary calculus concepts, step-by-step procedures, and tips to analyze the function thoroughly.

Understanding Critical Numbers

Before we dive into calculating the critical numbers for $g(y)$, it's important to understand what critical numbers are and why they matter.

What Are Critical Numbers?

Critical numbers (or critical points) of a function are the values of the independent variable (in this case, y) where the derivative of the function is either zero or undefined. These points are significant because:

- They can indicate local maximums or minimums.
- They can reveal points of inflection where the function's concavity changes.
- They are potential candidates for absolute extrema on an interval.

Formally, a critical number y_c of a function $g(y)$ satisfies:

- $g'(y_c) = 0$, or
- $g'(y_c)$ does not exist

Note: Not every critical number corresponds to a maximum or minimum; some may be points of inflection or other interesting behavior.

Step-by-Step Process to Find Critical Numbers of $g(y)$

The process involves differentiating the function, analyzing where the derivative is zero or undefined, and confirming the critical points.

Step 1: Understand the Function Structure

The given function is a rational function:

$$
g(y) = \frac{y - 1}{y^2 - y + 1}
$$

- The numerator $y - 1$ is linear.

- The denominator $y^2 - y + 1$ is quadratic.

Since the denominator cannot be zero (which would make the function undefined), we should first verify where the denominator is non-zero.

Check for points where the denominator is zero:

$$
y^2 - y + 1 = 0
$$

Discriminant:

$$
\Delta = (-1)^2 - 4 \times 1 \times 1 = 1 - 4 = -3 < 0
$$

Since the discriminant is negative, the quadratic has no real roots, meaning the denominator is never zero on the real line. Therefore, the function is defined for all real y, and the derivative will not have points of discontinuity due to the denominator.

Step 2: Find the Derivative $g'(y)$

Given $g(y) = \frac{u(y)}{v(y)}$, where:

- $u(y) = y - 1$
- $v(y) = y^2 - y + 1$

We will employ the quotient rule:

$$
g'(y) = \frac{u'(y) v(y) - u(y) v'(y)}{[v(y)]^2}
$$

Calculate each derivative:

- $u'(y) = 1$
- $v'(y) = 2y - 1$

Substitute into the quotient rule:

$$
g'(y) = \frac{(1)(y^2 - y + 1) - (y - 1)(2y - 1)}{(y^2 - y + 1)^2}
$$

Step 3: Simplify the Derivative Expression

Simplify numerator:

$$
N(y) = y^2 - y + 1 - (y - 1)(2y - 1)
$$

Expand the second term:

$$
(y - 1)(2y - 1) = y \times 2y - y \times 1 - 1 \times 2y + 1 \times 1 = 2y^2 - y - 2y + 1 = 2y^2 - 3y + 1
$$

Now rewrite the numerator:

$$
N(y) = y^2 - y + 1 - (2y^2 - 3y + 1) = y^2 - y + 1 - 2y^2 + 3y - 1
$$

Combine like terms:

$$
N(y) = (y^2 - 2y^2) + (- y + 3y) + (1 - 1) = - y^2 + 2 y + 0 = - y^2 + 2 y
$$

Therefore, the derivative simplifies to:

$$
g'(y) = \frac{- y^2 + 2 y}{(y^2 - y + 1)^2}
$$

Step 4: Find Values of y Where $g'(y) = 0$ or $g'(y)$ is Undefined

Since the denominator $(y^2 - y + 1)^2$ is always positive (as the quadratic has no real roots), the derivative is defined everywhere on the real line. Therefore, the only critical points occur where the numerator is zero:

$$
- y^2 + 2 y = 0
$$

Factor out y:

$$
y(- y + 2) = 0
$$

Set each factor equal to zero:

- $y = 0$
- $- y + 2 = 0 \Rightarrow y = 2$

Conclusion: The critical numbers are at $y = 0$ and $y = 2$.

Interpreting the Critical Numbers

Having identified the critical numbers, the next step involves analyzing the nature of these points—whether they are local maxima, minima, or points of inflection.

Step 5: Use the First or Second Derivative Test

- First Derivative Test: Check the sign of $g'(y)$ around the critical points.
- Second Derivative Test: Compute $g''(y)$ and evaluate at critical points.

Given the focus on critical numbers, the primary goal is to determine their significance for the function's behavior.

Additional Considerations and Tips

- Since the denominator $(y^2 - y + 1)^2$ is always positive, the critical points are solely determined by the numerator.
- The critical numbers $y = 0$ and $y = 2$ are valid for both the first derivative test and the second derivative test.
- To further analyze these points, compute $g(y)$ at these critical numbers:

$$
g(0) = \frac{0 - 1}{0^2 - 0 + 1} = \frac{-1}{1} = -1
$$

$$
g(2) = \frac{2 - 1}{(2)^2 - 2 + 1} = \frac{1}{4 - 2 + 1} = \frac{1}{3}
$$

These function values help visualize the function's graph and determine whether these points are local maxima or minima.

Summary of the Critical Numbers for $g(y)$

- Critical points: $y = 0$ and $y = 2$
- Function values at these points:

$$
g(0) = -1, \quad g(2) = \frac{1}{3}
$$

- Both points are within the domain of the function, and the derivative is zero at these points, making them critical numbers.

Conclusion

Finding the critical numbers of $g(y) = \frac{y - 1}{y^2 - y + 1}$ involves differentiating the function, simplifying the derivative, and analyzing where the derivative equals zero or is undefined. In this case, since the denominator is always positive, the critical points are solely where the numerator is zero—at $y = 0$ and $y = 2$. These points are instrumental in understanding the function's behavior, including identifying potential local extrema.

By mastering the process of calculating derivatives and analyzing critical points, you can deepen your comprehension of the function's graph and its key features. Whether for academic purposes or real-world applications, recognizing and analyzing critical numbers is a vital skill in calculus.

Additional Resources:

- Calculus textbooks on derivatives and critical points
- Online graphing tools to visualize \(g(y) \) and verify critical points
- Practice problems on rational functions and their critical points

Frequently Asked Questions

How do you find the critical numbers of the function $g(y) = (y - 1) / (y^2 - y + 1)$?

To find the critical numbers, first compute the derivative $g'(y)$, then set $g'(y) = 0$ and solve for y, and also check where $g'(y)$ is undefined. The solutions are the critical numbers.

What is the derivative of $g(y) = (y - 1) / (y^2 - y + 1)$?

Using the quotient rule, $g'(y) = [(1)(y^2 - y + 1) - (y - 1)(2y - 1)] / (y^2 - y + 1)^2$.

How do you determine where the derivative $g'(y)$ is undefined for $g(y)$?

The derivative $g'(y)$ is undefined where the denominator $(y^2 - y + 1)^2$ equals zero, which occurs when $y^2 - y + 1 = 0$. Since $y^2 - y + 1$ has no real roots, $g'(y)$ is defined everywhere.

Are there any critical numbers for $g(y) = (y - 1) / (y^2 - y + 1)$?

Yes. Critical numbers occur where $g'(y) = 0$, which can be found by solving the numerator of $g'(y) = 0$, since the denominator is never zero for real y. This leads to solving $(y^2 - y + 1) - (y - 1)(2y - 1) = 0$.

What are the critical numbers of $g(y)$, and how do you identify them in this case?

After computing $g'(y)$ and setting numerator equal to zero, the critical numbers are the solutions to that equation. In this case, solving $(y^2 - y + 1) - (y - 1)(2y - 1) = 0$ yields the critical points, since the derivative is undefined nowhere on real numbers.

Additional Resources

Find The Critical Numbers Of The Function $g(y) = (y - 1) / (y^2 - y + 1)$

Understanding the critical numbers of a function is fundamental in calculus, as they provide insight into the potential points where the function may have local maxima, minima, or points of inflection. In this comprehensive analysis, we will delve deeply into the process of finding the critical numbers of the function \(g(y) = \frac{y - 1}{y^2 - y + 1} \).

Introduction to Critical Numbers

Critical numbers (or critical points) of a function are the values of the independent variable at which the derivative is either zero or undefined, provided that the function itself is defined at those points. These points are essential because they mark potential locations where the function's behavior shifts — such as peaks, valleys, or points of inflection.

Formally, for a function $g(y)$, the critical numbers are all y in the domain where:

1. $g'(y) = 0$, or
2. $g'(y)$ does not exist.

To find these, we follow a systematic approach: determine the derivative $g'(y)$, identify where it equals zero, and find where it is undefined within the domain of g.

Step 1: Understanding the Function $g(y)$

The function in question is:

$$
g(y) = \frac{y - 1}{y^2 - y + 1}
$$

Let's analyze it:

- Numerator: $y - 1$, a linear polynomial.
- Denominator: $y^2 - y + 1$, a quadratic polynomial.

Domain of $g(y)$:

Since the denominator cannot be zero (division by zero is undefined), the domain is:

$$
\text{Domain} = \{ y \in \mathbb{R} \mid y^2 - y + 1 \neq 0 \}
$$

Let's check whether the quadratic $y^2 - y + 1$ has real roots:

$$
\Delta = (-1)^2 - 4 \times 1 \times 1 = 1 - 4 = -3 < 0
$$

Because the discriminant is negative, the quadratic has no real roots, meaning:

$$
y^2 - y + 1 \neq 0 \quad \text{for all real } y
$$

Therefore, the domain of $g(y)$ is all real numbers $\mathbb{R}$.

This simplifies our task: the derivative $g'(y)$ exists everywhere in $\mathbb{R}$, so the only critical points occur where $g'(y) = 0$.

Step 2: Computing the Derivative $g'(y)$

Given $g(y)$ as a quotient, the most straightforward approach is to use the quotient rule:

$$
\frac{d}{dy} \left[\frac{u(y)}{v(y)} \right] = \frac{u'(y) v(y) - u(y) v'(y)}{[v(y)]^2}
$$

where:

- $u(y) = y - 1$
- $v(y) = y^2 - y + 1$

Let's compute:

- $u'(y) = 1$
- $v'(y) = 2y - 1$

Applying the quotient rule:

$$
g'(y) = \frac{(1)(y^2 - y + 1) - (y - 1)(2y - 1)}{(y^2 - y + 1)^2}
$$

Step-by-step calculation:

1. Expand numerator:

$$
\text{Numerator} = y^2 - y + 1 - (y - 1)(2y - 1)
$$

2. Expand the second term:

$$
(y - 1)(2y - 1) = y \times 2y - y \times 1 - 1 \times 2y + 1 \times 1 = 2y^2 - y - 2y + 1 = 2y^2 - 3y + 1
$$

3. Combine numerator:

$$
y^2 - y + 1 - (2y^2 - 3y + 1) = y^2 - y + 1 - 2y^2 + 3y - 1
$$

4. Simplify:

$$
(y^2 - 2y^2) + (- y + 3y) + (1 - 1) = - y^2 + 2 y + 0
$$

So,

$$
g'(y) = \frac{- y^2 + 2 y}{(y^2 - y + 1)^2}
$$

Step 3: Finding Critical Numbers — Setting $g'(y) = 0$

Since the denominator $(y^2 - y + 1)^2$ is always positive (because the quadratic denominator has no real roots and is squared), the critical points are determined solely by the numerator:

$$
- y^2 + 2 y = 0
$$

Factor the numerator:

$$
- y^2 + 2 y = 0 \implies y (- y + 2) = 0
$$

Set each factor to zero:

1. $y = 0$
2. $- y + 2 = 0 \implies y = 2$

Therefore, the critical numbers are at $y = 0$ and $y = 2$.

Step 4: Confirming the Critical Points

Since the derivative exists everywhere in the domain, and the numerator equals zero at $y = 0$ and $y = 2$, these are the only critical points based on the derivative test.

Summary:

- Critical Numbers: $y = 0$ and $y = 2$

Step 5: Additional Verification — Are These Points in the Domain?

Recall, the domain is all real numbers because the quadratic denominator has no real roots. Therefore, both $y=0$ and $y=2$ are within the domain, confirming they are valid critical points.

Step 6: Interpretations and Further Analysis

Having identified the critical numbers, the next logical steps involve:

- Determining the nature of these critical points (local maxima, minima, or points of inflection).
- Investigating the behavior of $g(y)$ near these points.

While this extends beyond the original scope of finding critical numbers, such analysis typically involves:

- Computing the second derivative $g''(y)$.
- Applying the First or Second Derivative Test.
- Analyzing limits as $y \to \pm \infty$.

Summary and Key Takeaways

- The function $g(y) = \frac{y - 1}{y^2 - y + 1}$ has a domain of all real numbers.
- The derivative $g'(y)$ simplifies to:

$$
g'(y) = \frac{- y^2 + 2 y}{(y^2 - y + 1)^2}
$$

- Critical numbers occur where numerator $- y^2 + 2 y = 0$, which simplifies to $y(y - 2) = 0$.
- Critical numbers are therefore at $y = 0$ and $y = 2$.

Concluding Remarks

Finding critical numbers is a key step in understanding the overall behavior of a function, especially when analyzing for local extrema or points of inflection. In this particular case, the process involved:

- Recognizing the quotient form of the function;
- Applying the quotient rule diligently;
- Simplifying the derivative to a manageable form;
- Solving the resulting algebraic equation to find critical points.

This approach exemplifies standard calculus techniques and can be generalized to a wide class of rational functions. By mastering the process of deriving and solving for critical numbers, one gains powerful tools for analyzing the shape and behavior of functions across numerous mathematical contexts.

In essence, the critical numbers of $g(y) = \frac{y - 1}{y^2 - y + 1}$ are at $y = 0$ and $y = 2$, marking potential points of interest for further analysis such as identifying local maxima, minima, or inflection points.

Find The Critical Numbers Of The Function G Y Y 1 Y2 Y 1

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