Find The Discriminant Of 2x Square + X - 6 Is Equal To Zero?

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Understanding how to find the discriminant of a quadratic equation is fundamental in algebra. It allows students and mathematicians to determine the nature of the roots without solving the equation explicitly. In this article, we will explore the process of calculating the discriminant for the quadratic equation $2x^2 + x - 6 = 0$, explaining each step thoroughly. Whether you're preparing for exams, solving real-world problems, or just enhancing your mathematical skills, this guide will provide clarity and confidence in handling quadratic equations.

What Is a Quadratic Equation?

Before diving into the discriminant, it's essential to understand the structure of quadratic equations.

Definition of a Quadratic Equation

A quadratic equation is a second-degree polynomial equation in a single variable, generally written in the form:

$$ ax^2 + bx + c = 0 $$

where:
- a, b, and c are coefficients with $a \neq 0$,
- x is the variable.

In our specific example, the quadratic equation is:

$$ 2x^2 + x - 6 = 0 $$

Here:
- $a = 2$,
- $b = 1$,
- $c = -6$.

Why Is the Discriminant Important?

The discriminant helps us determine:
- Whether the quadratic equation has real roots,

- The nature of those roots (distinct or repeated),
- The number of solutions without solving the equation.

Understanding the Discriminant

Definition of the Discriminant

The discriminant (D) of a quadratic equation $ax^2 + bx + c = 0$ is given by the formula:

$$D = b^2 - 4ac$$

The value of D reveals the nature of the roots:
- If $D > 0$, the quadratic has two distinct real roots.
- If $D = 0$, the quadratic has exactly one real root (a repeated root).
- If $D < 0$, the quadratic has two complex conjugate roots.

Significance of the Discriminant's Value

Knowing the discriminant helps in:
- Quickly assessing the number of solutions,
- Determining whether solutions are real or complex,
- Planning for solving the quadratic more efficiently.

Calculating the Discriminant of $2x^2 + x - 6 = 0$

Let's proceed step-by-step to compute the discriminant for our specific quadratic equation.

Step 1: Identify the coefficients a, b, and c

From the equation:

$$2x^2 + x - 6 = 0$$

we have:

- $a = 2$,

- $b = 1$,
- $c = -6$.

Step 2: Apply the discriminant formula

Recall:

$$D = b^2 - 4ac$$

Plugging in the values:

$$D = (1)^2 - 4 \times 2 \times (-6)$$

Step 3: Simplify the expression

Calculate each component:
- $(1)^2 = 1$,
- $4 \times 2 \times (-6) = 8 \times (-6) = -48$.

So:

$$D = 1 - (-48)$$

which simplifies to:

$$D = 1 + 48 = 49$$

Step 4: Interpret the result

Since $D = 49$, which is greater than zero, we conclude:
- The quadratic equation has two distinct real roots.
- The roots are real and different.

Implications of the Discriminant Result

Having found that $D = 49$, let's explore what this means practically.

Number of Roots

- The quadratic equation has two real roots because $D > 0$.

Nature of Roots

- The roots are distinct and real.

Calculating the Roots

Although the main focus is on the discriminant, knowing how to find the roots can be useful.

The roots are given by the quadratic formula:

$$ x = \frac{-b \pm \sqrt{D}}{2a} $$

Applying this to our example:
- $b = 1$,
- $D = 49$,
- $a = 2$.

Calculate:

$$ x = \frac{-1 \pm \sqrt{49}}{2 \times 2} = \frac{-1 \pm 7}{4} $$

Thus, the two solutions are:

1. $x = \frac{-1 + 7}{4} = \frac{6}{4} = \frac{3}{2}$

2. $x = \frac{-1 - 7}{4} = \frac{-8}{4} = -2$

Additional Tips for Calculating Discriminants

Handling Different Coefficients

- Always identify the coefficients a, b, and c carefully.
- Be cautious with signs, especially when coefficients are negative.

Using Discriminant to Decide Next Steps

- If $D > 0$, proceed to find roots using the quadratic formula.
- If $D = 0$, there is one repeated root, which simplifies to $x = -b/(2a)$.
- If $D < 0$, roots are complex and involve imaginary numbers.

Practice Examples

To master the concept, practice calculating the discriminant for various quadratic equations, such as:
- $3x^2 - 4x + 1 = 0$,
- $-x^2 + 5x - 6 = 0$,
- $x^2 + 2x + 5 = 0$.

Applications of the Discriminant in Real-World Problems

The discriminant isn't just an academic concept; it has practical applications across numerous fields.

Engineering and Physics

- Analyzing projectile motion where quadratic equations model trajectories.
- Determining stability in systems modeled by quadratic equations.

Economics and Finance

- Calculating break-even points where profit functions are quadratic.
- Analyzing quadratic cost or revenue functions to find maximum or minimum points.

Computer Graphics and Design

- Detecting intersections between curves and surfaces.
- Rendering shapes accurately by solving quadratic equations.

Summary: How to Find the Discriminant of $2x^2 + x - 6 = 0$

To summarize, here's a quick step-by-step guide:

1. Write the quadratic in standard form: $ax^2 + bx + c = 0$.
2. Identify coefficients:
- $a = 2$,
- $b = 1$,

- $c = -6$.
3. Use the discriminant formula:

$$D = b^2 - 4ac$$

4. Substitute the values:

$$D = 1^2 - 4 \times 2 \times (-6) = 1 + 48 = 49$$

5. Interpret the discriminant:
- Since $D > 0$, the equation has two real roots.

Conclusion

Calculating the discriminant of quadratic equations like $2x^2 + x - 6 = 0$ is a straightforward yet powerful technique in algebra. It provides essential insights into the nature of the roots, saving time and effort when solving quadratic equations. Remember, the key steps involve identifying coefficients, applying the discriminant formula, and interpreting the result effectively.

Understanding the discriminant equips students and professionals with the ability to analyze quadratic equations quickly and accurately, which is invaluable in both academic settings and real-world applications. Practice regularly with different equations to strengthen your skills and deepen your understanding of this fundamental mathematical concept.

Keywords: Discriminant, quadratic equation, $2x^2 + x - 6$, roots, real solutions, quadratic formula, algebra, discriminant calculation, quadratic roots, properties of quadratic equations, solving quadratic equations.

Frequently Asked Questions

What is the quadratic equation given in the problem?

The quadratic equation is $2x^2 + x - 6 = 0$.

How do you identify the coefficients in the quadratic equation $2x^2 + x - 6 = 0$?

The coefficients are $a=2$, $b=1$, and $c=-6$.

What is the formula for the discriminant of a quadratic equation?

The discriminant D is given by $D = b^2 - 4ac$.

How do you calculate the discriminant for $2x^2 + x - 6 = 0$?

Calculate D as $D = (1)^2 - 4(2)(-6) = 1 - (-48) = 1 + 48 = 49$.

What does the value of the discriminant tell us about the roots?

If $D > 0$, the quadratic has two real and distinct roots; if $D = 0$, one real repeated root; if $D < 0$, two complex roots.

What is the discriminant of the quadratic equation $2x^2 + x - 6 = 0$?

The discriminant is 49.

Are the roots of the quadratic equation real or complex?

Since the discriminant is positive (49), the roots are real and distinct.

How can the discriminant be useful in solving quadratic equations?

The discriminant helps determine the nature and number of roots before solving the equation fully.

Additional Resources

Find The Discriminant Of 2x Square + X - 6 Is Equal To Zero?

Understanding the discriminant of a quadratic equation is fundamental in algebra, serving as a key indicator of the nature and number of solutions the equation possesses. When faced with the quadratic expression $2x^2 + x - 6 = 0$, the discriminant reveals whether the equation has real or complex solutions, and whether these solutions are distinct or repeated. This article delves into the concept of the discriminant, explores the specific quadratic in question, and provides a comprehensive analysis of its discriminant, including the implications for the solutions of the quadratic equation.

Introduction to Quadratic Equations and the Discriminant

What Is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation in a single variable x, generally expressed in the form:

$$ ax^2 + bx + c = 0 $$

where:

- a, b, c are coefficients with $a \neq 0$,
- x is the variable.

Quadratic equations are pervasive in mathematics, physics, engineering, and various applied sciences because they model phenomena where relationships are quadratic in nature, such as projectile motion, area calculations, and optimization problems.

The Role of the Discriminant

The discriminant, often denoted as D or Δ, is a specific algebraic expression derived from the coefficients of a quadratic equation:

$$ D = b^2 - 4ac $$

Its value provides critical insight into the nature of the roots (solutions) of the quadratic:

- If $D > 0$: The quadratic has two distinct real solutions.
- If $D = 0$: The quadratic has exactly one real solution (a repeated root).
- If $D < 0$: The quadratic has two complex conjugate solutions.

Understanding the discriminant helps mathematicians and students quickly analyze the solutions' nature without explicitly solving the equation — a powerful shortcut in algebra.

Analyzing the Quadratic Equation: $2x^2 + x - 6 =$

0

Identifying Coefficients

In the given quadratic:

$$ 2x^2 + x - 6 = 0 $$

the coefficients are:

- $a = 2$
- $b = 1$
- $c = -6$

These coefficients are crucial for calculating the discriminant.

Calculating the Discriminant

Using the standard formula:

$$ D = b^2 - 4ac $$

we substitute the known coefficients:

$$ D = (1)^2 - 4 \times 2 \times (-6) $$

Calculating step by step:

1. Square of b:

$$ (1)^2 = 1 $$

2. Product of $4, a, c$:

$$ 4 \times 2 \times (-6) = 8 \times (-6) = -48 $$

3. Putting it all together:

$$ D = 1 - (-48) = 1 + 48 = 49 $$

Thus, the discriminant of the quadratic $2x^2 + x - 6$ is 49.

Implications of the Discriminant Value

Nature of the Roots

Since the discriminant $D = 49$ is positive, it indicates that the quadratic equation has two distinct real roots. This is a common scenario in quadratic equations with positive discriminants, and it implies that the parabola described by the quadratic function intersects the x-axis at two points.

Calculating the Roots

Knowing the discriminant allows us to find the roots explicitly via the quadratic formula:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Applying this to our quadratic:

$$x = \frac{-1 \pm \sqrt{49}}{2 \times 2}$$

$$x = \frac{-1 \pm 7}{4}$$

Calculating the two solutions:

1. $x = \frac{-1 + 7}{4} = \frac{6}{4} = \frac{3}{2}$

2. $x = \frac{-1 - 7}{4} = \frac{-8}{4} = -2$

Therefore, the two roots are x = 1.5 and x = -2.

Graphical Interpretation and Significance

Parabola Characteristics

The quadratic $2x^2 + x - 6$ describes a parabola opening upwards because $a = 2 > 0$. The roots at $x = 1.5$ and $x = -2$ are the x-intercepts, where the parabola crosses the x-axis.

Vertex and Axis of Symmetry

The vertex of the parabola can be calculated as:

$$ x_{v} = -\frac{b}{2a} = -\frac{1}{2 \times 2} = -\frac{1}{4} $$

The y-coordinate of the vertex:

$$ y_{v} = 2 \left(-\frac{1}{4} \right)^2 + \left(-\frac{1}{4} \right) - 6 $$

$$ y_{v} = 2 \times \frac{1}{16} - \frac{1}{4} - 6 = \frac{2}{16} - \frac{1}{4} - 6 = \frac{1}{8} - \frac{2}{8} - 6 = -\frac{1}{8} - 6 \approx -6.125 $$

Thus, the vertex is at approximately $\left(-\frac{1}{4}, -6.125 \right)$.

This graphical insight confirms the parabola's shape and the position of roots relative to the vertex.

Why Is the Discriminant Important?

Predicting Solution Types

The discriminant allows mathematicians to anticipate the nature of solutions without solving the quadratic explicitly:

- Two real roots: When $D > 0$, as in our case, solutions are real and distinct.
- One real root (repeated): When $D = 0$, the quadratic touches the x-axis at one point.
- Complex roots: When $D < 0$, solutions involve imaginary numbers.

Applications in Real-World Contexts

Understanding the discriminant is not just an academic exercise; it has practical implications:

- Physics: In projectile motion, the discriminant determines if an object will hit the ground (real roots) or not (complex roots).
- Engineering: Structural analysis often involves quadratic equations where

the discriminant indicates stability or failure points.
- Economics: Optimization problems sometimes reduce to quadratic forms; the discriminant influences the nature of the solutions.

Mathematical Techniques and Efficiency

Calculating the discriminant offers an efficient way:

- To quickly assess the solvability of a quadratic.
- To determine whether further algebraic or numerical solutions are necessary.
- To analyze the behavior of quadratic functions graphically and analytically.

Conclusion and Summary

The quadratic equation $2x^2 + x - 6 = 0$ has a discriminant of 49, a positive value that signifies the presence of two distinct real roots. These roots are explicitly calculated as $x = 1.5$ and $x = -2$, and the vertex of the parabola lies at approximately $\left(-\frac{1}{4}, -6.125 \right)$.

Understanding how to compute and interpret the discriminant is essential for anyone studying algebra, as it provides immediate insights into the solutions' nature, saving time and guiding subsequent problem-solving steps. This knowledge is applicable across various scientific and engineering disciplines, highlighting the importance of mastering the concept of the discriminant in quadratic equations.

In summary:

- The discriminant $D = 49$ confirms two real, distinct solutions.
- The solutions are $x = 1.5$ and $x = -2$.
- The quadratic graph intersects the x-axis at these points.
- The discriminant serves as a quick diagnostic tool for solution analysis.

By mastering the calculation and interpretation of the discriminant, students and professionals can efficiently analyze quadratic equations and understand the broader implications of their solutions in different contexts.

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