# Find The Exact Length Of The Curve. Y = X3 3 1 4x , 1 X 2

**Find The Exact Length Of The Curve. Y = X3 3 1 4x , 1 X 2**

Calculating the exact length of a curve is a fundamental problem in calculus, often encountered in various fields such as physics, engineering, and mathematics. Finding the length of a curve described by a function involves understanding how to apply the principles of calculus to measure the cumulative distance along the path of the function between two points. In this article, we will explore the process of finding the exact length of the curve defined by the function \( y = x^3 + 4x \) for \( x \) in the interval [1, 2].

## Understanding the Problem

Before diving into the solution, it's essential to clarify the given problem statement and what it entails.

### The Given Function

The curve is described by the function:
\[
y = x^3 + 4x
\]
for \( x \) within the interval \( [1, 2] \).

### The Goal

Our goal is to determine the exact length of this curve between the points \( x = 1 \) and \( x = 2 \).

## Mathematical Background: Arc Length Formula

The length \( L \) of a curve \( y = f(x) \) from \( x = a \) to \( x = b \) can be calculated using the arc length formula:
\[
L = \int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

This formula stems from the Pythagorean theorem, considering an infinitesimal segment of the curve with differential changes $dx$ and $dy$.

## Applying the Formula

To apply the formula to our specific function, we need to:
1. Find the derivative $\frac{dy}{dx}$.
2. Square the derivative and add 1.
3. Integrate the square root of the sum from 1 to 2.

# Step-by-Step Solution

Let's now proceed step-by-step to compute the exact length of the curve.

## Step 1: Find the Derivative $\frac{dy}{dx}$

Given:
$$
y = x^3 + 4x
$$
Differentiate with respect to $x$:
$$
\frac{dy}{dx} = 3x^2 + 4
$$

## Step 2: Formulate the Integral

Substitute $\frac{dy}{dx}$ into the arc length formula:
$$
L = \int_1^2 \sqrt{1 + (3x^2 + 4)^2} \, dx
$$
Simplify inside the square root:
$$
(3x^2 + 4)^2 = 9x^4 + 24x^2 + 16
$$
Therefore:
$$
L = \int_1^2 \sqrt{1 + 9x^4 + 24x^2 + 16} \, dx
$$
Combine like terms:

$$
L = \int_1^2 \sqrt{9x^4 + 24x^2 + 17} \, dx
$$

## Step 3: Simplify the Expression Under the Square Root

The integral becomes:

$$
L = \int_1^2 \sqrt{9x^4 + 24x^2 + 17} \, dx
$$

Note that:

$$
9x^4 + 24x^2 + 17
$$

resembles a quadratic in $x^2$. Let's set:

$$
u = x^2
$$

which implies:

$$
du = 2x \, dx \quad \Rightarrow \quad dx = \frac{du}{2x}
$$

But since $x = \sqrt{u}$, when $x$ changes from 1 to 2, $u$ changes from 1 to 4.

Express the integral in terms of $u$:

$$
L = \int_{u=1}^{4} \sqrt{9u^2 + 24u + 17} \times \frac{1}{2\sqrt{u}} \, du
$$

However, this substitution complicates the integral. Instead, it's more straightforward to recognize that the expression under the square root is a quadratic in $x^2$ and attempt to factor or complete the square.

# Evaluating the Integral

The integral:

$$
L = \int_1^2 \sqrt{9x^4 + 24x^2 + 17} \, dx
$$

can be approached using substitution methods or numerical approximation techniques. However, for an exact symbolic solution, it's common to look for a substitution that simplifies the radical.

## Substitution Approach

Observe that:

$$
9x^4 + 24x^2 + 17
$$

might be rewritten as:

$$
(3x^2)^2 + 2 \times 3x^2 \times 4 + 4^2 - (2 \times 3x^2 \times 4 - 17)
$$

which suggests completing the square.

Alternatively, consider substitution:

$$
t = 3x^2 + 4
$$

which is the derivative of $y$. Then:

$$
dt = 6x \, dx
$$

or

$$
dx = \frac{dt}{6x}
$$

But since $t = 3x^2 + 4$, then:

$$
x^2 = \frac{t - 4}{3}
$$

and

$$
x = \pm \sqrt{\frac{t - 4}{3}}
$$

for $x \geq 0$, we take the positive root.

When $x = 1$:

$$
t = 3(1)^2 + 4 = 3 + 4 = 7
$$

When $x = 2$:

$$
t = 3(4) + 4 = 12 + 4 = 16
$$

Express the integral in terms of $t$:

$$
L = \int_{t=7}^{16} \sqrt{1 + t^2} \times \frac{1}{6x} \, dt
$$

But $x = \sqrt{\frac{t - 4}{3}}$, so:

$$
dx = \frac{dt}{6 \sqrt{\frac{t - 4}{3}}}
$$

which complicates the integral further.

Given the complexity, this integral does not simplify into elementary functions easily, and an exact antiderivative in closed form involves special functions such as elliptic integrals.

# Numerical Approximation and Practical Calculation

Since the integral involves a radical of a quartic polynomial, an exact symbolic solution is complicated. Instead, practitioners often turn to numerical methods to approximate the length.

## Method 1: Simpson's Rule or Trapezoidal Rule

- Divide the interval $[1, 2]$ into small subintervals.
- Calculate the integrand at these points.
- Apply numerical formulas to approximate the integral.

## Method 2: Use of Computational Tools

- Software such as WolframAlpha, MATLAB, or Python's SciPy library can evaluate the integral numerically with high precision.
- For example, in Python:

```python
import scipy.integrate as integrate
import numpy as np

def integrand(x):
return np.sqrt(1 + (3x2 + 4)2)

length, error = integrate.quad(integrand, 1, 2)
print(f"The approximate length of the curve is {length:.4f}")
```

This yields a practical and accurate approximation of the curve length.

## Summary and Final Remarks

Calculating the exact length of the curve $y = x^3 + 4x$ between $x = 1$ and $x = 2$ involves applying the arc length integral:

$$
L = \int_1^2 \sqrt{1 + (3x^2 + 4)^2} \, dx
$$

which simplifies to:

$$
L = \int_1^2 \sqrt{9x^4 + 24x^2 + 17} \, dx
$$

While an elementary antiderivative may not exist, this integral can be evaluated numerically to obtain an approximate length with high precision. For most practical purposes, numerical methods provide reliable and efficient solutions.

Key Takeaways:
- The derivative of the function is crucial in setting up the arc length integral.
- The integral involves a radical of a quartic polynomial, often requiring numerical approximation.
- Modern computational tools make it straightforward to find approximate values when exact solutions are complex.

By mastering these steps, you can confidently find the length of any curve defined by a function, enabling deeper insights into the geometry and analysis of curves in calculus.

## Frequently Asked Questions

### How do I set up the integral to find the length of the curve $Y = x^3 + 4x$ from $x = 1$ to $x = 2$?

To find the length of the curve, set up the integral as $L = \int$ from 1 to 2 of $\sqrt{1 + (dy/dx)^2}\, dx$. First, compute $dy/dx = 3x^2 + 4$, then square it to get $(3x^2 + 4)^2$, and substitute into the integral.

### What is the derivative dy/dx for the function $Y = x^3 + 4x$?

The derivative $dy/dx$ is $3x^2 + 4$.

### How do I evaluate the integral for the arc length of $Y = x^3 + 4x$

between x=1 and x=2?

You compute L = $\int_1^2$ √(1 + (3x^2 + 4)^2) dx, which simplifies to $\int_1^2$ √(1 + 9x^4 + 24x^2 + 16) dx. Then evaluate this integral, possibly using substitution or numerical methods if necessary.

## Can the arc length integral for Y = x^3 + 4x be simplified analytically?

In this case, the integrand becomes √(17 + 24x^2 + 9x^4), which does not simplify easily into elementary functions. Numerical methods or approximation techniques are typically used to evaluate the integral.

## What numerical methods can I use to approximate the exact length of the curve?

You can use numerical integration methods such as Simpson's rule, Trapezoidal rule, or computational tools like calculators or software (e.g., WolframAlpha, MATLAB, or Python) to approximate the value of the integral.

## What is the significance of finding the exact length of a curve like Y = x^3 + 4x between x=1 and x=2?

Finding the exact length helps in applications like engineering, physics, and geometry where precise measurements of curved paths are required, such as in designing roads, cables, or understanding the geometry of objects.

# Additional Resources

Find The Exact Length Of The Curve. Y = $X^3$ + 4x , 1 ≤ X ≤ 2

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Introduction

Calculating the length of a curve is a fundamental problem in calculus, with applications spanning physics, engineering, computer graphics, and more. The task involves determining the precise arc length of a given function over a specified interval. In this article, we explore the process of finding the exact length of the curve defined by the function:

\[ y = x^3 + 4x \quad \text{for} \quad 1 \leq x \leq 2 \]

This problem exemplifies the classical approach to arc length calculation, combining derivative computations, integral setup, and evaluation. We will proceed step-by-step, providing detailed explanations, relevant formulas, and thorough analysis to ensure complete understanding.

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Understanding the Arc Length Formula

The general formula for the length $L$ of a smooth curve $y = f(x)$ between $x = a$ and $x = b$ is:

$$
L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
$$

This formula emerges from the Pythagorean theorem applied infinitesimally along the curve, summing the small segments to find the total length.

---

Step 1: Deriving the Derivative $dy/dx$

Given:

$$
y = x^3 + 4x
$$

Compute the derivative:

$$
\frac{dy}{dx} = 3x^2 + 4
$$

This derivative describes the slope of the tangent to the curve at any point $x$ within the interval $[1, 2]$.

---

Step 2: Setting Up the Integral for Arc Length

Substituting the derivative into the arc length formula:

$$
L = \int_{1}^{2} \sqrt{1 + (3x^2 + 4)^2} \, dx
$$

Expanding the squared term:

$$
(3x^2 + 4)^2 = 9x^4 + 2 \times 3x^2 \times 4 + 16 = 9x^4 + 24x^2 + 16
$$

Hence, the integral becomes:

$$
L = \int_{1}^{2} \sqrt{1 + 9x^4 + 24x^2 + 16} \, dx
$$

Simplify inside the square root:

$$
1 + 9x^4 + 24x^2 + 16 = 9x^4 + 24x^2 + 17
$$

So,

$$
L = \int_{1}^{2} \sqrt{9x^4 + 24x^2 + 17} \, dx
$$

---

Step 3: Simplifying the Integral

The integral involves a quartic polynomial under the square root, which suggests trying to express it as a perfect square or a substitution to simplify integration.

Notice that:

$$
9x^4 + 24x^2 + 17
$$

could be expressed as:

$$
(3x^2 + a)^2 + b
$$

Let's attempt to write it as:

$$
(3x^2 + m)^2 + c
$$

Expanding:

$$
(3x^2 + m)^2 = 9x^4 + 6m x^2 + m^2
$$

Matching coefficients:

- For $x^4$: $9x^4$ matches.
- For $x^2$: $6m x^2 = 24x^2 \Rightarrow m = 4$
- Constant term: $m^2 + c = 17 \Rightarrow 16 + c = 17 \Rightarrow c = 1$

Thus,

$$
9x^4 + 24x^2 + 17 = (3x^2 + 4)^2 + 1
$$

Now, the integral becomes:

$$
L = \int_{1}^{2} \sqrt{(3x^2 + 4)^2 + 1} \, dx
$$

This is a significant simplification. The integrand resembles the form:

$$
\sqrt{u^2 + 1}
$$

where:

$$
u = 3x^2 + 4
$$

---

Step 4: Substitution to Simplify the Integral

Define:

$$
u = 3x^2 + 4
$$

Then,

$$
\frac{du}{dx} = 6x
$$

$$
dx = \frac{du}{6x}
$$

Express $x$ in terms of $u$:

$$
u = 3x^2 + 4 \Rightarrow x^2 = \frac{u - 4}{3}
$$

Since $x \in [1, 2]$:

- When $x = 1$:

$$
u_{1} = 3(1)^2 + 4 = 3 + 4 = 7
$$

- When $x = 2$:

$$
u_{2} = 3(2)^2 + 4 = 3 \times 4 + 4 = 12 + 4 = 16
$$

Now, rewrite the integral in terms of $u$:

$$
L = \int_{u=7}^{16} \sqrt{u^2 + 1} \cdot \frac{dx}{du} \, du
$$

But we need $dx$ in terms of $du$:

$$
dx = \frac{du}{6x}
$$

Recall $x = \sqrt{\frac{u - 4}{3}}$, so:

$$
dx = \frac{du}{6 \sqrt{\frac{u - 4}{3}}} = \frac{du}{6} \times \sqrt{\frac{3}{u - 4}} = \frac{\sqrt{3}}{6} \times \frac{1}{\sqrt{u - 4}} \, du
$$

Substituting into the integral:

$$
L = \int_{7}^{16} \sqrt{u^2 + 1} \times \frac{\sqrt{3}}{6} \times \frac{1}{\sqrt{u - 4}} \, du
$$

Factor out constants:

$$
L = \frac{\sqrt{3}}{6} \int_{7}^{16} \frac{\sqrt{u^2 + 1}}{\sqrt{u - 4}} \, du
$$

---

Step 5: Tackling the Integral

The integral:

$$
I = \int_{7}^{16} \frac{\sqrt{u^2 + 1}}{\sqrt{u - 4}} \, du
$$

is nontrivial but can be approached via substitution methods or advanced techniques such as hyperbolic substitution.

Substitution:

Let:

\[

t = \sqrt{u - 4}
\Rightarrow u = t^2 + 4
\]

Then:

\[
du = 2t \, dt
\]

Express \(\sqrt{u^2 + 1}\):

\[
u = t^2 + 4
\]
\[
u^2 + 1 = (t^2 + 4)^2 + 1 = t^4 + 8t^2 + 16 + 1 = t^4 + 8t^2 + 17
\]
\[
\sqrt{u^2 + 1} = \sqrt{t^4 + 8t^2 + 17}
\]

Our integral becomes:

\[
I = \int_{t_{u=7}}^{t_{u=16}} \frac{\sqrt{t^4 + 8t^2 + 17}}{t} \times 2t \, dt
\]

Note that:

\[
t_{u} = \sqrt{u - 4}
\]

- When \(u=7\):

\[
t_{1} = \sqrt{7 - 4} = \sqrt{3}
\]

- When \(u=16\):

\[

t_{2} = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}
\]

And the \(t\) in numerator and denominator cancels:

\[
I = 2 \int_{\sqrt{3}}^{2\sqrt{3}} \sqrt{t^4 + 8t^2 + 17} \, dt
\]

Express the radicand as a quadratic in \(t^2\):

\[
t^4 + 8t^2 + 17
\]

Let:

\[
z = t^2
\]

then:

\[
\sqrt{z^2 + 8z + 17}
\]

which is similar in structure to previous steps, but solving this integral explicitly involves advanced techniques such as completing the square or hyperbolic substitution.

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Summary of the Integral Path

The integral reduces to a form involving \(\int \sqrt{z^2 + 8z + 17} \, dz\), which can be approached

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