

Find The Exact Values Of The Following. A) $\sin 315^\circ$ B) $\tan 322^\circ$

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Understanding how to find the exact values of trigonometric functions like sine and tangent is fundamental in mathematics, especially in the study of angles and their properties. Whether you're a student preparing for exams or someone looking to deepen your understanding of trigonometry, this guide will help you grasp the process of calculating exact values for specific angles, such as $\sin 315^\circ$ and $\tan 322^\circ$ (which likely refers to $\tan 322^\circ$). We will explore these values step-by-step, providing clear explanations and useful tips.

Understanding Trigonometric Functions and Their Values

Before diving into the specific calculations, it's important to review some basic concepts:

What Are Sine and Tangent?

- **Sine (sin):** For an angle θ in a right-angled triangle, sine is the ratio of the length of the side opposite the angle to the hypotenuse.
- **Tangent (tan):** For an angle θ , tangent is the ratio of the sine of the angle to the cosine of the angle, or equivalently, the ratio of the opposite side to the adjacent side in a right triangle.

Exact Values and Special Angles

- Some angles have well-known exact values, especially those related to the special angles in the unit circle, such as 0° , 30° , 45° , 60° , and 90° .
- Angles like 315° and 322° are not standard special angles, but their values can be derived using symmetry and the unit circle.

Calculating Sin 315°

Let's begin with the first value: Sin 315°.

Understanding the Angle 315°

- 315° is located in the fourth quadrant of the unit circle.
- It is equivalent to $360^\circ - 45^\circ$, meaning it has the same sine value as -45° , but with a positive or negative sign depending on the quadrant.

Using the Unit Circle for Exact Values

The unit circle is a circle with a radius of 1 centered at the origin. The coordinates of points on the circle correspond to $(\cos \theta, \sin \theta)$.

- Since $315^\circ = 360^\circ - 45^\circ$, and 45° is a well-known angle, the sine value at 45° is $\sqrt{2}/2$.

- In the fourth quadrant, sine values are negative because the y-coordinate is negative.

Therefore:

1. $\sin 315^\circ = -\sin 45^\circ$
2. Since $\sin 45^\circ = \sqrt{2}/2$, we get:
3. $\sin 315^\circ = -\sqrt{2}/2$

Final Value for Sin 315°

The exact value of Sin 315° is $-\sqrt{2}/2$.

Calculating Tan 322°

Next, let's analyze Tan 322°. This angle is in the fourth quadrant, as it is greater than 270° but less than 360° .

Understanding the Angle 322°

- Since $322^\circ = 360^\circ - 38^\circ$, it corresponds to a 38° angle in the fourth quadrant.
- The tangent function's sign depends on the quadrant: in the fourth quadrant, tangent is negative.

Using the Reference Angle

- The reference angle for 322° is 38° , calculated as $360^\circ - 322^\circ$.
- To find $\tan 322^\circ$, we find $\tan 38^\circ$, then adjust the sign based on the quadrant.

Finding $\tan 38^\circ$ Exactly

- Unlike 45° , 30° , or 60° , 38° does not have a standard exact value expressible with simple radicals.
- Therefore, $\tan 38^\circ$ is typically approximated or expressed in terms of a calculator.
- However, for the purpose of exact value calculations, we can express $\tan 38^\circ$ as $\tan(38^\circ)$ directly.

Expressing $\tan 322^\circ$

- Since tangent is positive in the third and first quadrants and negative in the second and fourth, and 322° is in the fourth quadrant:

$$1. \tan 322^\circ = -\tan 38^\circ$$

- Therefore, the exact value of $\tan 322^\circ$ is:

$$\tan 322^\circ = -\tan(38^\circ)$$

- Because $\tan(38^\circ)$ does not simplify to a radical expression, the most precise exact value is expressed as above.

Approximate Value

- Using a calculator, $\tan 38^\circ \approx 0.7813$.
- Hence, $\tan 322^\circ \approx -0.7813$.

Summary of Exact Values

- $\sin 315^\circ = -\sqrt{2}/2$
- $\tan 322^\circ = -\tan(38^\circ)$

Additional Tips for Finding Exact Values

Use of the Unit Circle

- Understanding the unit circle is key to finding exact values for common angles.
- Angles related to 30° , 45° , and 60° often have radical expressions for their sine, cosine, and tangent values.

Symmetry and Reference Angles

- Angles can be related to each other using symmetry across quadrants.
- Reference angles help simplify the process of finding exact values for angles outside the primary special angles.

Using Radical Expressions

- Values like $\sqrt{2}/2$, $\sqrt{3}/2$, and $1/2$ are common for exact trigonometric functions at special angles.
- For other angles, such as 38° , exact radical expressions are generally not available, and approximation or calculator use is necessary.

Conclusion

In summary, the exact value of $\sin 315^\circ$ is $-\sqrt{2}/2$, derived from the unit circle and symmetry principles. For $\tan 322^\circ$, since 38° does not have a standard radical expression, the exact value is best expressed as $-\tan(38^\circ)$. These methods underscore the importance of understanding the unit circle, reference angles, and the properties of trigonometric functions across different quadrants.

By mastering these concepts, you can confidently find the exact values of various trigonometric functions for a wide range of angles, enhancing your problem-solving skills and deepening your understanding of trigonometry.

Frequently Asked Questions

What is the exact value of $\sin 315^\circ$?

The exact value of $\sin 315^\circ$ is $\sqrt{2}/2$ or $\sqrt{2}$ divided by 2.

How can I find the exact value of $\sin 315^\circ$ without a calculator?

Since 315° is in the fourth quadrant, $\sin 315^\circ = -\sin 45^\circ = -\sqrt{2}/2$.

What is the value of $\tan 322^\circ$ in degrees?

$\tan 322^\circ$ (or $3^\circ 22'$) is approximately 0.0594.

How do I convert 322° to a decimal for tangent calculation?

Convert $3^\circ 22'$ to decimal degrees: $3 + 22/60 = 3.3667^\circ$, then find $\tan(3.3667^\circ)$.

Is there a simple exact value for $\tan 322^\circ$?

No, 322° is a non-standard angle, so its tangent value is typically approximated, around 0.0594.

Can I use a calculator to find $\tan 322^\circ$?

Yes, using a calculator you can directly find $\tan(322^\circ)$ or 3.3667° for an approximate value.

What is the significance of knowing exact values of trigonometric functions?

Exact values are essential for precise calculations in mathematics, engineering, and physics, especially when solving equations analytically.

Are there any special angles related to 315° and $3/22^\circ$?

Yes, 315° is a standard angle with known exact values, while $3/22^\circ$ is a non-standard angle, often requiring approximation.

Additional Resources

Find The Exact Values Of The Following. A) $\sin 315^\circ$ B) $\tan 3/22$

In the realm of trigonometry, understanding how to determine the exact values of sine, cosine, and tangent functions at specific angles is fundamental. These values not only facilitate solving complex mathematical problems but also underpin applications across physics, engineering, and computer science. Today, we delve into two specific trigonometric evaluations: the sine of 315 degrees and the tangent of $3/22$ radians. Through a clear, detailed exploration, we aim to demystify these calculations, illustrating the methods and concepts that make such evaluations straightforward and intuitive.

Understanding the Foundation: Trigonometric Functions and Their Significance

Before tackling specific angles, it's essential to grasp the core concepts of sine and tangent functions:

- Sine ($\sin$): For an angle in a right-angled triangle, sine is the ratio of the length of the side opposite the angle to the hypotenuse.
- Tangent ($\tan$): The ratio of the length of the side opposite the angle to the adjacent side; equivalently, tangent is sine divided by cosine.

These functions extend beyond right triangles into the unit circle—a circle with radius 1 centered at the origin in the coordinate plane—which provides a powerful visual and analytical tool for evaluating trigonometric functions at any angle, positive or negative, measured in degrees or radians.

Part A: Finding the Exact Value of $\sin 315^\circ$

1. The Significance of 315 Degrees in the Unit Circle

The angle 315° is located in the fourth quadrant of the unit circle, where sine values are negative, and cosine values are positive. It is related to the more familiar 45° , 90° , or 180° angles but is situated 45° below the x-axis in the fourth quadrant.

2. Converting Degrees to a More Recognizable Reference

Since 315° can be expressed as:

$$- (360^\circ - 45^\circ)$$

This relationship is crucial because it connects to known reference angles and their sine and cosine values.

3. Using Reference Angles to Find Exact Values

The key to evaluating $\sin 315^\circ$ lies in the concept of reference angles. The reference angle for 315° is:

$$- (360^\circ - 315^\circ = 45^\circ)$$

Knowing that sine of 45° is:

$$- (\sin 45^\circ = \frac{\sqrt{2}}{2})$$

and considering the quadrant (fourth), where sine is negative, the sine of 315° becomes:

$$- (\sin 315^\circ = - \sin 45^\circ = - \frac{\sqrt{2}}{2})$$

4. Final Expression

Thus, the exact value of $\sin 315^\circ$ is:

$$\boxed{- \frac{\sqrt{2}}{2}}$$

Part B: Finding the Exact Value of $\tan 32^\circ$

1. Clarifying the Angle Measurement: Is it in Degrees or Radians?

The notation " 32° " can be ambiguous. It could represent:

- 3 degrees and 22 minutes (a common notation in angle measurement)
- 3 radians and 22 units (less common)
- A typo or formatting error

Given the context and common mathematical notation, the most plausible interpretation is 3 degrees 22 minutes, which is a standard way of expressing angles in degrees and minutes.

Assumption: The angle is $3^\circ 22'$ (3 degrees and 22 minutes).

2. Converting Degrees and Minutes to Decimal Degrees

To work with the angle in calculations, convert $3^\circ 22'$ to decimal degrees:

- 1 degree = 60 minutes
- $(22' = \frac{22}{60})$ degrees

Calculating:

$$[3^\circ + \frac{22}{60}^\circ = 3 + 0.3667 = 3.3667^\circ]$$

Alternatively, for precise calculations, keep the fractional form:

$$[3^\circ + \frac{11}{30}^\circ]$$

since:

$$[22' = \frac{22}{60} = \frac{11}{30}]$$

3. Converting Degrees to Radians

Since tangent calculations are often easier in radians:

- 1 degree = $(\frac{\pi}{180})$ radians

Therefore,

$$[3.3667^\circ \times \frac{\pi}{180} \approx 3.3667 \times 0.0174533 \approx 0.0587 \text{ radians}]$$

or, more precisely:

$$[\frac{11}{30} \times \frac{\pi}{180} = \frac{11 \pi}{30 \times 180} = \frac{11 \pi}{5400}]$$

which simplifies to:

$$[\frac{11 \pi}{5400}]$$

4. Calculating the Exact Tangent Value

Given the small size of the angle, the tangent value can be approximated or expressed in terms of sine and cosine:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

But to find an exact value, especially for such an angle, we rely on the known sine and cosine of $3^\circ 22'$, or their series expansions.

Alternatively, recognizing that $3^\circ 22'$ is not a standard angle with a well-known radical form, the most common approach is to use the tangent of the decimal or radian measure calculated earlier.

5. Approximating Tan $3^\circ 22'$

Using a calculator:

$$\theta \approx 0.0587 \text{ radians}$$

Then,

$$\tan 0.0587 \approx 0.0588$$

This approximation is very close, but it's not an exact radical value.

6. Expressing Tan $3^\circ 22'$ in Exact Terms

Since $3^\circ 22'$ does not correspond to a standard special angle, the exact value of $\tan 3^\circ 22'$ cannot be expressed in simple radicals like $\frac{\sqrt{2}}{2}$ or $\frac{\sqrt{3}}{2}$. Instead, the exact value is best represented in terms of sine and cosine:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

where:

$$\begin{aligned} \sin \theta &= \sin \left(\frac{11\pi}{540} \right) \\ \cos \theta &= \cos \left(\frac{11\pi}{540} \right) \end{aligned}$$

Therefore, the exact value is:

$$\boxed{\frac{\sin \left(\frac{11\pi}{540} \right)}{\cos \left(\frac{11\pi}{540} \right)}}$$

which can be left as a precise expression or computed numerically for practical purposes.

Summary of Results

Expression	Exact Value	Notes
A) $\sin 315^\circ$	$-\frac{\sqrt{2}}{2}$	Derived from reference angle 45° , in the fourth quadrant.
B) $\tan 3^\circ 22'$	$\frac{\sin \left(\frac{11\pi}{540} \right)}{\cos \left(\frac{11\pi}{540} \right)}$	Expressed in terms of sine and cosine of the radian measure; approximate value ≈ 0.0588 .

Final Thoughts: The Power of Trigonometry in Precise Evaluations

While some angles yield neat radical expressions—like 45° , 30° , and 60° —others, such as $3^\circ 22'$, often require approximate calculations or expressing the value in terms of sine and cosine functions. The key to mastering these evaluations lies in understanding reference angles, unit circle properties, and conversion techniques between degrees, minutes, and radians.

These methods are invaluable not just in academic settings but also across various scientific disciplines where precise angular measurements underpin accurate calculations. As technology advances, the ability to convert and approximate these values accurately continues to be a vital skill for mathematicians, engineers, and scientists alike.

In essence, understanding how to find the exact values of trigonometric functions at specific angles enhances both theoretical knowledge and practical problem-solving skills, allowing for a deeper appreciation of the elegant relationships within mathematics.

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