Find The Length Of The Curve. R(t) = 6t, T2, 1 9 T3 , 0 T 1

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Understanding how to find the length of a curve defined parametrically is a fundamental concept in calculus, especially in the fields of physics, engineering, and mathematics. In this article, we will explore the process of calculating the length of the given parametric curve $R(t) = 6t, T2, 1 9 T3$ over the interval $0 \leq t \leq 1$. We will break down each step systematically, ensuring clarity and comprehensiveness, so you can apply these methods to similar problems confidently.

Introduction to Curve Length and Parametric Equations

Before delving into the specific problem, it is essential to understand what is meant by the length of a curve and how parametric equations describe curves in space.

What Is the Length of a Curve?

The length of a curve, often called the arc length, measures the distance along the curve from one point to another. If the curve is smooth and continuous, the arc length between two points can be calculated using integral calculus. This is particularly useful when the curve is expressed parametrically, as it allows for a flexible way to handle complex shapes.

Parametric Equations of a Curve

A parametric curve in three-dimensional space is described by three functions:

$$
x(t), \quad y(t), \quad z(t)
$$

where t is a parameter that varies over a specific interval. The entire geometric shape of the curve is traced out as t moves from the initial to the final value.

In our case, the given curve appears to be expressed as:

$$
R(t) = (x(t), y(t), z(t))
$$

with the components:

- $x(t) = 6t$
- $y(t) = T2$ (which appears to be a typo or unclear notation; assuming it might be $y(t) = 2$)
- $z(t) = 1.9 t^3$

and the parameter t varies from 0 to 1.

Note: Clarify any ambiguous notation before proceeding. Based on context, it seems the components are:

$$
x(t) = 6t, \quad y(t) = 2, \quad z(t) = 1.9 t^3
$$

which is a reasonable interpretation.

Step-by-Step Process to Find the Curve Length

Calculating the length of a parametric curve involves the following steps:

1. Identify the component functions $x(t), y(t), z(t)$.
2. Compute the derivatives $x'(t), y'(t), z'(t)$.
3. Calculate the integrand $\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$.
4. Integrate the integrand over the interval $[a, b]$, where $a = 0$ and $b = 1$.

The arc length L is given by:

$$
L = \int_{a}^{b} \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt
$$

Component Functions and Their Derivatives

Let's explicitly define the component functions:

$$
x(t) = 6t
$$
$$
y(t) = 2
$$
$$

$$
z(t) = 1.9 t^3
$$

Now, compute their derivatives:

$$
x'(t) = \frac{d}{dt} (6t) = 6
$$
$$
y'(t) = \frac{d}{dt} (2) = 0
$$
$$
z'(t) = \frac{d}{dt} (1.9 t^3) = 1.9 \times 3 t^2 = 5.7 t^2
$$

Formulating the Arc Length Integral

Substituting the derivatives into the arc length formula:

$$
L = \int_{0}^{1} \sqrt{(6)^2 + (0)^2 + (5.7 t^2)^2} \, dt
$$

Simplify the integrand:

$$
L = \int_{0}^{1} \sqrt{36 + 0 + (5.7)^2 t^4} \, dt
$$

Calculate $(5.7)^2$:

$$
(5.7)^2 = 32.49
$$

Thus,

$$
L = \int_{0}^{1} \sqrt{36 + 32.49 t^4} \, dt
$$

Evaluating the Integral for Arc Length

The integral:

$$
L = \int_{0}^{1} \sqrt{36 + 32.49 t^4} \, dt
$$

is a non-trivial integral due to the t^4 term inside the square root. To evaluate it, consider the following approaches:

Numerical Methods

Since the integral does not have a straightforward antiderivative, numerical approximation methods such as Simpson’s rule or Gaussian quadrature are practical.

Analytical Approximation

Alternatively, if an exact answer is necessary, a substitution or special functions might be used, but these techniques are advanced and may involve elliptic integrals.

Numerical Approximation of the Curve Length

Given the integral's complexity, let's illustrate how to approximate the length numerically using Simpson’s rule with a small number of subdivisions for simplicity.

Procedure:

1. Divide the interval $[0, 1]$ into n equal parts. For simplicity, choose $n = 4$.

2. Calculate the t points:

$$
t_0 = 0, \quad t_1 = 0.25, \quad t_2 = 0.5, \quad t_3 = 0.75, \quad t_4 = 1
$$

3. Compute the integrand $f(t) = \sqrt{36 + 32.49 t^4}$ at each point:

- $f(0) = \sqrt{36 + 0} = 6$
- $f(0.25) = \sqrt{36 + 32.49 \times (0.25)^4}$

Calculate $(0.25)^4 = 0.00390625$:

$$
f(0.25) = \sqrt{36 + 32.49 \times 0.00390625} \approx \sqrt{36 + 0.127} \approx \sqrt{36.127} \approx 6.009
$$

\]

- \(f(0.5) = \sqrt{36 + 32.49 \times (0.5)^4} \)

Calculate \((0.5)^4 = 0.0625 \):

\[
f(0.5) = \sqrt{36 + 32.49 \times 0.0625} = \sqrt{36 + 2.0306} = \sqrt{38.0306} \approx 6.169
\]

- \(f(0.75) = \sqrt{36 + 32.49 \times (0.75)^4} \)

Calculate \((0.75)^4 = 0.31640625 \):

\[
f(0.75) = \sqrt{36 + 32.49 \times 0.31640625} \approx \sqrt{36 + 10.283} \approx \sqrt{46.283} \approx 6.800
\]

- \(f(1) = \sqrt{36 + 32.49 \times 1^4} = \sqrt{36 + 32.49} = \sqrt{68.49} \approx 8.283
\]

4. Apply Simpson’s rule:

\[
L \approx \frac{\Delta t}{3} \left[f(t_0) + 4f(t_1) + 2f(t_2) + 4f(t_3) + f(t_4) \right]
\]

where \(\Delta t = 0.25 \).

Plugging in the values:

\[
L \approx \frac{0.25}{3} \left[6 + 4(6.009) + 2(6.169) + 4(6.800) + 8.283 \right]
\]

Calculate inside the brackets:

\[
6 + 4(6.009) = 6 + 24.036 = 30.036
\]
\[
2(6.169) = 12.338
\]
\[
4(6.800) = 27.200
\]
\[
\text{Sum} = 30.036 + 12.338 + 27.200 + 8.283 = 77.857
\]

Finally,

\[
L \approx \frac{

Frequently Asked Questions

What is the formula for finding the length of a space curve R(t)?

The length L of a space curve R(t) from t = a to t = b is given by the integral $L = \int_a^b |R'(t)|\,dt$, where R'(t) is the derivative of R(t).

Given $R(t) = \langle 6t, t^2, t^3 \rangle$ for $0 \leq t \leq 1$, how do we compute R'(t)?

Differentiate each component: $R'(t) = \langle d/dt(6t), d/dt(t^2), d/dt(t^3) \rangle = \langle 6, 2t, 3t^2 \rangle$.

How do we find the magnitude |R'(t)| in this case?

Calculate $|R'(t)| = \sqrt{6^2 + (2t)^2 + (3t^2)^2} = \sqrt{36 + 4t^2 + 9t^4}$.

What is the integral expression for the length of the curve $R(t) = \langle 6t, t^2, t^3 \rangle$ from t=0 to t=1?

$L = \int_0^1 \sqrt{36 + 4t^2 + 9t^4}\,dt$.

Is the integral for the length of the curve solvable analytically?

The integral $\int_0^1 \sqrt{36 + 4t^2 + 9t^4}\,dt$ is complex and may not have a simple antiderivative, often requiring numerical methods for an approximate solution.

What numerical methods can be used to approximate the length of this curve?

Methods such as Simpson's rule, trapezoidal rule, or numerical integration techniques like Gaussian quadrature can be used to approximate the integral.

What is the significance of finding the length of a curve in calculus?

Finding the length of a curve helps in understanding the geometry of the curve, calculating distances traveled along paths, and in applications like engineering and physics where path length is important.

Additional Resources

Find The Length Of The Curve: A Comprehensive Guide to Calculating Arc Length for R(t) = (6t, t^2, 1 + $9t^3$), with t in [0, 1]

Understanding how to determine the length of a curve defined parametrically is a fundamental skill in calculus, especially in the study of vector functions and their applications in physics, engineering, and computer graphics. In this detailed exploration, we will analyze the parametric curve R(t) = (6t, t^2, 1 + $9t^3$) over the interval t ∈ [0,1], and derive its length step-by-step.

Introduction to Curve Length in Parametric Form

The concept of the length of a curve can be viewed as the total distance traveled along a path as the parameter t varies from an initial to a final value. When a curve is expressed parametrically as:

$$ R(t) = (x(t), y(t), z(t)), \quad t \in [a, b], $$

the arc length L from t = a to t = b is given by the integral:

$$ L = \int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt. $$

This formula generalizes the Pythagorean theorem to a continuous setting, summing infinitesimal distances along the curve.

Understanding the Given Parametric Curve

Let's analyze the specific vector function:

$$ R(t) = (6t, t^2, 1 + 9t^3), \quad t \in [0, 1]. $$

Breaking down the components:

- x(t) = 6t: a linear function, indicating a straightforward increase along the x-axis.
- y(t) = t^2: a quadratic function, representing a parabolic growth along the y-axis.
- z(t) = 1 + $9t^3$: a cubic function, starting at 1 when t=0 and increasing more rapidly as t approaches 1.

This combination results in a smooth, continuous curve in three-dimensional space, with increasing complexity due to the cubic term in z(t).

Step-by-Step Derivation of the Arc Length

Let's proceed systematically:

Step 1: Find the derivatives of each component with respect to t

Calculating derivatives:

- $\frac{dx}{dt} = \frac{d}{dt}(6t) = 6$,
- $\frac{dy}{dt} = \frac{d}{dt}(t^2) = 2t$,
- $\frac{dz}{dt} = \frac{d}{dt}(1 + 9t^3) = 27t^2$.

Step 2: Write the integrand for the arc length

The integrand is:

$$
\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} = \sqrt{6^2 + (2t)^2 + (27t^2)^2}.
$$

Simplify term by term:

- $6^2 = 36$,
- $(2t)^2 = 4t^2$,
- $(27t^2)^2 = 729 t^4$.

Thus, the integrand becomes:

$$
\sqrt{36 + 4t^2 + 729 t^4}.
$$

Step 3: Write the integral expression for the length

The total arc length L from $t=0$ to $t=1$ is:

$$
L = \int_0^1 \sqrt{36 + 4t^2 + 729 t^4} \, dt.
$$

Analyzing and Simplifying the Integral

The integral:

$$
L = \int_0^1 \sqrt{36 + 4t^2 + 729 t^4} \, dt,
$$

appears complex due to the polynomial inside the square root. To evaluate this integral, consider factorization or substitution strategies to reduce it to a more manageable form.

Factorization of the Radicand

Observe that:

$$
36 + 4t^2 + 729 t^4,
$$

can be rearranged as:

$$
(6)^2 + (2t)^2 + (27 t^2)^2,
$$

which we've already noted. Let's explore whether this quadratic form can be expressed as a perfect square or a combination thereof.

Attempt at Polynomial Substitution

Because the polynomial involves only even powers of t, a substitution involving $u = t^2$ might help. Let:

$$
u = t^2, \quad du = 2t \, dt,
$$

or equivalently,

$$
dt = \frac{du}{2t} = \frac{du}{2 \sqrt{u}}.
$$

But note that the integral is from $t=0$ to $t=1$, so u varies from 0 to 1 as well.

Express the integrand in terms of u:

$$
36 + 4u + 729 u^2,
$$

and the integral becomes:

$$
L = \int_{u=0}^{1} \sqrt{36 + 4u + 729 u^2} \times \frac{du}{2 \sqrt{u}}.
$$

This introduces a square root in the denominator, complicating the integral. Therefore, alternative approaches might be more effective, such as recognizing the quadratic form and completing the square or applying substitution strategies.

Evaluating the Integral: Approaches and Techniques

Given the integral's complexity, several methods could be applied:

Method 1: Numerical Approximation

Since the integral involves a quartic polynomial under a square root, closed-form solutions are complicated or may not exist in elementary functions. Numerical methods like Simpson's rule or Gaussian quadrature can provide approximate values with high accuracy.

- Advantages: Quick and practical for specific applications.
- Disadvantages: Does not provide a symbolic expression; only an approximate numerical value.

Method 2: Recognize the Polynomial as a Perfect Square

Attempt to express:

$$
36 + 4t^2 + 729 t^4
$$

as a perfect square of a quadratic in t, i.e.,

$$
(A t^2 + B)^2 = A^2 t^4 + 2AB t^2 + B^2.
$$

Compare coefficients:

$$
A^2 = 729 \Rightarrow A = \pm 27,
$$
$$
2AB = 4 \Rightarrow 2 \times 27 \times B = 4 \Rightarrow 54 B = 4 \Rightarrow B = \frac{4}{54} = \frac{2}{27},
$$
$$
B^2 = 36 \Rightarrow \left(\frac{2}{27}\right)^2 = \frac{4}{729} \neq 36,
$$

so it's not a perfect square. Similarly, try with $A = -27$.

- For $A = -27$:

$$
2 \times (-27) \times B = 4 \Rightarrow -54 B = 4 \Rightarrow B = -\frac{4}{54} = -\frac{2}{27},
$$
$$
B^2 = \frac{4}{729} \neq 36,
$$

so the polynomial isn't a perfect square of a quadratic.

Method 3: Completing the Square or Polynomial Decomposition

Attempt to write the radicand as:

$$
(\alpha t^2 + \beta)^2 + \text{additional terms}.
$$

\]

Alternatively, express:

\[
36 + 4 t^2 + 729 t^4 = 729 t^4 + 4 t^2 + 36,
\]

and note that:

\[
(27 t^2)^2 + 2 \times 27 t^2 \times \frac{2}{27} + \left(\frac{2}{27}\right)^2,
\]

which we've already checked doesn't match.

Numerical Approximation of the Arc Length

Given the algebraic complexity and the absence of an elementary antiderivative, the most practical approach is to approximate the integral numerically.

Step 1: Discretize the interval [0, 1]

Divide the interval into n subintervals (e.g., n=1000 for high accuracy):

\[
\Delta t = \frac{1 - 0}{n} = \frac{1}{n}.
\]

Step 2: Compute the integrand at each t-value

For each \(t_i = i \times \Delta t,\quad i=0,1,\dots,n:\)

\[
f(t

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