Find The Slope Of The Tangent Line For F(x)=65x^2 When X=1

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Understanding the slope of the tangent line to a function at a specific point is a fundamental concept in calculus. It provides insight into the instantaneous rate of change of the function at that point, which is crucial in various applications such as physics, engineering, economics, and data analysis. In this comprehensive guide, we will explore how to determine the slope of the tangent line for the quadratic function $f(x) = 65x^2$ at the point where $x = 1$.

This process involves calculating the derivative of the function, which represents the slope of the tangent line at any point along the curve. We will discuss the theoretical background, step-by-step calculation, and real-world applications to ensure a thorough understanding.

Understanding the Concept of the Tangent Line and Its Slope

What Is a Tangent Line?

A tangent line to a curve at a particular point is a straight line that just touches the curve at that point without crossing it locally. It represents the best linear approximation of the function around that specific point. The tangent line provides valuable information about the behavior of the function near that point, such as whether it is increasing or decreasing.

The Significance of the Slope of a Tangent Line

The slope of the tangent line at a point on a function graph indicates how rapidly the function's output value changes with respect to its input. A positive slope signifies an increasing function at that point, while a negative slope indicates a decreasing function. A zero slope signifies a stationary point, which could be a maximum, minimum, or saddle point.

Mathematical Foundations: Derivatives and Their Role

What Is a Derivative?

The derivative of a function $f(x)$, denoted as $f'(x)$ or $\frac{df}{dx}$, measures the rate at which $f(x)$ changes as x varies. Geometrically, it corresponds to the slope of the tangent line to the function at any point.

How Derivatives Are Calculated

Derivatives are calculated using rules of differentiation, such as the power rule, product rule, quotient rule, and chain rule. For polynomial functions like $65x^2$, the power rule is typically used.

Step-by-Step Calculation of the Slope for $f(x) = 65x^2$ at $x=1$

Step 1: Write the function

Given:
$$
f(x) = 65x^2
$$

This is a quadratic function scaled by a factor of 65.

Step 2: Find the derivative $f'(x)$

Using the power rule:
$$
\frac{d}{dx}[ax^n] = a \cdot n \cdot x^{n-1}
$$
where a is a constant, and n is the power of x.

Applying this to $f(x) = 65x^2$:
$$
f'(x) = 65 \times 2 \times x^{2-1} = 130x
$$

Step 3: Evaluate the derivative at \(x=1 \)

Substitute \(x=1 \) into \(f'(x) \):
\[
f'(1) = 130 \times 1 = 130
\]

This value, 130, is the slope of the tangent line to the curve at \(x=1 \).

Conclusion of Calculation

The slope of the tangent line to the function \(f(x) = 65x^2 \) at the point where \(x=1 \) is 130.

Interpreting the Result and Its Implications

What Does a Slope of 130 Mean?

A slope of 130 indicates that at \(x=1 \), the function \(f(x) \) is increasing rapidly. For every unit increase in \(x \), the output \(f(x) \) increases by approximately 130 units near that point.

Graphical Representation

If you plot \(f(x) = 65x^2 \), the tangent line at \(x=1 \) will have a steep positive inclination, reflecting the high rate of change. The tangent line equation can be written as:
\[
y = f(1) + f'(1) \times (x - 1)
\]
where:
- \(f(1) = 65 \times 1^2 = 65 \)
- \(f'(1) = 130 \)

Thus:
\[
y = 65 + 130 \times (x - 1)
\]

This line will touch the parabola at \((1, 65) \) and match the slope at that point.

Applications of Finding the Slope of the Tangent Line

In Physics

The slope of a position-time graph indicates velocity. For example, if $f(x)$ models position over time, then $f'(x)$ gives instantaneous velocity at x.

In Economics

The derivative helps analyze marginal cost or revenue, indicating how costs or revenues change with production levels.

In Engineering and Data Analysis

Understanding the rate of change enables engineers and analysts to optimize systems and predict future behavior based on current trends.

Additional Considerations and Advanced Topics

Second Derivative and Concavity

While the first derivative indicates the slope, the second derivative $f''(x)$ reveals the concavity of the function:

$$f''(x) = \frac{d}{dx} (130x) = 130$$

Since $f''(x) = 130 > 0$, the parabola is concave upward at all points, confirming the shape of $f(x)$.

Implications of the Derivative's Sign

- Positive derivative: function increasing
- Negative derivative: function decreasing
- Zero derivative: potential maximum or minimum

Using Derivatives for Optimization

Finding where $f'(x) = 0$ helps identify critical points, which could be maxima, minima, or saddle points.

Summary and Key Takeaways

- The derivative of $f(x) = 65x^2$ is $f'(x) = 130x$.
- Evaluating at $x=1$, the slope of the tangent line is 130.
- This indicates a steep positive slope, meaning the function is increasing rapidly at that point.
- The tangent line equation at $x=1$:

$$
y = 65 + 130 \times (x - 1)
$$

- Understanding tangent slopes helps in various real-world applications, from physics to economics.

Final Thoughts

Calculating the slope of a tangent line at a specific point is a foundational skill in calculus, enabling us to analyze and predict the behavior of functions. For the quadratic function $f(x) = 65x^2$, determining the slope at $x=1$ involves straightforward differentiation and substitution, resulting in a slope of 130. This process exemplifies how derivatives serve as powerful tools for understanding the dynamics of functions, with broad relevance across scientific and engineering disciplines.

By mastering these techniques, students and professionals alike can interpret the behavior of complex systems, optimize processes, and make informed decisions based on precise mathematical analysis.

Frequently Asked Questions

What is the function given for finding the slope of the tangent line?

The function is $f(x) = 65x^2$.

How do you find the slope of the tangent line at a specific point for a function?

You find the derivative of the function and then evaluate it at the given x-value.

What is the derivative of f(x) = 65x^2?

The derivative is f'(x) = 130x.

What is the slope of the tangent line to the function at x = 1?

The slope is f'(1) = 130 1 = 130.

Why do we use derivatives to find the slope of a tangent line?

Because the derivative represents the instantaneous rate of change of the function at a specific point, which is the slope of the tangent line.

Can you explain how to evaluate the derivative at x=1?

Yes, substitute x=1 into the derivative f'(x) = 130x, giving f'(1) = 130 1 = 130.

What is the significance of the slope value 130 at x=1 for this function?

It indicates that the tangent line to the curve at x=1 has a steepness or inclination of 130.

Is the slope of the tangent line the same as the slope of the secant line passing through two points?

No, the tangent line's slope is the limit of secant slopes as the two points approach each other; it represents the instantaneous slope.

How would the slope change if x increased to 2?

At x=2, the slope would be f'(2) = 130 2 = 260, which is steeper than at x=1.

Additional Resources

Find The Slope Of The Tangent Line For F(x)=65x^2 When X=1

In the realm of calculus, understanding the behavior of functions at specific points is fundamental to grasping how these functions change and operate within various contexts. One of the core concepts in this field is the tangent line to a curve at a given point, which provides a linear approximation of the function's behavior locally. When asked to find the slope of the tangent line for a particular function at a specific value of x, such as $x=1$ for the function $f(x) = 65x^2$, we are essentially seeking the derivative of the function evaluated at that point. This process not only deepens our comprehension of the function's instantaneous rate of change but also illustrates the practical applications of calculus in fields ranging from physics to economics. In this comprehensive review, we will explore the mathematical foundations of derivatives, the method for calculating the slope of the tangent line, and the broader implications of these calculations in real-world scenarios.

Understanding the Concept of a Tangent Line and Its Slope

What Is a Tangent Line?

A tangent line to a curve at a specific point is a straight line that just "touches" the curve at that point without crossing it in an immediate vicinity. Geometrically, it represents the best linear approximation of the curve near that point. The significance of the tangent line lies in its ability to describe the instantaneous rate at which the function's output changes with respect to its input.

For example, imagine a roller coaster track; the tangent line at a particular point on the track indicates the immediate direction and slope of the track at that point. If the track is steep, the tangent line has a high slope; if it is flat, the slope is zero.

The Slope of the Tangent Line

The slope of the tangent line at a point $x=a$ on the function $f(x)$ is mathematically expressed as the derivative $f'(a)$. This slope quantifies how rapidly the function's value is changing at that point.

Formally:

$$
\text{Slope of tangent line at } x=a = f'(a)
$$

This derivative is a limit process that measures the rate of change of the function as the interval around a becomes infinitesimally small.

Calculating the Derivative of $f(x) = 65x^2$

The Power Rule for Differentiation

To find the slope of the tangent line at $x=1$, we first need to compute the derivative of the function $f(x) = 65x^2$. One of the most straightforward methods for differentiating polynomial functions like this is the power rule, which states:

$$
\frac{d}{dx} [x^n] = n x^{n-1}
$$

Applying this rule to $f(x) = 65x^2$:
- The constant multiple rule allows us to factor out the constant:
$$
f'(x) = 65 \times \frac{d}{dx} [x^2]
$$
- Differentiating x^2 using the power rule:
$$
\frac{d}{dx} [x^2] = 2x
$$
- Combining these results:
$$
f'(x) = 65 \times 2x = 130x
$$

This derivative function $f'(x) = 130x$ represents the slope of the tangent line to the curve at any point x.

Interpreting the Derivative Formula

The derivative $f'(x) = 130x$ indicates that the slope of the tangent line depends linearly on x. When x increases, the slope increases proportionally. Conversely, at $x=0$, the slope is zero, meaning the tangent line is horizontal at that point.

This linear relationship simplifies the process of calculating the tangent line's slope at specific points, such as $x=1$.

Evaluating the Derivative at $x=1$

Plugging in the Value $x=1$

To find the specific slope of the tangent line at $x=1$, substitute this value into the derivative:

$$
f'(1) = 130 \times 1 = 130
$$

This result tells us that the slope of the tangent line to the curve $f(x) = 65x^2$ at the point where $x=1$ is 130.

Implication of the Result

A slope of 130 is notably steep. It indicates that at $x=1$, the function $f(x)$ is increasing very rapidly. Since the function is quadratic and opens upward (positive leading coefficient), this steep slope aligns with the expectation that the function's rate of change increases as x moves away from zero.

The result also provides a critical insight into the function's behavior at that point:
- The tangent line's equation at $x=1$ can now be formulated, which aids in approximating the function near this point.
- It reflects the instantaneous rate of change, which is essential in physics for velocity calculations or in economics for marginal analysis.

Formulating the Equation of the Tangent Line at $x=1$

Point-Slope Form of a Line

Once the slope $m = 130$ is known, the next step is to find the equation of the tangent line itself. Recall the point-slope form:

$$
y - y_1 = m (x - x_1)
$$

where:
- (x_1, y_1) is the point of tangency,
- m is the slope at that point.

Calculating the Point of Tangency

To find y_1, evaluate $f(x)$ at $x=1$:

$$
f(1) = 65 \times 1^2 = 65
$$

So, the point of tangency is $(1, 65)$.

Writing the Equation

Substitute into the point-slope form:

$$
y - 65 = 130 (x - 1)
$$

which simplifies to:

$$
y = 130 (x - 1) + 65
$$

This is the equation of the tangent line at $x=1$. It can be used for approximate predictions of the function's behavior near that point.

Broader Context and Applications

Practical Significance of the Slope Calculation

Calculating the slope of the tangent line at a specific point has significant implications:

- Physics: The derivative represents velocity if $f(x)$ is a position function. For $f(x) = 65x^2$, the slope at $x=1$ (which is 130) indicates the instantaneous velocity at that moment.

- Economics: The derivative can represent marginal cost or revenue. Understanding how these change at specific production levels allows businesses to optimize profits or minimize costs.

- Engineering: Tangent slopes help analyze stress-strain relationships and material behaviors at specific points.

Visualizing the Function and Its Tangent

Graphing $f(x) = 65x^2$ alongside its tangent line at $x=1$ reveals the local linear approximation. The tangent line provides a close estimate of the function's value for values of x near 1, illustrating the practical utility of derivatives.

Conclusion: The Power of Calculus in Analyzing Functions

The process of finding the slope of the tangent line for $f(x)=65x^2$ at $x=1$ encapsulates a fundamental principle of calculus: the derivative as a measure of instantaneous rate of change. Through the application of the power rule, we determined the derivative $f'(x)=130x$, and by substituting $x=1$, established that the tangent line's slope is 130.

This calculation exemplifies the seamless integration of mathematical theory and real-world applications. Whether assessing velocity in physics, marginal costs in economics, or rates of change in engineering, the derivative remains an indispensable tool. Moreover, understanding the geometric and algebraic aspects of tangent lines enhances our ability to analyze complex functions efficiently.

In summary, the slope of the tangent line at $x=1$ for the function $f(x) = 65x^2$ is 130, a result that encapsulates the function's rapid increase at that point. This exercise not only demonstrates the mechanics of differentiation but also underscores the profound implications of calculus in interpreting and modeling the dynamic world around us.

[Find The Slope Of The Tangent Line For F X 65x 2 When X 1]

Find The Slope Of The Tangent Line For F X 65x 2 When X 1

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