Find The Sum Of The First 32 Terms Of The AP : 0.5, 0.75, 1

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Understanding arithmetic progressions (AP) is fundamental in mathematics, especially when dealing with sequences and series. In this article, we'll explore how to find the sum of the first 32 terms of the given AP: 0.5, 0.75, 1, and provide a comprehensive step-by-step explanation to help you grasp the concept thoroughly.

Introduction to Arithmetic Progression (AP)

What Is an AP?

An arithmetic progression (AP) is a sequence of numbers in which the difference between any two consecutive terms is constant. This constant difference is called the common difference, usually denoted by d.

General Form of AP

A typical arithmetic progression can be written as:

a, a + d, a + 2d, a + 3d, ..., a + (n - 1)d

Where:
- a = the first term
- d = the common difference
- n = number of terms

Analyzing the Given Sequence

Sequence Details

The sequence provided is:

0.5, 0.75, 1, ...

Let's identify the key parameters:

- First term, **a** = 0.5
- Second term = 0.75
- Third term = 1

Calculate the Common Difference

The common difference d is:

d = second term - first term = 0.75 - 0.5 = 0.25

To verify, check the difference between the third and second terms:

1 - 0.75 = 0.25

Since the difference is consistent, d = 0.25.

Formulating the Sequence and the Sum

General Term (n-th Term)

The formula for the n-th term (an) of an AP is:

an = a + (n - 1)d

Substituting the known values:

an = 0.5 + (n - 1) × 0.25

Sum of First n Terms (S_n)

The sum of the first n terms of an AP is given by:

Sn = (n / 2) × (a + an)

Alternatively,

Sn = (n / 2) × [2a + (n - 1)d]

We will use the second formula for convenience.

Calculating the Sum of First 32 Terms

Step 1: Identify n, a, and d

- $n = 32$
- $a = 0.5$
- $d = 0.25$

Step 2: Find the 32nd Term (a_{32})

Use the formula:

$an = a + (n - 1)d$

Plugging in the values:

$a32 = 0.5 + (32 - 1) \times 0.25$
$a32 = 0.5 + 31 \times 0.25$
$a32 = 0.5 + 7.75$
$a32 = 8.25$

Step 3: Apply the Sum Formula

Using:

$Sn = (n / 2) \times [2a + (n - 1)d]$

Calculate:

$S32 = (32 / 2) \times [2 \times 0.5 + (32 - 1) \times 0.25]$
$S32 = 16 \times [1 + 31 \times 0.25]$
$S32 = 16 \times [1 + 7.75]$
$S32 = 16 \times 8.75$
$S32 = 140$

Therefore, the sum of the first 32 terms of the sequence is 140.

Additional Insights and Applications

Why Is Understanding AP Important?

- Mathematical problem-solving: AP concepts help solve real-world problems involving sequences, such as calculating total savings, distances, or any repetitive additive process.
- Foundation for series: Many advanced topics like geometric series and calculus build upon the understanding of AP and its sums.

Real-World Examples of AP

- Savings over time: If you deposit a fixed amount each month, your total savings over a period form an AP.
- Distance covered: If a vehicle accelerates uniformly, the distances covered over equal time intervals follow an AP.

Summary and Key Takeaways

- Sequence analysis involves identifying the first term and common difference.
- The nth term of an AP is given by $a_n = a + (n - 1)d$.
- The sum of the first n terms is calculated using $S_n = (n / 2) \times [2a + (n - 1)d]$.
- For the AP 0.5, 0.75, 1, the sum of the first 32 terms is 140.

Conclusion

Calculating the sum of a specified number of terms in an arithmetic progression involves understanding the sequence's fundamental parameters and applying the appropriate formula. The sequence 0.5, 0.75, 1, with a common difference of 0.25, demonstrates a straightforward application of AP sum formulas. Mastering these concepts enhances problem-solving skills and provides a robust foundation for further mathematical exploration.

FAQs

Q1: How do I find the sum of the first n terms of any AP?

A: Use the formula $S_n = (n / 2) \times [2a + (n - 1)d]$, where a is the first term and d is the common difference.

Q2: Can AP be decreasing?

A: Yes, if the common difference d is negative, the sequence decreases. The formulas still apply.

Q3: What are some real-life scenarios where AP is applicable?

A: Examples include savings plans, escalated payments, or the distribution of tasks over time.

Q4: How does this knowledge help in advanced mathematics?

A: AP forms the basis for series, calculus, and many algebraic concepts, facilitating the understanding of more complex mathematical structures.

By understanding the step-by-step process to find the sum of the first 32 terms of this AP, you can confidently approach similar sequence problems in your studies and real-world applications.

Frequently Asked Questions

What is the common difference in the AP: 0.5, 0.75, 1?

The common difference is 0.25.

How do you find the sum of the first 32 terms of the AP: 0.5, 0.75, 1?

Use the formula for the sum of an AP: $S_n = n/2 (2a + (n - 1)d)$. Plug in n=32, a=0.5, d=0.25 to find the sum.

What is the value of the 32nd term in this AP?

The nth term is given by $a_n = a + (n - 1)d$. For n=32, $a_{32} = 0.5 + (32 - 1)0.25 = 0.5 + 310.25 = 0.5 + 7.75 = 8.25$.

Calculate the sum of the first 32 terms of the AP: 0.5, 0.75, 1.

Using the sum formula: $S_{32} = 32/2 [20.5 + (32 - 1)0.25] = 16 [1 + 7.75] = 16 8.75 = 140$.

Is this AP arithmetic, and why?

Yes, because the difference between consecutive terms is constant (0.25), which defines an arithmetic progression.

Additional Resources

Find The Sum Of The First 32 Terms Of The AP: 0.5, 0.75, 1, ...

When delving into the fascinating world of arithmetic progressions (AP), one of the fundamental problems students and enthusiasts encounter is calculating the sum of a specified number of terms. This exploration will focus on a specific AP sequence: 0.5, 0.75, 1, ... and guide you step-by-step through understanding its structure, deriving the sum of its first 32 terms, and appreciating the underlying principles of APs.

Understanding the Basics of Arithmetic Progressions (AP)

Before jumping into calculations, it's essential to grasp what an arithmetic progression entails.

What is an AP?

An arithmetic progression is a sequence of numbers where the difference between any two successive terms is constant. This constant difference is known as the common difference, denoted by d.

Formally, an AP can be written as:
$$ a, a + d, a + 2d, a + 3d, ... $$
where:
- a = first term
- d = common difference

Characteristics of AP

- Linear growth: Each term increases or decreases by a fixed amount.
- Explicit formula for nth term:
$$ T_n = a + (n - 1)d $$
- Sum of first n terms:
$$ S_n = \frac{n}{2} \times (2a + (n - 1)d) $$

Understanding these core principles allows for efficient calculations and deeper insights into the sequence's behavior.

Analyzing the Given Sequence: 0.5, 0.75, 1, ...

Let's analyze the specific AP sequence provided:

Sequence: 0.5, 0.75, 1, ...

Identifying the First Term (a)

- The first term, a, is clearly 0.5.

Determining the Common Difference (d)

- The difference between the second and first term:
$$ 0.75 - 0.5 = 0.25 $$
- The difference between the third and second term:
$$ 1 - 0.75 = 0.25 $$

Since these are equal, the sequence has a common difference:

$$ d = 0.25 $$

Expressing the Sequence

- The sequence can be expressed as:

$$ T_n = a + (n - 1)d = 0.5 + (n - 1) \times 0.25 $$

This explicit formula allows us to find any term in the sequence directly.

Calculating the Sum of the First 32 Terms: Methodology

Calculating the sum of a certain number of terms in an AP involves using the standard sum formula:

$$ S_n = \frac{n}{2} \times (2a + (n - 1)d) $$

In our case:
- $n = 32$
- $a = 0.5$
- $d = 0.25$

Let's walk through the calculation thoroughly.

Step-by-Step Calculation

Step 1: Plugging the values into the sum formula:

$$
S_{32} = \frac{32}{2} \times [2 \times 0.5 + (32 - 1) \times 0.25]
$$

Step 2: Simplify the expression inside the brackets:

$$
2 \times 0.5 = 1
$$
$$
(32 - 1) \times 0.25 = 31 \times 0.25 = 7.75
$$

Step 3: Sum the terms inside the brackets:

$$
1 + 7.75 = 8.75
$$

Step 4: Multiply by $\frac{32}{2} = 16$:

$$
S_{32} = 16 \times 8.75 = 140
$$

Result: The sum of the first 32 terms is 140.

Deeper Insights and Alternative Approaches

While the direct application of the formula is straightforward, understanding alternative perspectives enriches comprehension.

Using the Explicit Formula for nth Term

Recall:

$$ T_n = 0.5 + (n - 1) \times 0.25 $$

The sum of the first n terms can also be expressed as:

$$
S_n = \sum_{k=1}^n T_k = \sum_{k=1}^n [0.5 + (k - 1) \times 0.25]
$$

This summation can be split into two parts:

$$
S_n = \sum_{k=1}^n 0.5 + \sum_{k=1}^n (k - 1) \times 0.25
$$

Calculating each:

- $\sum_{k=1}^n 0.5 = 0.5 \times n$
- $\sum_{k=1}^n (k - 1) = \frac{n(n - 1)}{2}$

Therefore:

$$
S_n = 0.5n + 0.25 \times \frac{n(n - 1)}{2}
$$

Plugging in $n=32$:

$$
S_{32} = 0.5 \times 32 + 0.25 \times \frac{32 \times 31}{2}
$$
$$
= 16 + 0.25 \times 496
$$
$$
= 16 + 124
$$
$$
= 140
$$

Again, confirming the previous result.

Implications and Applications

Understanding how to compute the sum of terms in an AP has broad applications across different fields.

Real-World Contexts

- Financial calculations: Computing cumulative interest or payments over time.
- Physics: Analyzing uniformly increasing velocities or displacements.
- Engineering: Summing discrete signals or energy levels with regular increments.

Mathematical Significance

- Demonstrates the power of formulas to simplify summations.
- Illustrates the importance of explicit formulas and summation techniques.
- Provides foundational skills for more advanced topics like geometric progressions, series, and calculus.

Key Takeaways and Summary

- The sequence 0.5, 0.75, 1, ... is an arithmetic progression with:
- First term $a = 0.5$
- Common difference $d = 0.25$
- The explicit formula for the nth term:

$$T_n = 0.5 + (n - 1) \times 0.25$$

- The sum of the first n terms:

$$S_n = \frac{n}{2} \times [2a + (n - 1)d]$$

- For the first 32 terms, the sum is 140.

Additional Tips for AP Calculations

- Always verify the common difference before using summation formulas.
- When sequences involve fractional or decimal numbers, maintain consistency in decimal places to avoid errors.
- Cross-verify results using different methods (direct formula vs. summation) for accuracy.
- Practice with different sequences to develop intuition about APs and their properties.

Conclusion

Calculating the sum of the first 32 terms of the AP 0.5, 0.75, 1, ... exemplifies the elegance and utility of arithmetic progression formulas. By understanding the sequence's structure, applying the sum formula, and verifying through alternative methods, you not only solve a specific problem but also reinforce foundational mathematical concepts. Mastery of such techniques paves the way for tackling more complex problems involving series, sequences, and their numerous applications in academics and real life.

In summary, the sum of the first 32 terms of the sequence 0.5, 0.75, 1, ... is 140. This result emerges from recognizing the sequence as an AP, identifying its parameters, and systematically applying the sum formula — a testament to the power of mathematical principles in solving structured problems efficiently and accurately.

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