Find The TWO Integers Whos Product Is -12 And Whose Sum Is 1

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When tackling algebraic problems involving integers, a common challenge is to find two specific numbers based on their sum and product. In this article, we will explore how to find two integers whose product is -12 and whose sum is 1. This problem not only enhances problem-solving skills but also provides insight into the relationships between numbers, factors, and algebraic equations. Whether you're a student preparing for tests or someone interested in mathematical puzzles, understanding this problem's solution will deepen your grasp of integers and their properties.

Understanding the Problem

Before diving into the solution, let's clearly understand what the problem is asking:

- Find two integers, let's call them x and y.
- Their product (x y) equals -12.
- Their sum (x + y) equals 1.

Mathematically, this can be summarized as:

$$
\begin{cases}
x \times y = -12 \\
x + y = 1
\end{cases}
$$

Our goal is to determine the values of x and y that satisfy both conditions simultaneously.

Approach to Solve the Problem

There are various methods to solve such problems, including:

- Factoring
- Using substitution
- Applying algebraic formulas

For this particular problem, factoring is often the simplest method, especially when dealing with integer solutions.

Step 1: List the Factor Pairs of -12

Since the product of the integers is -12, the first step is to list all pairs of integers whose multiplication results in -12.

Factor pairs of -12:

1. (-1, 12)
2. (1, -12)
3. (-2, 6)
4. (2, -6)
5. (-3, 4)
6. (3, -4)

Note: These are ordered pairs, but since we're looking for two integers without regard to order, the pairs are considered as sets.

Step 2: Find the Pair Whose Sum Is 1

Next, check each pair to see if their sum equals 1:

- (-1 + 12) = 11 → Not 1
- (1 + -12) = -11 → Not 1
- (-2 + 6) = 4 → Not 1
- (2 + -6) = -4 → Not 1
- (-3 + 4) = 1 → Yes!
- (3 + -4) = -1 → Not 1

Conclusion: The pair (-3, 4) has a product of -12 and a sum of 1.

Answer:

The two integers are -3 and 4.

Verification of the Solution

To ensure the solution is correct, verify both conditions:

- Product: $-3 \times 4 = -12$ □
- Sum: $-3 + 4 = 1$ □

Both conditions are satisfied, confirming that the integers are indeed -3 and 4.

Alternative Methods to Find the Solution

While listing factors is straightforward, other methods can also be used:

Method 1: Using Algebraic Equations

Suppose the integers are x and y.

Given:

$$
x + y = 1
$$
$$
xy = -12
$$

Express y in terms of x:

$$
y = 1 - x
$$

Substitute into the second equation:

$$
x(1 - x) = -12
$$
$$
x - x^2 = -12
$$
$$
x^2 - x - 12 = 0
$$

Now, solve the quadratic:

$$
x^2 - x - 12 = 0
$$

Using quadratic formula:

$$
x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-12)}}{2 \times 1}
$$
$$
x = \frac{1 \pm \sqrt{1 + 48}}{2}
$$

\]
\[
x = \frac{1 \pm \sqrt{49}}{2}
\]
\[
x = \frac{1 \pm 7}{2}
\]

Calculate both options:

- \(x = \frac{1 + 7}{2} = \frac{8}{2} = 4\)
- \(x = \frac{1 - 7}{2} = \frac{-6}{2} = -3\)

Corresponding y values:

- When \(x = 4\), \(y = 1 - 4 = -3\)
- When \(x = -3\), \(y = 1 - (-3) = 4\)

Again, the pair is (-3, 4), confirming our earlier solution.

Additional Insights and Related Problems

Understanding how to find such pairs is fundamental in algebra and helps in solving quadratic equations, factoring, and understanding number properties.

Related Problems:

1. Find two integers whose product is 24 and their sum is 11.
2. Find two integers whose product is 35 and whose sum is -2.
3. Find two integers whose product is 0 and whose sum is 5.

Solutions involve similar steps:

- Listing factor pairs
- Applying algebraic formulas
- Using factoring methods

Practical Applications of Finding Integer Pairs

Identifying pairs of integers with specific sum and product conditions is relevant in various fields:

- Cryptography: Factoring large integers for encryption algorithms.
- Number Theory: Understanding properties of integers and their factors.
- Computer Science: Algorithm design involving pairings and combinations.
- Mathematics Education: Developing problem-solving skills and algebraic

reasoning.

Summary and Key Takeaways

- To find two integers with a specific product and sum, list the factor pairs of the product.
- Check each pair to see if their sum matches the given condition.
- Alternatively, set up a system of equations and solve using algebraic methods.
- For this problem, the integers are -3 and 4, satisfying both conditions.

Key points:

- The process involves understanding factors, sums, and algebra.
- Multiple methods can be used, but listing factors is often the most straightforward for small integers.
- Verifying solutions is essential to confirm correctness.

Conclusion

Finding two integers based on their product and sum is a fundamental skill in algebra. In the specific case where the product is -12 and the sum is 1, the solution involves listing factor pairs and verifying the sum. The pair (-3, 4) meets both criteria, illustrating how simple algebraic techniques can solve seemingly complex problems. Mastering such problems enhances overall mathematical reasoning and prepares you for more advanced topics in algebra and number theory. Keep practicing similar problems to develop a strong foundation in integer properties and algebraic problem-solving.

Frequently Asked Questions

What are the two integers whose product is -12 and sum is 1?

The two integers are 4 and -3.

How can I find two integers with a specific product and sum?

You can set up equations for the integers, such as x y = -12 and x + y = 1, then solve the system to find the pair.

Are the integers 4 and -3 the only solution for the problem?

Yes, 4 and -3 are the only integers that satisfy both conditions: their product is -12 and their sum is 1.

Can this problem be solved using factoring?

Yes, factoring the quadratic equation $x^2 - x - 12 = 0$ helps find the integers, which are 4 and -3.

What quadratic equation represents this problem?

The quadratic equation is $x^2 - (\text{sum})\, x + (\text{product}) = 0$, which becomes $x^2 - x - 12 = 0$.

How do I verify the solution for these integers?

Check that their product is -12 and their sum is 1; for 4 and -3, 4 -3 = -12 and 4 + (-3) = 1, confirming the solution.

Additional Resources

Find The TWO Integers Whose Product Is -12 And Whose Sum Is 1

When tackling algebraic problems, especially those involving two variables, it can sometimes feel like solving a puzzle. One such intriguing problem is: find the two integers whose product is -12 and whose sum is 1. This question combines basic arithmetic operations with logical reasoning, making it an excellent exercise for developing problem-solving skills. In this comprehensive guide, we will explore how to approach this problem step-by-step, examining potential solutions, and understanding the underlying mathematical principles involved.

Understanding the Problem

Before diving into calculations, it’s crucial to understand what the problem is asking for:

- Two integers: The solution must be two whole numbers, not fractions or decimals.
- Product is -12: When these two integers are multiplied, the result should be -12.
- Sum is 1: When these two integers are added, the result should be 1.

Restating the problem in algebraic terms:

Let the two integers be x and y.

Then,

- $x y = -12$
- $x + y = 1$

Our goal is to find the values of x and y that satisfy both these equations simultaneously.

Mathematical Approach to the Problem

Step 1: Express One Variable in Terms of the Other

From the second equation, $x + y = 1$, we can express y in terms of x:

$y = 1 - x$

Step 2: Substitute into the First Equation

Substituting $y = 1 - x$ into $x y = -12$:

$x (1 - x) = -12$

Expanding this:

$x 1 - x x = -12$

which simplifies to:

$x - x^2 = -12$

Rearranged into a standard quadratic form:

$x^2 - x - 12 = 0$

Solving the Quadratic Equation

The quadratic equation derived is:

$x^2 - x - 12 = 0$

Step 1: Factor the Quadratic

To factor, look for two numbers that multiply to -12 and add to -1.

Factors of -12:

- 1 and -12
- -1 and 12
- 2 and -6
- -2 and 6
- 3 and -4
- -3 and 4

Now, check their sums:

- $1 + (-12) = -11$
- $-1 + 12 = 11$
- $2 + (-6) = -4$
- $-2 + 6 = 4$
- $3 + (-4) = -1$ ← This matches our middle coefficient!
- $-3 + 4 = 1$

Since we're looking for a sum of -1, the pair 3 and -4 fit.

Therefore, the quadratic factors as:

$$(x + 3)(x - 4) = 0$$

Step 2: Find the Roots

Set each factor equal to zero:

- $x + 3 = 0 \rightarrow x = -3$
- $x - 4 = 0 \rightarrow x = 4$

Step 3: Find Corresponding y Values

Recall $y = 1 - x$:

- If $x = -3$, then $y = 1 - (-3) = 1 + 3 = 4$
- If $x = 4$, then $y = 1 - 4 = -3$

Verifying the Solutions

Let's verify the two pairs:

1. $x = -3$, $y = 4$

- Product: $(-3)\ 4 = -12$ ✔️
- Sum: $-3 + 4 = 1$ ✔️

2. $x = 4$, $y = -3$

- Product: $4\ (-3) = -12$ ✔️
- Sum: $4 + (-3) = 1$ ✔️

Both pairs satisfy the original conditions.

Final Answer

The two integers whose product is -12 and sum is 1 are:

- -3 and 4

or equivalently,

- 4 and -3

Additional Insights and Variations

Are These the Only Solutions?

Given the quadratic solutions, these are the only integer pairs that satisfy the conditions. Since the quadratic factorization led to two specific roots, the solutions are unique in integers.

Why Only Integers?

The problem specifies integers, which restricts us to whole numbers. If fractions or real numbers were allowed, additional solutions might exist, but in this case, only these two integer solutions exist.

Extending the Problem

Suppose the problem asked for real numbers instead of integers, or included different conditions. The general approach would still involve forming a quadratic equation and solving it, but the solutions might include non-integer values.

Summary: Step-by-Step Approach

To systematically find the two integers whose product is -12 and whose sum is 1:

1. Define variables: x and y.
2. Write equations: $x y = -12$; $x + y = 1$.
3. Express one variable: $y = 1 - x$.
4. Substitute into the product equation: $x(1 - x) = -12$.
5. Form a quadratic: $x^2 - x - 12 = 0$.
6. Factor the quadratic: $(x + 3)(x - 4) = 0$.
7. Find roots: $x = -3$ or $x = 4$.

8. Calculate corresponding y: y = 4 or y = -3.
9. Verify solutions: Check the original conditions.

Conclusion

The problem of finding two integers with a specific product and sum is a classic algebraic exercise that reinforces the power of substitution, factoring, and quadratic equations. The solutions -3 and 4 demonstrate how algebra simplifies seemingly complex puzzles. Whether you're a student honing your skills or a math enthusiast exploring problem-solving strategies, understanding the step-by-step process provides a solid foundation for tackling similar problems. Remember, breaking down the problem into manageable parts and systematically solving the equations is key to mastering algebraic challenges like this.

In summary, the two integers whose product is -12 and whose sum is 1 are -3 and 4.

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