Find The Value Of The Derivative For The Given Function.

Find The Value Of The Derivative For The Given Function is a fundamental task in calculus that helps us understand how a function behaves at any given point. Derivatives are essential for analyzing rates of change, slopes of tangent lines, and the behavior of functions in various fields such as physics, engineering, economics, and more. Whether you're a student beginning to explore calculus or a professional seeking a refresher, mastering how to find the derivative of a function is crucial. This article provides a comprehensive guide on how to find the derivative for different types of functions, with step-by-step instructions, tips, and examples to enhance your understanding.

Understanding the Concept of Derivatives

Before diving into the methods of calculating derivatives, it's important to grasp what a derivative represents.

What Is a Derivative?

A derivative measures the rate at which a function changes at any given point. Formally, the derivative of a function $f(x)$ at a point x is defined as the limit:

$$
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
$$

This limit, if it exists, gives the slope of the tangent line to the curve at x.

Physical Interpretation

In physics, the derivative often corresponds to velocity if $f(t)$ represents position over time. In economics, it can represent marginal cost or revenue. Understanding the physical or practical interpretation helps to contextualize the importance of derivatives.

Basic Rules for Differentiation

Mastering the fundamental rules of differentiation simplifies calculating derivatives of complex functions. Here are the most common rules:

Power Rule

- For any real number n, the derivative of x^n is:

$$
\frac{d}{dx} x^n = n x^{n-1}
$$

- Example: $\frac{d}{dx} x^3 = 3x^2$

Constant Rule

- The derivative of a constant c is zero:

$$
\frac{d}{dx} c = 0
$$

Constant Multiple Rule

- The derivative of a constant multiplied by a function:

$$
\frac{d}{dx} [c \cdot f(x)] = c \cdot f'(x)
$$

Sum Rule

- The derivative of a sum of functions:

$$
\frac{d}{dx} [f(x) + g(x)] = f'(x) + g'(x)
$$

Difference Rule

- The derivative of a difference:

$$
\frac{d}{dx} [f(x) - g(x)] = f'(x) - g'(x)
$$

Product Rule

- For the product of two functions:

$$
\frac{d}{dx} [f(x) \cdot g(x)] = f'(x) g(x) + f(x) g'(x)
$$

Quotient Rule

- For the quotient of two functions:

$$
\frac{d}{dx} \left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) g(x) - f(x) g'(x)}{[g(x)]^2}
$$

Chain Rule

- Used for composite functions:

$$
\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)
$$

Step-by-Step Guide to Find the Derivative of a Function

To find the derivative of a function effectively, follow these steps:

Step 1: Simplify the Function

- Combine like terms.
- Factor the function if possible.
- Rewrite the function in a form suitable for applying differentiation rules.

Step 2: Identify the Type of Function

- Determine whether the function is polynomial, rational, exponential, logarithmic, or composite.
- Recognizing the type guides which rules to apply.

Step 3: Apply Appropriate Differentiation Rules

- Use basic rules for simple functions.
- For composite or complex functions, apply the chain rule, product rule, or quotient rule as needed.

Step 4: Simplify the Derivative

- After differentiation, simplify the resulting expression.
- Factor or combine terms for clarity.

Step 5: Verify the Derivative

- Check the derivative for correctness.
- Consider plugging in specific x values to compare the derivative with the average rate of change over an interval.

Examples of Finding Derivatives

To solidify understanding, let's work through some examples.

Example 1: Derivative of a Polynomial Function

Find $f'(x)$ for $f(x) = 4x^3 - 3x^2 + 2x - 5$.

Solution:
- Apply power rule to each term:
- $\frac{d}{dx} 4x^3 = 12x^2$
- $\frac{d}{dx} (-3x^2) = -6x$
- $\frac{d}{dx} 2x = 2$
- $\frac{d}{dx} -5 = 0$

Result:

$$
f'(x) = 12x^2 - 6x + 2
$$

Example 2: Derivative of a Rational Function

Find the derivative of $g(x) = \frac{2x^2 + 3}{x - 1}$.

Solution:
- Use the quotient rule:
- $f(x) = 2x^2 + 3$, $f'(x) = 4x$
- $g(x) = x - 1$, $g'(x) = 1$

Applying quotient rule:
$$
g'(x) = \frac{(4x)(x - 1) - (2x^2 + 3)(1)}{(x - 1)^2}
$$

Simplify numerator:
$$
4x(x - 1) - (2x^2 + 3) = 4x^2 - 4x - 2x^2 - 3 = (4x^2 - 2x^2) - 4x - 3 = 2x^2 - 4x - 3
$$

Final derivative:
$$
g'(x) = \frac{2x^2 - 4x - 3}{(x - 1)^2}
$$

Example 3: Derivative Using Chain Rule

Find the derivative of $h(x) = \sqrt{5x^2 + 4}$.

Solution:
- Rewrite as $h(x) = (5x^2 + 4)^{1/2}$.
- Let $u = 5x^2 + 4$, then $h(x) = u^{1/2}$.

Applying chain rule:
$$
h'(x) = \frac{1}{2} u^{-1/2} \cdot u'
$$
where $u' = 10x$.

Thus:
$$
h'(x) = \frac{1}{2} (5x^2 + 4)^{-1/2} \times 10x = \frac{10x}{2 \sqrt{5x^2 + 4}} = \frac{5x}{\sqrt{5x^2 + 4}}
$$

Tips for Effective Differentiation

- Always double-check your application of rules: Misapplication can lead to incorrect derivatives.
- Simplify functions before differentiating: It reduces errors and simplifies calculations.
- Practice with different types of functions: Polynomial, exponential, logarithmic, and composite functions.
- Use derivatives to analyze functions: Find critical points, inflection points, and analyze behavior for better understanding.

Applications of Derivatives

Understanding how to find derivatives is not just an academic exercise; it has practical applications:

- **Optimizing functions:** Maximize profits, minimize costs, or determine optimal dimensions.
- **Analyzing motion:** Calculate velocity and acceleration in physics problems.
- **Economics:** Find marginal cost, revenue, or profit.
- **Engineering:** Design systems with optimal parameters.
- **Biology:** Model population growth or decay rates.

Common Mistakes to Avoid

- Forgetting to apply the chain rule for composite functions.
- Mixing up the quotient rule and product rule.
- Not simplifying the function before differentiating.
- Sign errors in

Frequently Asked Questions

How do I find the derivative of a polynomial function?

To find the derivative of a polynomial function, apply the power rule to each term: if f(x) = ax^n, then f'(x)

= nax^(n-1). Sum the derivatives of all terms to get the overall derivative.

What is the derivative of a composite function, and how do I compute it?

The derivative of a composite function is found using the chain rule. If y = f(g(x)), then dy/dx = f'(g(x)) g'(x). First, find the derivatives of the outer and inner functions, then multiply them.

How can I find the derivative of a function using the limit definition?

Using the limit definition, the derivative at a point x is given by f'(x) = lim (h→0) [f(x+h) - f(x)] / h. Substitute the function into this limit and evaluate as h approaches zero.

What is the derivative of exponential and logarithmic functions?

The derivative of e^x is e^x. For logarithmic functions, the derivative of ln(x) is 1/x. Use these rules to differentiate exponential and log functions accordingly.

How do I find the derivative of a function at a specific point?

First, find the derivative function f'(x). Then, substitute the specific point's x-value into f'(x) to get the slope of the tangent line at that point, which is the derivative value.

What are common rules to simplify finding derivatives for complex functions?

Common rules include the product rule, quotient rule, chain rule, and power rule. Use these systematically to differentiate products, quotients, and composed functions efficiently.

Additional Resources

Find The Value Of The Derivative For The Given Function

When delving into the fascinating world of calculus, one of the foundational concepts that frequently emerges is the derivative. The derivative essentially measures how a function changes at any given point, providing critical insights into the function's behavior, slope, and rate of change. Whether you're a student grappling with the basics or a seasoned mathematician seeking clarity, understanding how to find the value of a derivative for a given function is an invaluable skill.

In this comprehensive article, we will explore the concept of derivatives in-depth, examine various methods to compute derivatives, and provide step-by-step guidance on how to determine their values for different types of functions. Think of this as your expert guide—detailing each aspect with clarity, precision, and real-world relevance.

Understanding the Derivative: The Core Concept

What Is a Derivative?

At its essence, a derivative represents the rate at which a function's output changes concerning its input. Imagine driving a car: your speed at any moment signifies how quickly your position changes over time. Similarly, the derivative of a function at a point indicates the slope of the tangent line to the function's graph at that point.

Mathematically, the derivative of a function $f(x)$ at a point $x=a$ is defined as:

$$
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
$$

This limit, known as the difference quotient, captures the instantaneous rate of change, provided it exists.

Why Is Finding Derivatives Important?

Understanding derivatives unlocks multiple analytical possibilities:

- Determining slopes of curves: Crucial in graphing and understanding the shape of functions.
- Optimization problems: Identifying maximum and minimum points for various applications.
- Physics applications: Calculating velocities and accelerations.
- Economics and finance: Assessing marginal costs and revenues.
- Engineering design: Analyzing system behaviors and response rates.

Methods for Calculating Derivatives

Different functions require different techniques for computing derivatives. The main methods include:

- Power Rule

- Product Rule
- Quotient Rule
- Chain Rule
- Implicit Differentiation
- Logarithmic Differentiation

Let's explore each method in detail, along with practical examples.

Fundamental Differentiation Techniques

Power Rule

Overview: The power rule is among the simplest yet most powerful tools. It states that for any real number n:

$$
\frac{d}{dx} [x^n] = n x^{n-1}
$$

Application: Use this when your function is a simple power of x.

Example:

Find the derivative of $f(x) = 5x^3$.

Solution:

$$
f'(x) = 5 \times 3 x^{3-1} = 15x^2
$$

Product Rule

Overview: When a function is the product of two functions, $u(x)$ and $v(x)$, the derivative is:

$$
\frac{d}{dx} [u(x) v(x)] = u'(x) v(x) + u(x) v'(x)
$$

Application: Useful for functions like $f(x) = x^2 \sin x$.

Example:

Find the derivative of $f(x) = x^2 \sin x$.

Solution:

- $u(x) = x^2 \Rightarrow u'(x) = 2x$
- $v(x) = \sin x \Rightarrow v'(x) = \cos x$

Applying product rule:

$$
f'(x) = 2x \sin x + x^2 \cos x
$$

Quotient Rule

Overview: For a function expressed as a quotient $\frac{u(x)}{v(x)}$:

$$
\frac{d}{dx} \left[\frac{u(x)}{v(x)}\right] = \frac{u'(x) v(x) - u(x) v'(x)}{[v(x)]^2}
$$

Application: Suitable for functions like $f(x) = \frac{x^3}{\sin x}$.

Example:

Find the derivative of $f(x) = \frac{x^3}{\sin x}$.

Solution:

- $u(x) = x^3 \Rightarrow u'(x) = 3x^2$
- $v(x) = \sin x \Rightarrow v'(x) = \cos x$

Applying quotient rule:

$$
f'(x) = \frac{3x^2 \sin x - x^3 \cos x}{\sin^2 x}
$$

Chain Rule

Overview: When a function is composed of functions, i.e., $f(g(x))$, the derivative is:

$$
\frac{d}{dx} f(g(x)) = f'(g(x)) \times g'(x)
$$

Application: Essential for nested functions like $f(x) = \sqrt{1 + x^2}$ or $f(x) = \sin(x^2)$.

Example:

Find the derivative of $f(x) = \sin(x^2)$.

Solution:

- Outer function: $f(u) = \sin u$, so $f'(u) = \cos u$
- Inner function: $g(x) = x^2$, so $g'(x) = 2x$

Applying chain rule:

$$
f'(x) = \cos(x^2) \times 2x = 2x \cos(x^2)
$$

Implicit Differentiation

Overview: Used when the function isn't explicitly solved for y, i.e., equations relate x and y implicitly.

Method: Differentiate both sides with respect to x, applying the chain rule where necessary.

Example:

Find $\frac{dy}{dx}$ if $x^2 + y^2 = 25$.

Solution:

Differentiate both sides:

$$
2x + 2y \frac{dy}{dx} = 0
$$

Solve for $\frac{dy}{dx}$:

$$
2y \frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{y}
$$

Step-by-Step Approach to Find the Derivative Value

Having explored the techniques, let's now focus on the process of calculating the derivative's value at a specific point.

Step 1: Identify the Function and Point of Interest

Suppose we are given $f(x)$ and asked to find $f'(a)$.

Step 2: Choose the Appropriate Differentiation Method

Based on the form of the function, select the method(s):

- Power Rule for polynomial expressions
- Product or Quotient Rule for products or ratios
- Chain Rule for composite functions
- Other techniques as needed

Step 3: Compute the Derivative Expression

Apply the chosen method to derive a general formula for $f'(x)$.

Step 4: Evaluate the Derivative at $x=a$

Substitute $x = a$ into $f'(x)$ to find the specific value.

Practical Examples and Applications

Let's work through some real-world inspired examples to demonstrate how to find the derivative's value for specific functions.

Example 1: Polynomial Function

Function: $f(x) = 3x^4 - 5x^2 + 2$

Find: $f'(2)$

Solution:

- Derivative:

$$
f'(x) = 12x^3 - 10x
$$

- Evaluate at $x=2$:

$$
f'(2) = 12 \times 8 - 10 \times 2 = 96 - 20 = 76
$$

Interpretation: At $x=2$, the rate of change of the function is 76.

Example 2: Rational Function

Function: $f(x) = \frac{x^2 + 1}{x - 1}$

Find: $f'(3)$

Solution:

- Use quotient rule:

$$
f'(x) = \frac{(2x)(x - 1) - (x^2 + 1)(1)}{(x - 1)^2}
$$

Simplify numerator:

$$
2x(x - 1) - (x^2 + 1) = 2x^2 - 2x - x^2 - 1 = x^2 - 2x - 1
$$

- At $x=3$:

\[
f'(3) = \frac{3^2 - 2 \times 3 - 1}{(3 - 1)^2} = \frac{9 -

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